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Simple receiver with soft decision forward error correction for binary amplitude modulation

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Abstract. **A simple receiver with soft decision forward error correction (FEC) based on two comparators is proposed. It is shown that the gain in the optical signal-to-noise ratio (OSNR), as compared with conventional receivers with hard decision FEC, can reach 0.5 dB. Some design variants of such detectors are presented.**

Keywords: *receiver with soft decision FEC, error correction encoding, binary communication channel, mutual information, channel capacity, Q-factor.*

1. Introduction

Intensive informatization of the society, development of the Internet, growth in the number of subscribers of mobile networks, increasing the number of video conferences, etc., have conditioned an exponential growth in the volume of transmitted information and, as a consequence, the need for a corresponding increase in the capacity of optical networks and communication systems at all levels [1, 2]. To increase the capacity of optical communication channels, conventional telecommunication systems using amplitude modulation with the channel rate of 10 Gbit s^{-1} are being replaced with the communication systems with phase modulation and the channel rate of 40 Gbit s^{-1} [3, 4], and also with the coherent communication systems with the channel rate of 100 Gbit s^{-1} [5, 6]. In addition, the urban communication networks and access networks are being modernised.

Currently, 100G-systems become dominant in long-haul networks. At the same time, the demand for 10G DWDM-channels continues to grow in urban and regional networks, where the issue of equipment cost reduction plays a key role. In this connection, the solutions are needed to ensure improving specifications without the use of expensive components. The main difficulty to be overcome by designers in increasing the capacity of optical communication networks is degradation of the optical signal quality due to accumulation of the amplifier noise, as well as due to linear and nonlinear distortions. Forward error correction (FEC) and digital processing of the received signals have become to play a key role in today's high-speed communication networks by improving the quality of the services provided and reducing the costs for equipment and its operation [7, 8].

In accordance with a method of digitising the binary optical signal coming on a receiver and the principle of encoding/decoding of messages, transceivers can be divided into two main categories [9] (Fig. 1): transceivers with hard decision FEC, or HD-FEC and transceivers with soft decision FEC, or SD-FEC. In a receiver of a binary optical signal with HD-FEC, the value of each input symbol (0 or 1) is determined by comparing the amplitude (level) of the signal with a threshold level in the comparator. If the input signal level exceeds the comparator threshold level, the comparator takes a hard (i.e. unambiguous) decision that the value of the signal received is '1' (otherwise '0'). The HD-FEC method, which employs a single comparator, is characterised by a moderate price and constructive simplicity of the receiver; however, since the error correction algorithms must be equally rigorous in analysing all the symbols received, this restricts their possibilities.

In a receiver of binary optical signal with SD-FEC, the value of each input symbol is determined by comparing the amplitude (level) of the signal with multiple thresholds. For this purpose, the analogue-digital converters (ADC) or several comparators with different levels of comparison are used. Typically, there exists a central threshold signal level, after comparison with which an approximate decision concerning each input symbol is made. However, whilst the receiver with HD-FEC makes a final primary decision (we call it final decision prior to FEC) as to which value should be assigned to the received symbol (0 or 1), the receiver with SD-FEC, after comparing the signal with additional thresholds, provides additional information about the degree of confidence in the decision correctness. This additional information allows one to detect and correct great amount of errors. In particular, typical receivers with HD-FEC at the redundancy of 15% ensure reduction in the bit-error-rate (BER) coefficient after FEC to the level of 10^{-12} (at the BER value of 10^{-4} prior to FEC). At the same 15% redundancy, the receivers with SD-FEC can provide the BER after FEC at a level of 10^{-12} at the BER value of 10^{-2} prior to FEC. The performance of the receivers with SD-FEC is improved by means of using more expensive comparators or ADCs than in HD-FEC, as well as at the expense of larger computing resources used in decoding.

