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Transient radiation from a ring resonant medium excited by an ultrashort superluminal pulse

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Abstract. **We report some specific features of transient radiation from a periodic spatially modulated one-dimensional medium with a resonant response upon excitation by an ultrashort pulse. The case of ring geometry (with particle density distributed along the ring according to the harmonic law) is considered. It is shown that the spectrum of scattered radiation contains (under both linear and nonlinear interaction), along with the frequency of intrinsic resonance of the medium, a new frequency, which depends on the pulse velocity and the spatial modulation period. The case of superluminal motion of excitation, when the Cherenkov effect manifests itself, is also analysed.**

Keywords: *Cherenkov radiation, ultrashort pulse, superluminal motions, optical Bloch equations.*

1. Introduction

In studying radiation from a straight-line system of harmonic oscillators with spatially modulated density, excited by a moving point source, we have revealed some nontrivial spectral features related to transient processes [1, 2]: along with radiation at the eigenfrequency of oscillators, which is typical for resonant systems, spectral components characterising the spatial distribution of oscillator density are also excited. In particular, in the case of periodic density modulation, the spectrum contains new quasi-monochromatic components, whose frequency depends on the modulation period and the obervation direction. Propagation of the excitation with a rate exceeding the speed of light in vacuum c is accompanied by emission of Cherenkov radiation.

The motion of a physical object with a velocity exceeding the speed of light in vacuum, at which a signal (information) is transferred, is forbidden by the theory of relativity. However, there are many situations in physics and, in particular, optics, where superluminal motion of a localised physical perturbation occurs without signal transfer. Heaviside and Sommerfeld [3–5] were interested in these superluminal motions and related radiation even at the turn of the 19th and 20th centuries, before the theory of relativity was developed. The problem of possibility of superluminal motion has been repeatedly raised throughout the 20th century and nowadays [1–6]. In particular, the existence of superluminal particles – tachyons – was considered in the 1960s–1970s [6].

In the 1930s, a study of the motion of an electric charge in a medium with a velocity exceeding the phase speed of light in this medium led to discovery of Cherenkov radiation [7–12]. This radiation is related to the displacement of local polarisation of the medium [9] characterised by instantaneous response to external perturbations. Radiations from different sources moving with superluminal velocity were considered in [13–16]. Radiation of superluminal charges was studied in [17]. The possibility of manifestation of effects due to superluminal motions in different fields of physics was also investigated in [4]. Many recent studies reported the generation of Cherenkov radiation in the form of the second harmonic with respect to the incident light wave frequency in media with both random [18] and periodic [19] distributions of susceptibility. Radiation generated during motion of charged particles along a periodic structure (known as the Purcell–Smith radiation) was also considered in [20] and experimentally demonstrated for the first time in [21].

Examples of superluminal objects are widely known in optics. In the 1960s–1970s, after the development of lasers, the possibility of propagation of high-power ultrashort light pulses (USLPs) in a nonlinear medium with a velocity exceeding the speed of light was shown in [22–26]. We considered the displacement of the region of intersection of a flat USLP with a straight line or a plane in [1, 2, 27]. In particular, the scattering of a flat superluminal USLP propagating over a flat surface with a spatially modulated density of scattering centres, positioned along a straight line or a plane, was investigated in [27]. Only the case of instantaneous relaxation of scattering centres was analysed. The obtained time dependence of scattered-radiation amplitude reflects the law of change in the scattering-centre density; note that the time scale of the corresponding signal depends on the observation angle.

However, the most popular example is the situation considered in [13–15]: motion of a light spot from a projector (or even a pulsar), rotating with a constant angular velocity, on a distant screen. Modern methods for controlling circular scan of light or electron beams provide readily their rotation with frequencies as high as $\sim 10^{10}$ Hz or even higher. Correspondingly, the linear velocity of rotation of the excitation region formed at the point of intersection of a pencil-like rotating beam from a point emitter with a plane orthog-

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onal to the cone axis can be controlled in very wide limits (from subluminal to superluminal) by simple displacement of the plane along the cone axis.

