PACS numbers: 42.55.Rz; 42.65.Ky; 42.25.Ja
DOI: 10.1070/QE2015v045n02ABEH015607

Polarisation analysis of optimal conditions for stationary second-harmonic generation in a solid-state laser

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Abstract. **The Jones matrix method is used to study the optimal conditions for steady-state generation through intracavity frequency conversion in a solid-state laser under type-II phase matching based on a weakly anisotropic model of an active medium (amplitude and phase anisotropy) and a nonlinear element. The optimal rotation angles of the nonlinear element are found.**

Keywords: *solid-state laser with intracavity frequency doubling, phase anisotropy, amplitude anisotropy, polarisation mode, pump-induced gain anisotropy, Fabry – Perot resonator, Jones matrix method.*

From a practical point of view, of particular importance is the problem of the stability of the intracavity-doubled multimode solid-state laser output. In experiments with multimode Nd:YAG lasers with an intracavity-doubling KTP crystal, Baer showed [1] that the mode coupling in the process of intracavity frequency doubling leads to instability of steady-state generation (green problem). Analysis of the balanced model of such a laser confirmed the existence of dynamic instability in a certain range of parameters [1 – 4]. It has been shown that this instability is attributed to sum-frequency generation, which usually accompanies the process of second harmonic generation in intracavity-doubled multimode lasers.

Two types of phase matching of light waves in a nonlinear crystal are possible through intracavity frequency doubling: in the case of type-I phase matching, laser mode frequencies of the same polarisation are summed, and in the case of type-II phase matching, which we consider in this paper, the process of the nonlinear frequency conversion involves waves with orthogonal polarisations. When polarisations of the laser modes coincide with the directions of the birefringence axes of a nonlinear element (NE), sum-frequency generation of orthogonally polarised modes occurs, resulting in unsteady-state generation if the nonlinear conversion efficiency exceeds a certain critical value [5]. The discrepancy between the laser eigenpolarisations and the directions of the NE axes leads to the appearance of frequency-doubling, which enhances the stability of the steady-state generation process. The maximum stability is obtained when the axes are rotated through 45°, which is achieved by inserting an additional quarter-wave phase plate into the cavity [6].

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Received 30 June 2014; revision received 22 October 2014
Kvantovaya Elektronika **45** (2) 128 – 130 (2015)
Translated by I.A. Ulitkin

The present work is devoted to lasers with weakly anisotropic active media, such as yttrium aluminium garnet doped with neodymium ions, and a nonlinear element for intracavity frequency doubling. The phase anisotropy may be due to the small residual birefringence in the crystal of the active element, while the amplitude anisotropy of the active medium – by the gain anisotropy induced by linearly polarised pump radiation [7]. It is shown that the change in orientation of the NE axes (rotation in a plane perpendicular to the cavity axis through 45° relative to the direction of mode polarisations of a bipolarisation laser) without additional phase plates can result in optimal conditions for steady-state generation.

A solid-state laser with a weakly anisotropic Fabry – Perot cavity and a nonlinear element is schematically shown in Fig. 1. The active medium is represented in the form of a partial polariser P, which can generally be rotated through an angle α in the xy plane (relative to the x axis), and a phase-anisotropic element (phase plate) PP1 with a phase difference δ_a, oriented by fast and slow axes along x and y. The nonlinear element, which provides frequency conversion under type-II phase matching conditions is also represented in the form of a phase-anisotropic element PP2 with a phase difference $\Delta_n = 2\pi m + \delta_n$, where m is an integer and δ_n is the additional phase difference, which is assumed to be small ($\delta_n \ll 1$; this condition is satisfied by an appropriate choice of the length of the nonlinear crystal and its slight tilt in the xy plane).