The decoding process in the receivers with SD-FEC can be described in the following way. The ADC converts the input signal pulse into a multiple-bit symbol. The first bit indicates a decision-making (prior to FEC), whilst the subsequent bits (confidence bits) provide additional information as to the correctness of the decision made. The coherent systems

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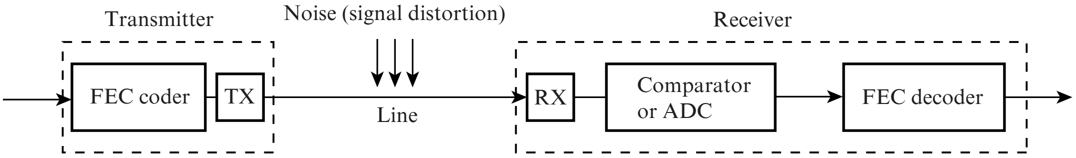

Figure 1. Transmission system with FEC.

100G DP-QPSK [10] employ the SD-FEC receivers with an eight-bit ADC ($n = 8$, the number of states is 256). Despite the implementation complexity, the use of eight-bit ADCs in the coherent systems is justified since they ensure the fulfilment of a multitude of necessary operations over the received and digitised signal. Such operations include the compensation of chromatic and polarisation mode dispersion, compensation of detuning and phase drift of the transmitting laser relative to the frequency and phase of the reference laser, etc. [11, 12]. In the receivers with direct detection, the use of such sophisticated equipment is less justified. In most cases, such receivers still use the simplest single-threshold comparators and the HD-FEC. Earlier, two- and three-bit SD-FEC receivers ($n = 2, 3$) were also examined, with the number of states 4 and 8 per each received signal pulse, respectively [13, 14].

In this paper, we propose and study the simplest version of the receiver with SD-FEC, which has been designed with the use of two comparators only. Our analysis is based on the Shannon theory and identifies the potential opportunities to improve the transmission characteristics when replacing the HD-FEC receiver with the simplest type of the SD-FEC receiver suggested.

2. Receiver structure and operation principle

A block diagram of the SD-FEC receiver with two comparators is shown in Fig. 2. A binary optical signal (in the OOK format) is fed to the photodiode, converted into an electric current, and then supplied to the electric divider. The divider transmits two identical voltages, the magnitude of which is proportional to the input current, to the inputs of two comparators. Different thresholds $I_{\text{th}0}$ and $I_{\text{th}1}$ are set on the comparators as the comparison levels. Thus, each comparator converts the input voltage into a binary symbol 0 or 1. The state of comparators can be described using a pair of numbers, first – the comparator indication K0, then – K1. In the general case, the following comparator indications are possible: 00, 01, 10, and 11.

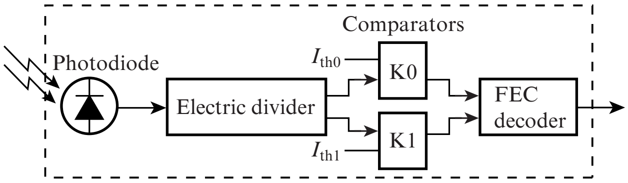

Figure 2. Structure of the SD-FEC receiver with two comparators.

Assuming that the main source of noise can only be located in the optical tract and in the photodiode receiver, and neglecting the electric divider noises, namely the noises that can be a cause of different voltages on the comparators, we can exclude, provided that $I_{\text{th}0} < I_{\text{th}1}$, the situation when the comparators give the indication 01. This indication actually becomes contradictory and is only possible if the comparator thresholds have been incorrectly set ($I_{\text{th}0} > I_{\text{th}1}$). We assume that the decoder processes the symbols that can only take three states: 00 (definitely 0), 10 (uncertain state), and 11 (definitely 1). Thus, two comparators play a role of the ADC with three output states. Information from the comparator is supplied to the decoder, which, after processing the information received, takes a final decision as to the sequence of binary symbols in the message transmitted.

3. A model of the optical transmission channel with Gaussian broadening of levels 0 and 1

Apart from the signal distortions, one of the main causes of errors (erroneous reception of a binary symbol) is the receiver noises.