In all above cases, the radiation source in the medium is its dynamic polarisation induced by an incident photon or electron beam. In [1, 2], we emphasised an interesting feature of radiation from a linear medium. This is generation of coherent Cherenkov-type radiation at a frequency that may differ from the resonant frequency of the medium. It is assumed that the distribution of oscillator density in space obeys a harmonic law. The occurrence of a mode with a new frequency in the emission spectrum of the medium is related to the fact that the scattered wave arriving at the observation point is a transient process arising due to the finite time delay of the waves emitted by different regions of the medium and the ones arriving at the observation point. Such a delay is characteristic of both superluminal and subluminal propagation of excitation. Frequency Ω_1 of radiation generated upon scattering is independent of the resonant frequency of oscillators but depends on observation angle α, spatial period Λ_z of the oscillator arrangement along the z axis, and velocity V of an excitation USLP [1, 2]:

$$\Omega_1 = 2\pi \frac{V/\Lambda_z}{|(V/c)\cos\alpha - 1|}. \qquad (1)$$

Note that relation (1) corresponds to the case where the oscillators of the medium are arranged along a straight line and nonlinear effects are disregarded.

Here, we consider a circular arrangement of oscillators. In our opinion, the analysis of the results of interaction between a USLP and a resonant medium under conditions where the region of the USLP effect on the medium moves in the latter with a velocity exceeding the speed of light in vacuum is of particular methodical (and, possibly, practical) interest. The optical response of the medium can be used to detect such motions; it may have interesting properties, differing from the typical ones of the Cherenkov radiation [1, 2],.

In this study, we consider (both disregarding nonlinear effects and taking into account the nonlinearity of the medium) the cases where oscillators of the medium are located along a circle and the density of particle distribution changes according to the harmonic law. The lifetime of Cherenkov radiation excited in the system by a short superluminal pulse corresponds to the time during which the beam describes the circle; this radiation is resonant. That is why we use the term 'transient' in the title of the paper. It is shown that the frequency of generated radiation differs from the resonant frequency of the medium. Possible application of this effect is discussed.

2. Analytical theory of the phenomenon in the case of circular motion of excitation, and radiation detection at the centre of the system. The case of low excitation power (linear response of the medium)

Let us consider a scattering medium having a one-dimensional ring geometry (Fig. 1). Let oscillators be located along a circle of radius R and the oscillator distribution density depend harmonically on the polar angle φ and be determined by the expression

$$N_\varphi(\varphi) = \tfrac{1}{2}(1 + \cos\kappa\varphi), \qquad (2)$$

where $\kappa = 2\pi/\Lambda_\varphi$ is the dimensionless angular spatial frequency of oscillator distribution and Λ_φ is the angular period of this distribution. The angle φ is counted counterclockwise from the y axis, as in a polar coordinate system. The medium is excited by some physical object (e.g., light, electron beam, USLP, etc.) propagating along a circle and running this circle one time with a constant linear velocity V. We assume also (as in [2]) that the interaction time of the excitation beam with the excitation region is shorter than the oscillation period of the oscillators of the medium (or is comparable with it); correspondingly, the spectrum of the interaction pulse contains not only the resonant frequency of oscillators, but is also rather wide and flat. Optical pulses with these characteristics can be obtained, in particular, in the THz range upon excitation of a gas medium (see reviews [28–31]). Examples of oscillators are nanoantennas [32, 33], two-level atoms, or quantum dots [34, 35].

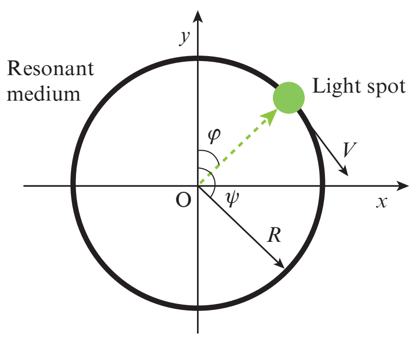

Figure 1. Ring geometry of a resonant medium excited by a light spot propagating along it with velocity V. The oscillators of the medium are characterised by periodic density distribution along the circumference. Radiation is detected either at the centre of the circle or at the point with angular coordinate ψ.