The polarisations of eigenwaves of an anisotropic cavity can be found using the Jones matrix method. Its application to calculate the eigenstates of the cavity polarisations consists in constructing a cavity round-trip matrix M [8] and in finding the eigenvectors $\boldsymbol{u}$ and eigenvalues λ of the matrix from the equation:

$$M\boldsymbol{u} = \lambda \boldsymbol{u}. \quad (1)$$

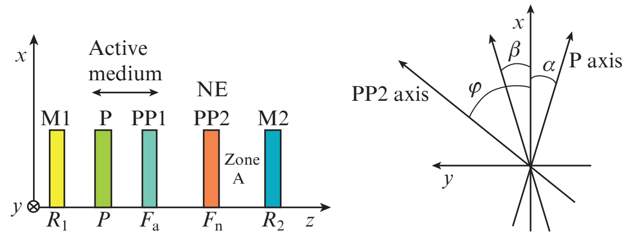

Figure 1. Schematic of a solid-state laser with a weakly anisotropic Fabry – Perot cavity and a nonlinear element.

The matrix M of an anisotropic cavity in zone A (at the laser output) can be written in the form

$$M = R_1 R_2 S(\varphi) F_n^2 S(-\varphi) F_a S(\alpha) P^2 S(-\alpha) F_a, \quad (2)$$

where

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 1-b \end{pmatrix}$$

is the Jones matrix of a partial polariser (the value of $b < 1$ determines the amplitude anisotropy);

$$S = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$$

is the rotation matrix through an angle φ;

$$F_a = \begin{pmatrix} \exp(i\delta_a/2) & 0 \\ 0 & \exp(-i\delta_a/2) \end{pmatrix}, \; F_n = \begin{pmatrix} \exp(i\delta_n/2) & 0 \\ 0 & \exp(-i\delta_n/2) \end{pmatrix}$$

are the Jones matrices of the phase plates PP1 and PP2, simulating the active medium and the nonlinear element, respectively; and $R_{1,2}$ are the reflection coefficients of mirrors M1 and M2.

We consider the eigenvector in the form

$$\boldsymbol{u} = E_x \begin{pmatrix} 1 \\ \chi \end{pmatrix}.$$

Here, $\chi = E_y/E_x$ is a complex-valued polarisation parameter, which allows one to determine ellipticity ε (the ratio of the minor axis to the major axis of a polarisation ellipse) and azimuth β (the angle between the semimajor axis of the polarisation ellipse and the x axis) in the form:

$$\varepsilon = \tan\left[\frac{1}{2}\arcsin\left(\frac{2\,\mathrm{Im}\chi}{1+|\chi|^2}\right)\right],$$
$$\beta = \arctan\left(\frac{2\,\mathrm{Re}\chi}{1+|\chi|^2}\right) + \frac{n\pi}{2}, \quad (3)$$

where $n = 0$ for one polarisation mode and $n = 1$ for the other.

Equation (1) has a solution in the form of two eigenvectors $\boldsymbol{u}_{1,2}$ and, consequently, two eigenvalues $\lambda_{1,2}$. The matrix elements M_{ij} make it possible to determine the eigenvalues

$$\lambda_{1,2} = \mathrm{Tr}M/2 \pm \sqrt{\mathrm{Tr}^2 M/4 - \det M} \quad (4)$$

and complex-valued polarisation parameters

$$\chi_{1,2} = \frac{\lambda_{1,2} - M_{11}}{M_{12}} = \frac{M_{21}}{\lambda_{1,2} - M_{22}}, \quad (5)$$

where $\mathrm{Tr}M = M_{11} + M_{22}$ and $\det M = M_{11}M_{22} - M_{12}M_{21}$.

We studied in detail [9] the mutual influence of phase and amplitude anisotropy of the active medium on the orientation of the cavity polarisation modes. Introduction of the phase anisotropy of a NE, δ_n, changes the orientation of the polarisation modes. We carried out numerical calculations of the polarisation states of the eigenwaves in the cavity in zone A and considered the effect of NE rotation in the xy plane on the orientation of the eigenpolarisations (or rather, the major axis of polarisation ellipses of weakly elliptic waves) with respect to the ordinary and extraordinary axes of the NE, the change in azimuth of eigenpolarisations being significantly dependent on the ratio between δ_n and the values of δ_a and b. We denote by $\psi = \varphi - \beta$ the angle between the ordinary axis of a nonlinear crystal and azimuth β of the eigenpolarisation of one of the waves (azimuth of the eigenpolarisation of the other polarisation mode differs by up to 90°).