For BER calculation, a specific form of the error distribution functions must be set. In the lines with optical amplifiers, along with thermal and shot noises, there are also noises caused by the spontaneous emission of optical amplifiers. The thermal and beat noises of a signal with spontaneous radiation are distributed according to the normal (Gaussian) law; the shot noises obey the Poisson distribution, whilst the beat noises of the spectral components of spontaneous radiation have the χ^2 (chi-square) distribution. In case of a large signal-to-noise ratio, the main contribution is produced by the beat noises of the signal with spontaneous radiation. These noises are distributed according to the Gaussian law because they represent a linear combination of the Gaussian variables [15]. Within such a model, the electrical signal applied to the comparator input is described by the following values: the average values I_0 and I_1, and the signal variances σ_0^2 and σ_1^2 upon receiving the '0' and '1', respectively.

In the binary channel with hard decision FEC, a single comparator with the comparison level I_{th} is used. From the input sequence X of two-level (binary) symbols, a two-level symbol sequence Y is formed (Fig. 3a). At the output of the receiver with two comparators having the levels $I_{\text{th}0}$ and $I_{\text{th}1}$, a three-level symbol sequence Y is formed (Fig. 3b).

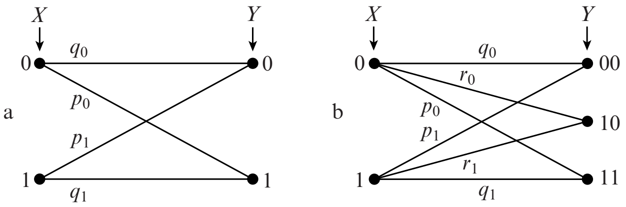

Figure 3. Transition diagrams for a hard-decision system (binary symbol at the decoder input) (a) and a soft-decision system (three-valued symbol at the decoder input) (b).

Table 1. The probability relations for the receivers with hard (HD-FEC) and soft (SD-FEC, three output states) decisions.

Description	HD-FEC	SD-FEC
Error probability in reception of '0' (0 → 1)/(0 → 11)	$p_0 = \frac{1}{2}\mathrm{erfc}\left(\frac{I_{\mathrm{th}} - I_0}{\sigma_0\sqrt{2}}\right)$	$p_0 = \frac{1}{2}\mathrm{erfc}\left(\frac{I_{\mathrm{th1}} - I_0}{\sigma_0\sqrt{2}}\right)$
Probability of error-free transmission of '0' (0 → 0)/(0 → 00)	$q_0 = 1 - p_0$	$q_0 = \frac{1}{2}\mathrm{erfc}\left(\frac{I_0 - I_{\mathrm{th0}}}{\sigma_0\sqrt{2}}\right)$
Probability of transition to the 'average' state in reception of '0'' (0 → 10)	–	$r_0 = 1 - p_0 - q_0$
Error probability in reception of '1' (1 → 0)/(1 → 00)	$p_1 = \frac{1}{2}\mathrm{erfc}\left(\frac{I_1 - I_{\mathrm{th}}}{\sigma_1\sqrt{2}}\right)$	$p_1 = \frac{1}{2}\mathrm{erfc}\left(\frac{I_1 - I_{\mathrm{th0}}}{\sigma_1\sqrt{2}}\right)$
Probability of error-free transmission of '1' (1 → 1)/(1 → 11)	$q_1 = 1 - p_1$	$q_1 = \frac{1}{2}\mathrm{erfc}\left(\frac{I_{\mathrm{th1}} - I_1}{\sigma_1\sqrt{2}}\right)$
Probability of transition to the 'average' state in reception of '1' (1 → 10)	–	$r_1 = 1 - p_1 - q_1$

The capacity of a communication channel with noises is determined through maximum of the mutual information of the X and Y sequences when varying the system parameters. We assume that a photodetector receives on average an equal number of '0' and '1'. Thus, the variable parameters represent a single comparison level of the comparators in HD-FEC and two comparison levels of the comparators in SD-FEC.