We analyse two cases below. In the first case, an observer detects radiation, being located at the centre of the circle along which the medium is distributed. In the second case, an observer is located directly on the circle and detects radiation propagating along the circle and arriving at the observation point (see Fig. 1).

Under an USLP, oscillators begin to emit at their eigenfrequency. The radiation field of an oscillator located on the circle at the point corresponding to polar angle φ is determined (accurate to a constant factor) by the expression

$$E(t,\varphi) = \exp\left[-\frac{\gamma}{2}\left(t - \frac{R}{V}\varphi\right)\right] \times \cos\left[\omega_0\left(t - \frac{R}{V}\varphi\right)\right]\Theta\left(t - \frac{R}{V}\varphi\right). \qquad (3)$$

Here, ω_0 is the resonant frequency of the oscillators of the medium; γ is the field damping rate; and $\Theta(t - R\varphi/V)$ is the Heaviside step function, which takes into account the fact that an oscillator located at point φ begins to emit at the instant $R\varphi/V$ (when excitation arrives at the point with coordinate φ).

The total field at the centre of the circle is expressed in terms of the integral of (3) over the circumference, with allowance for the spatial oscillator distribution density (1) and the radiation propagation time from a point on the circumference to the centre of the circle R/c:

$$E(t) = \int_0^{2\pi} N_\varphi(\varphi)\exp\left[-\frac{\gamma}{2}\left(t - \frac{R}{V}\varphi - \frac{R}{c}\right)\right]$$

$$\times\cos\left[\omega_0\left(t - \frac{R}{V}\varphi - \frac{R}{c}\right)\right]\Theta\left(t - \frac{R}{V}\varphi - \frac{R}{c}\right)\mathrm{d}\varphi. \quad (4)$$

Calculation of integral (4) shows that the spectrum of the transient process contains, along with the fundamental frequency of the medium (ω_0), a new frequency [2] [see also expression (A1) in the Appendix]:

$$\Omega_2 = 2\pi\frac{V/\Lambda_\varphi}{R} = \kappa\frac{V}{R}, \quad (5)$$

which depends on the angular period Λ_φ of oscillator distribution, circle radius R, and excitation propagation velocity. Relation (5) can be rewritten in the form

$$\Omega_2 = \kappa\Omega_{\text{exc}}, \quad (5a)$$

where $\Omega_{\text{exc}} = V/R$ is the angular velocity of excitation. In this notation, the value Ω_2 has a physical meaning of the excitation rate of a system of periodically arranged oscillators (for example, lines of a curvilinear spatial diffraction grating located along a circle) by an excitation pulse. This process will have an apparent frequency Ω_2 for an observer detecting radiation at the centre of the circle.

Obviously, when the resonance condition

$$R\Lambda_\varphi/\lambda_0 = V/c \quad (5b)$$

is satisfied, it follows from formulas (5) and (5a) that $\omega_0 = \Omega_2$; i.e., the new frequency coincides with the fundamental frequency, and the observer located at the centre of the circle detects radiation only at the resonant frequency of the medium.

Note that the above-considered situation coincides in general features with the case of a medium having a linear geometry, where radiation is detected in a direction perpendicular to the medium [1, 2]. Indeed, formulas (1) and (5) for the new frequency in this and previous studies coincide at $R = 1$ and $\varphi = \pi/2$.

Below we consider some numerical examples. Let the excitation point describe a circumstance one time and the following conditions be satisfied: $V/c = 3.75$, $R\Lambda_\varphi/\lambda_0 = 2$, and $\omega_0/\gamma = 22.22$; then, $\omega_0/\Omega_2 = 0.53$. The result of calculating integral (4) and the electric-field spectrum are shown in Figs 2a and 2b, respectively.

An analysis of the solution (Fig. 2a) demonstrates that emission from point O begins at the instant when the field emitted by the first excited oscillator of the medium arrives (corresponds to $R/c = 1$ at these parameters). Then radiation from other oscillators of the medium arrives at point O. As a result, a complex transient process develops to yield a new frequency in the emission spectrum of the medium (see Fig. 2b). After the end of the transient process, the observer located at point O observes conventional damping oscillations of the medium.