Figure 2 shows the dependence of ψ on the NE rotation angle φ for the case when the active medium lacks phase anisotropy ($\delta_a = 0$), but has amplitude anisotropy with a fixed orientation along the x axis ($\alpha = 0$, see Fig. 1); in this case, the phase shift is $\delta_n = 1°$ (0.0175 rad). One can see that at a low amplitude anisotropy ($b < \delta_n$), the value of ψ is less than 45° at any φ, because eigenpolarisation azimuths of the cavity track the NE rotation. At the same time, at strong amplitude anisotropy ($b > \delta_n$), we have $\psi = 45°$ at $\varphi_{opt} = 45°$ and $\psi = -45°$ at $\varphi_{opt} = 135°$.

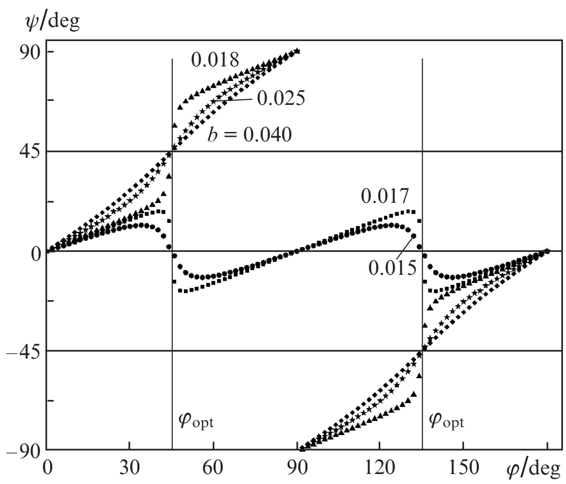

Figure 2. Dependences of the angle $\psi = \varphi - \beta$ on the nonlinear element rotation angle φ in the case of amplitude anisotropy ($\alpha = 0$) at $\delta_n = 1°$ and different values of b.

Figure 3 shows the calculation results for the case of weak ($b = 0.015$) amplitude anisotropy (the partial polariser is not rotated, $\alpha = 0$ and large [$\delta_a = 5°$ (0.0873 rad)] phase anisotropy of the active medium at different δ_n. For sufficiently small δ_n ($\delta_n \ll \delta_a$), the angle φ_{opt}, at which $\psi_{opt} = \pm 45°$, is either more than 45°, or less than 135°, but it is sufficiently close to these values; with the growth of δ_n the value of φ_{opt} approaches 90° (when the phase shift δ_n becomes comparable with the phase anisotropy δ_a). In the region, where $\delta_n > \delta_a$, the value of ψ is less than the optimal one (± 45°).

A change in the partial polariser orientation (angle α) causes a change in the optimal rotation angle φ_{opt} of the NE. Figure 4 shows the calculation results for the case when the amplitude anisotropy is comparable to the phase anisotropy: $b = 0.08$, $\delta_a = 5°$ (0.0873 rad) for different values of δ_n and partial polariser rotation angles $\alpha = 0$ and 30°. It can be seen that there remains a principle possibility to find an NE orientation, which allows one to optimise the process of intracavity nonlinear frequency conversion when the phase shift in the NE, which is below a certain critical value, is comparable with the values of the phase and amplitude anisotropy of the active medium.