In the case of predetermined comparison levels of the comparators I_{th} (I_{th0}, I_{th1}), the transition probabilities (p, r and q) indicated in Fig. 3 are determined by the relations [16] given in Table 1. A description of these probabilities is also presented in Table 1.

The general formula for calculating the mutual information reads as [16]

$$I(X, Y) = \sum_i p(x_i) \sum_j p(y_j \mid x_i) \log \frac{p(y_j \mid x_i)}{\sum_k p(y_j \mid x_k) p(x_k)}, \qquad (1)$$

where $p(x)$ is the prior probability that the event x occurs, and $p(y|x)$ is the conditional probability that the event y occurs provided that the event x has already occurred.

In the case of equal transmission probability of '0' and '1', formula (1) takes the form

$$I(X, Y) = 1 + \frac{1}{2}\left[\sum_j p(y_j \mid 0) \log\left(\frac{p(y_j \mid 0)}{p(y_j \mid 0) + p(y_j \mid 1)}\right)\right] + \frac{1}{2}\left[\sum_j p(y_j \mid 1) \log\left(\frac{p(y_j \mid 1)}{p(y_j \mid 0) + p(y_j \mid 1)}\right)\right], \qquad (2)$$

where $j = 1, 2$ for HD-FEC and 1, 2, 3 for SD-FEC.

4. Capacity of binary channels with hard and soft decisions

The maximum value of mutual information that can be obtained by optimising the transmission system with respect to free parameters is called the channel capacity. For a binary channel with hard decision FEC, the channel capacity is

$$C = \max_{I_{\mathrm{th}}} I(X, Y). \qquad (3)$$

In the receiver with HD-FEC

$$I(X, Y) = 1 + \frac{1}{2}(p_0 \log p_0 + q_0 \log q_0 + p_1 \log p_1 + q_1 \log q_1) - \frac{1}{2}[(p_0 + q_1)\log(p_0 + q_1) + (q_0 + p_1)\log(q_0 + p_1)]. \qquad (4)$$

If a binary channel is symmetric ($\sigma_0 = \sigma_1 = \sigma$), the maximum is reached at $I_{\mathrm{th}} = (I_0 + I_1)/2$. For simplicity, we assume that $I_0 = 0$, $I_1 = 1$, i.e. $I_{\mathrm{th}} = 0.5$ (all parameters are dimensionless). In this case, the hard-decision channel capacity takes the form

$$C = 1 + p\log p + q\log q, \qquad (5)$$

where the bit-error probability is

$$p = p_0 = p_1 = \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sigma_0 2\sqrt{2}}\right),$$

and $q = q_0 = q_1$ and $p + q = 1$.

The mutual information in a receiver with SD-FEC is

$$I(X, Y) = 1 + \frac{1}{2}(p_0 \log p_0 + q_0 \log q_0 + r_0 \log r_0 + p_1 \log p_1 + q_1 \log q_1 + r_1 \log r_1) - \frac{1}{2}[(q_0 + p_1)\log(q_0 + p_1) + (r_0 + r_1)\log(r_0 + r_1) + (p_0 + q_1)\log(p_0 + q_1)]. \qquad (6)$$

In the case of a binary symmetric channel ($\sigma_0 = \sigma_1 = \sigma$) similar to that considered above, we have

$$I(X, Y) = p + q + p\log p + q\log q - (p + q)\log(p + q), \qquad (7)$$

where $p_0 = p_1 = p$, $q_0 = q_1 = q$, $r_0 = r_1 = r$, $p + q + r = 1$, and

$$p = \frac{1}{2}\mathrm{erfc}\left(\frac{\Delta + 1}{\sigma 2\sqrt{2}}\right), \quad q = \frac{1}{2}\mathrm{erfc}\left(\frac{\Delta - 1}{\sigma 2\sqrt{2}}\right), \quad \Delta = I_{\mathrm{th1}} - I_{\mathrm{th0}}.$$

The dependences of the mutual information $I(X, Y)$ on the difference between the thresholds Δ for different values of σ, calculated numerically from formula (7), are shown in Fig.4. It can be seen that with a growth of Δ starting from 0, the mutual information increases (the growth is not observed visually only when $\sigma \leqslant 0.1$) and reaches its maximum at a certain value of Δ.