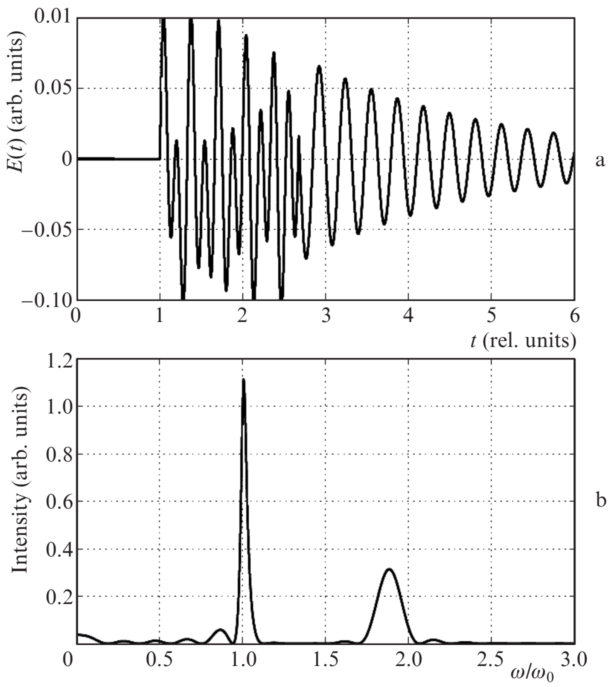

Figure 2. (a) Behaviour of electric field $E(t)$ at the centre of the circle in the linear case and (b) the spectrum of $E(t)$.

The above consideration also holds true for the case where excitation propagates with speed of light c or with a velocity smaller than c. The spectrum of radiation detected at the centre of the circle also contains a new frequency (5) in this case.

3. Case of high-power excitation pulse. Nonlinear dynamics

The above-described situation corresponds to the conditions under which the pump pulse has low power and the nonlinear effects due to the interaction of this short pulse with a resonant medium can be disregarded. However, if the pump pulse has sufficiently high power, the response of the system becomes nonlinear.

To calculate the dynamics of the population difference, $N(t,\varphi) = N_0 w(t,\varphi)$, and the polarisation of the medium, $P(t,\varphi) = d_{12}N_0[u(t,\varphi)\cos\omega t + v(t,\varphi)\sin\omega t]$, under a high-power pulse field, we used optical Bloch equations for two-level atoms [36]:

$$\frac{\mathrm{d}}{\mathrm{d}t}u(t,\varphi) = -\Delta\omega v(t,\varphi) - \frac{1}{T_2}u(t,\varphi), \quad (6)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}v(t,\varphi) = \Delta\omega u(t,\varphi) - \frac{1}{T_2}v(t,\varphi) + \Omega_{\mathrm{R}}(t,\varphi)w(t,\varphi), \quad (7)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}w(t,\varphi) = -\frac{1}{T_1}(w + 1) - \Omega_{\mathrm{R}}(t,\varphi)v(t,\varphi). \quad (8)$$

Here, N_0 is the particle concentration in the medium; $u(t,\varphi)$ and $v(t,\varphi)$ are, respectively, the in-phase and quadrature (with respect to the external field) components of polarisation of the medium per atom; $w(t,\varphi)$ is the difference in the level populations in a single atom; $\Delta\omega$ is the frequency mismatch between the field frequency and transition frequency ω_0 for two-level particles; d_{12} is the transition dipole moment; T_1 is the relaxation time of the population difference; T_2 is the polarisation relaxation time; and $d_{12}\varepsilon(t)/\hbar$ is the Rabi frequency of the pump field. Let the pump pulse envelope be Gaussian: $\varepsilon(t) = E_0\exp(-t^2/\tau^2)$.