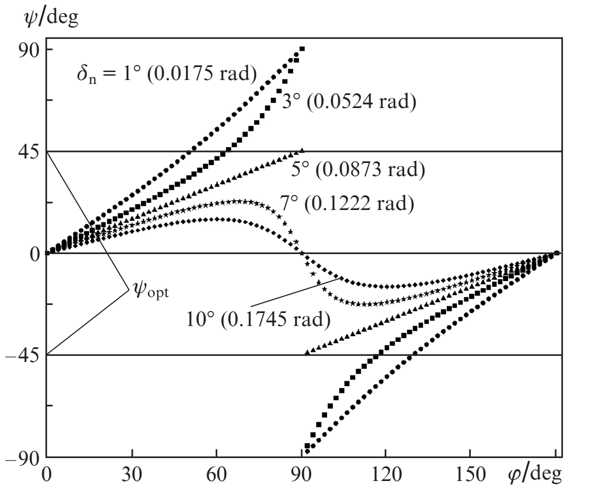

Figure 3. Dependences of the angle ψ on the nonlinear element rotation angle φ in the case of amplitude ($\alpha = 0$, $b = 0.015$) and phase ($\delta_a = 5°$) anisotropy at different values of δ_n.

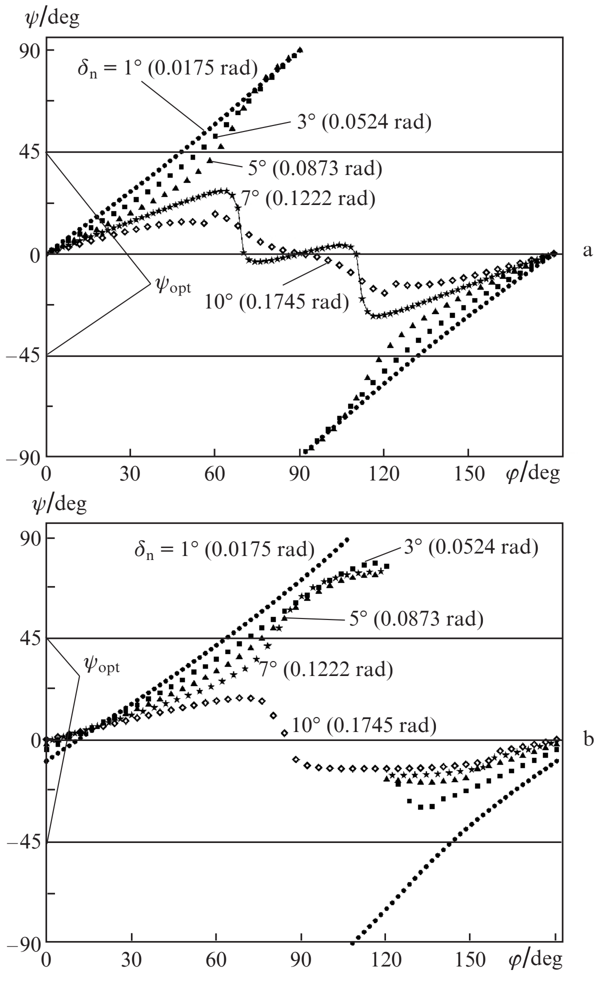

Figure 4. Dependences of the angle ψ on the nonlinear element rotation angle φ in the case of amplitude ($b = 0.08$) and phase ($\delta_a = 5°$) anisotropy at different values of δ_n for the partial polariser rotation angles α = (a) 0 and (b) 30°.

Thus, the calculations performed allow for the following conclusions. Optimal conditions for steady-state second-harmonic generation in a solid-state laser, which are considered on the basis of the polarisation analysis, can be achieved by a corresponding rotation of the nonlinear element only if the phase shift δ_n is smaller than the phase anisotropy δ_a or the amplitude anisotropy b of the active medium. This can be done by selecting properly the thickness of the NE and its inclination (with respect to the cavity axis), as noted above. The angle, though which it is required to turn the NE, changes from 45° to 135° depending on the relationship between the above-mentioned parameters of the active medium (such as amplitude and phase anisotropies and orientation of a partial polariser modelling the amplitude anisotropy) and the NE. Eigenpolarisations for the cases presented in this study are, in general, elliptical, but because of the smallness of the amplitude and phase anisotropy the ellipticity is small enough ($\varepsilon < 0.1$), we have considered only the problem of the orientation of the major axis of the polarisation ellipse with respect to fast and slow axes of the nonlinear element.

Acknowledgements. This work was supported by the RF President's Grants Council (Support to Leading Scientific Schools Programme, Grant No. NSh-2001.2014.2).

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