The dependences of the standard deviation σ on the distance Δ for the given values of the mutual information $I(X, Y)$ (Fig. 5) allow us to estimate the range of values Δ (the interval from 0.2 to 0.3 corresponds to the values of I from 0.75 to 0.85), within which the values of σ are close to their maxima.

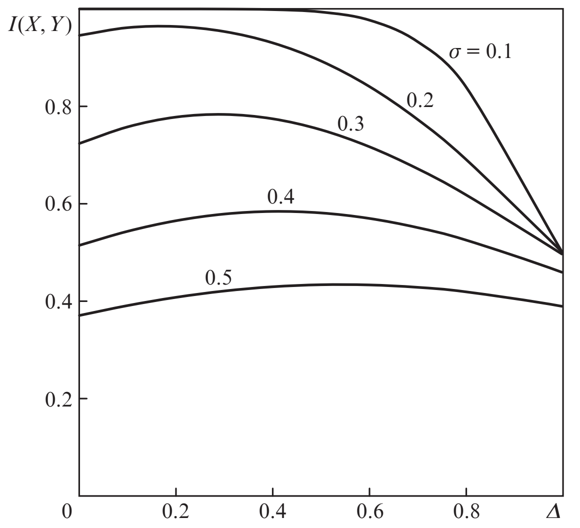

Figure 4. Dependences of the mutual information on the distance between the comparison levels for the given values of σ.

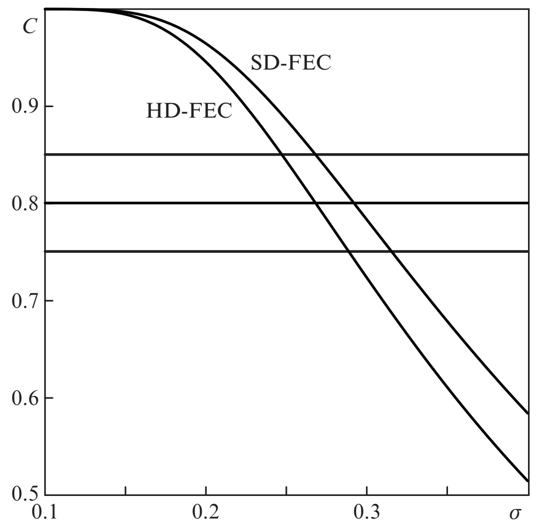

Figure 6. Dependences of the channel capacity on the mean-square deviation.

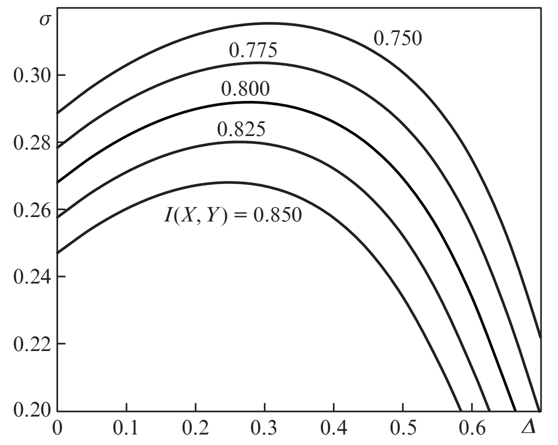

Figure 5. Dependences of the mean-square deviation on the distance between the comparison levels for given mutual information values.