In a particular case, where frequency mismatch $\Delta\omega = 0$ and relaxation is absent ($T_1 = T_2 = \infty$), system of equations (6)–(8) is solved analytically by introducing the local pulse area [23, 36]

$$\Phi(t,\varphi) \equiv \frac{d_{12}}{\hbar}\int_{-\infty}^{t}\varepsilon(t',\varphi)\,\mathrm{d}t'. \quad (9)$$

A solution of this system of equations yields an expression for the population difference per unit volume, $N(t,x)$, and the polarisation of the medium, $P(t,x)$, which are determined as [23, 36]

$$P(t,\varphi) = d_{12}N_0\sin[\Phi(t,\varphi)]\sin\omega_0 t, \quad (10)$$

$$N(t,\varphi) = N_0\cos[\Phi(t,\varphi)]. \quad (11)$$

Note that the system of equations (6)–(8) has a simple physical interpretation [23, 36]. A change in N and P can be presented as a rotation of a unit Bloch vector in the xy plane. In this case, the x and y components of the vector correspond, respectively, to N/N_0 and $-P/d_{12}N_0$. Then function Φ is the rotation angle of this Bloch vector. The total pulse area $\Phi = \pi$ (π pulse) corresponds to the complete transition of particles from the lower level to the upper level, while $\Phi = 2\pi$ corresponds to the transition from the lower level to the upper and complete return to the lower level (2π pulse).

With allowance for (4) and (10), the electric field at the centre of the circle in the nonlinear case can be written as

$$E(t) \simeq \int_0^{2\pi} N_\varphi(\varphi)\,P\Big(t - \frac{R}{V}\varphi - \frac{R}{c}\Big). \quad (12)$$

An example of nonlinear dynamics of the field and its spectrum are presented in Fig. 3 ($V/c = 2.3$, $R\Lambda_\varphi/\lambda_0 = 2$, $\omega_0/\gamma = 22.22$, $\tau = 2T_0$, where $T_0 = 2\pi/\omega_0$ is the period of natural oscillations of the oscillator, $\Omega_{\mathrm{R}} = 0.07\omega_0$, and the total pulse area $\Phi_\infty = \pi/2$). With these parameters, $\Omega_2/\omega_0 = 1.15$.

The spectrum of the observed oscillation is presented in Fig. 3b, where the excitation-pulse spectrum is shown by a dashed line. It can clearly be seen that, along with the fundamental frequency, there is a new frequency in the emission spectrum of the medium.

A dependence of the emission spectrum on the total pulse area is shown in Fig. 4. When solving the problem, the pulse area changed due to the change in the Rabi frequency (pulse amplitude) at a constant pulse duration $\tau = 2T_0$. The other parameters are the same as in the previous example. As can be seen in Fig. 4, the spectrum of the oscillation excited by a high-power pulse contains a branch corresponding to the resonance frequency of the medium, ω_0, and a branch corresponding to the frequency shift Ω_2. The small shift of the maximum of the fundamental oscillation of resonators from ω_0 is due to the oscillator damping; it is independent of the external field magnitude and the shift of the resonant oscillation frequency and is observed for all excitation pulse areas. Note the absence of radiation at the resonant

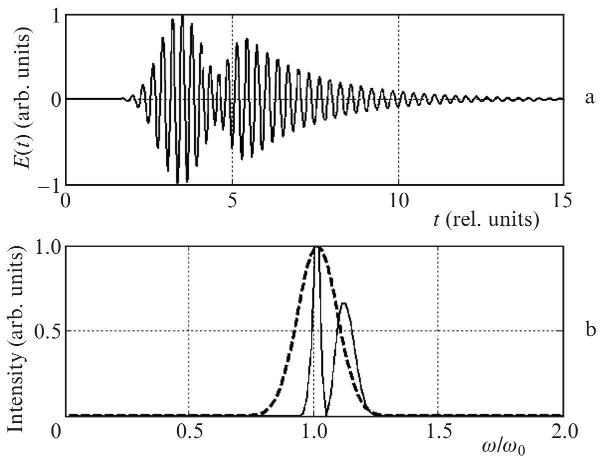

Figure 3. (a) Behaviour of electric field $E(t)$ at the centre of the circle in the nonlinear case and (b) the spectra of (solid line) $E(t)$ and (dashed line) the pump pulse.

frequency if the pulse area is multiple of 2π. These points correspond simultaneously to maxima of the shifted-frequency amplitude.