The channel capacity (mutual information), optimised with respect to the free parameter Δ, was calculated numerically:

$$C = \max_{I_{\text{th0}}, I_{\text{th1}}} I(X, Y) = \max_{\Delta} I(X, Y).$$

The calculation results in the form of dependences of the capacity C on σ for HD-FEC and SD-FEC cases are shown in Fig. 6. The horizontal lines correspond to different channel capacities. In this case, the values of C in the range from 0.75 to 0.85 correspond to the error coefficients in the range from 0.4 to 0.2 in a communication channel with HD-FEC. This range of C is typical for a large number of implementations of the fibre-optic communication lines.

5. OSNR gain

The most important characteristic of a transponder is the required value of the optical signal-to-noise ratio OSNR_R [17]. The use of the proposed type of the receiver with SD-FEC provides the operability of a communication system for lesser values of the required OSNR (OSNR_RS) compared to the conventional hard-decision receivers (OSNR_RH).

We define the gain in OSNR_R in the form of the ratio

$$V_{\text{OSNR}} = \frac{\text{OSNR}_\text{RH}}{\text{OSNR}_\text{RS}}. \tag{8}$$

The value of V_OSNR is expressed in decibels.

The gain in OSNR is ensured by means of maintaining the channel operability with SD-FEC at a high noise level, and, consequently, at large σ. A relative measure of the noise level is the quality parameter, or the Q-factor,

$$Q = \frac{I_1 - I_0}{\sigma_0 + \sigma_1} = \frac{1}{2\sigma}. \tag{9}$$

At zero extinction ($r = I_0/I_1 = 0$), the required OSNR is calculated using the formula [15]:

$$\text{OSNR}_\text{R} = \frac{B_\text{e}}{B_\text{r}}\left(Q^2 + Q\sqrt{\frac{B_\text{o}}{B_\text{e}}}\right), \tag{10}$$

where B_r = 12.5 GHz (or 0.1 nm) is the reference band; B_o = 78.75 GHz (or 0.63 nm) is the optical filter band; and B_e = 8.955 GHz (or 0.07164 nm) is the electric filter band. This implies that, with taking into account the typical values for the transponder 10G,

$$\text{OSNR}_\text{R} = 0.7164(Q^2 + 2.965Q).$$

In Table 2, the calculated values of the gain in OSNR_R for three values of redundancy are presented on the basis of two dependences of the channel capacitance on the signal dispersion as applied to the HD-FEC and SD-FEC receivers. Thus, we obtain that the gain amounts to 0.5 dB in a wide range of the redundancy values.

6. Conclusions

In this paper, we propose a simple implementation of the SD-FEC receiver to be used in conventional communication

Table 2. The gain in the required OSNR for different redundancy values.

Capacity/redundancy (%)	σ_{H}	σ_{S}	Q_{H} HD-FEC	Q_{S} SD-FEC	$\mathrm{OSNR}_{\mathrm{RH}}$/dB	$\mathrm{OSNR}_{\mathrm{RS}}$/dB	V_{OSNR}/dB
0.85/17.65	0.247	0.268	2.023	1.865	8.591	8.097	0.49
0.8/25.00	0.268	0.292	1.865	1.712	8.096	7.588	0.51
0.75/33.00	0.289	0.315	1.731	1.585	7.653	7.132	0.52

systems with power modulation and direct detection. The above-suggested receiver scheme employs a single additional level of comparison; it does not require changes to the existing communication line infrastructure. The addition of the second comparator is not so burdensome as the replacement with an eight-bit ADC. The SD-FEC receiver with a small number of output states allows employing the decoding schemes that do not require the high-performance DSP blocks. Thus, it is shown that such a soft-decision receiver possesses definite advantages over conventional receivers with hard decision FEC. A variant of the SD-FEC receiver implementation is suggested. Under optimal values of the noise-level-dependent thresholds, the gain in OSNR can reach 0.5 dB, which corresponds to an increase in the maximum length of a multihop DWDM line by 25%.

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