The new frequency, excited under an external effect, coincides with the result of calculation within the weak-field approximation only at a small pulse area. An increase in field leads to an approximately linear increase in the shift; this process has a periodic saw-tooth character. It can be seen in Fig. 4 that, in the strongly nonlinear regime, when the pulse has a large area, the emission intensity at the new frequency Ω_2 exceeds the emission intensity at the fundamental frequency ω_0. The spectrum of the response of a two-level system to an external field is known to contain a spectral component at frequency $\omega_+ = \omega_0 + \Omega_{\mathrm{R}}$. In this example, $\omega_+ \approx \Omega_2$; the larger the pump field amplitude, the more accurate the approximate equality. An increase in the pump field amplitude leads to a linear increase in the radiation intensity at frequency ω_+, as follows from Fig. 4.

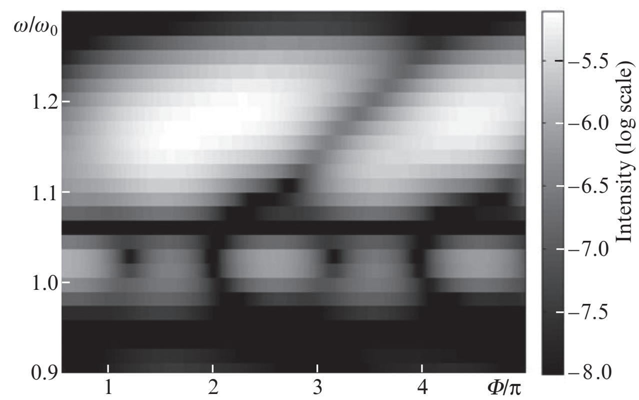

Figure 4. Dependence of the emission spectrum on Φ/π.

The periodic structure observed in Fig. 4 can be explained by the periodic dependence of the polarisation of the medium on the pulse area [the term $\sin\Phi$ entering expression (10) for polarisation $P(t,\varphi)$].

Note that we observed a similar dependence in [2] for a medium with linear geometry. Small minor oscillations of the intensity of spectral components are due to the well-known effect of sharp limitation of the time interval during which the medium is excited, while their relatively large amplitude in Fig. 4 is due to only logarithmic representation.

4. Elementary theory of the phenomenon for the case where radiation is detected on a circumference

Let us now assume that radiation from a medium can only propagate along a circumference (as in the case of whispering gallery modes [37, 38]) rather than along a chord and is detected not in the centre of the medium but at some point lying on a ring and specified by polar angle ψ (see Fig. 1). In this case, by means of considerations similar to those reported in the previous section, one can easily obtain an expression for the electric field at point ψ (with $\gamma = 0$):

$$E(t,\psi) = \int_0^{2\pi} N_\varphi(\varphi)\cos\left\{\omega_0\left[t - \frac{R}{V}\varphi - \frac{R}{c}(\psi - \varphi)\right]\right\}$$

$$\times\Theta\left[t - \frac{R}{V}\varphi - \frac{R}{c}(\psi - \varphi)\right]\mathrm{d}\varphi. \qquad (13)$$

Within our model, light can propagate only along the circumference; therefore, the term $(R/c)(\psi - \varphi)$ in the integrand in (13) corresponds to the propagation time of radiation from the oscillator located at the point characterised by polar angle φ to the observation point, specified by polar angle ψ.

Calculation of this integral shows that the spectrum of the transient process contains again, along with the fundamental frequency of the medium, a new frequency [see (A3) in the Appendix]:

$$\Omega_3 = 2\pi\frac{V/\Lambda_\varphi}{|1 - V/c|}. \qquad (14)$$

In contrast to the previous case, the new frequency depends on the V/c ratio. If the excitation propagates with a speed of light, the denominator in formula (14) becomes zero for the new frequency. In this case, radiation from any point of the medium arrives at the observation point simultaneously with excitation. No transient process is observed; correspondingly, there is not any new frequency in the radiation spectrum. Directly after the pulse arrival, one observes damping eigenoscillations of oscillators at frequency ω_0.

Let us now discuss the physical meaning of formula (14). As in the previous case, this formula can be rewritten as

$$\Omega_3 = \frac{\kappa\Omega_{\rm exc}}{|1 - V/c|}. \qquad (14a)$$

The numerator in expression (14a) has a meaning of the frequency of excitation of a system of periodically located oscillators (for example, lines of a curvilinear spatial diffraction grating) by an incident pulse. For an observer located at a point with angular coordinate ψ, this process would have apparent frequency Ω_3 (the corresponding correction is taken into account by the denominator). Note that in the case of complex (but periodic) distribution of particles over the circumference, excitation of frequency Ω_3 will be accompanied by excitation of multiple harmonics (diffraction orders higher than the first).

When the resonance condition (5b) is satisfied, formula (14) has the form

$$\Omega_3 = \frac{\omega_0}{1 - V/c} \qquad (14b)$$

and coincides with the formula for the frequency shift caused by the Doppler effect, where a source moves towards an immobile detector.

If the excitation velocity exceeds the speed of light, radiation from the medium is detected at the observation point at the same time when the excitation pulse arrives at this point, as is clearly indicated by the results of our numerical calculation based on formula (13). The dependences of the emission spectrum of the system on the parameters of the problem (velocity V of the excitation point and circle radius R) are presented in Figs 5a and 5b, respectively. In Fig. 5 one can see a branch, corresponding to the resonant frequency of the medium, $\omega = \omega_0$, and a branch of additional frequency Ω_3, determined by formula (14). The calculation was performed for the observer's angular coordinate $\psi = 2\pi$. All other parameters of the problem were the same as in Fig. 2. It follows from Fig. 5a that frequency Ω_3 increases with an increase in V at $V < c$ and decreases at $V > c$. When the excitation velocity tends to the speed of light c, frequency Ω_3 tends to infinity [see (14)]. As one would expect, frequency Ω_3 decreases with an increase in the circle radius R (Fig. 5) at a fixed linear velocity of the excitation point.

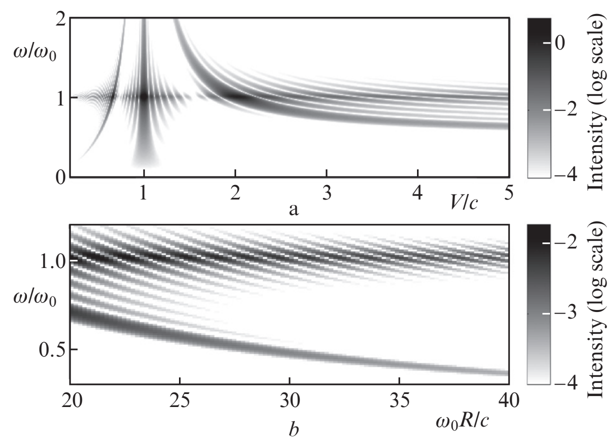

Figure 5. Dependences of the emission spectrum on (a) V/c and (b) circle radius R.

The situation changes if the excitation velocity is smaller than the speed of light. In this case, excitation lags behind radiation from the medium. Therefore, if the polar angle ψ of the observation point is nonzero, radiation is detected at the observation point not immediately but after the time $R\psi/c$, which is necessary for light to reach the observation point.

To conclude, we should note again that formula (14) for the new frequency also holds true for excitation with subluminal velocity. In this case, it is only the aforementioned character of the transient process that changes.

Our elementary consideration of this version of light propagation disregards spatial nonlinear effects that may arise when a short pulse affects an extended resonant medium. Generally, calculations must be performed within the more rigorous semiclassical theory of interaction of light with a resonant material [36]. This question, being a subject of individual consideration, is beyond the scope of our study. However, even based on general considerations, one can qualitatively understand the processes occurring during superluminal propagation of a short light pulse through, e.g., a straight-line resonant medium. Let a short pulse (with a duration shorter than the relaxation times of the medium, T_1 and T_2) excite a resonant optically dense medium by moving through it with some velocity (for example, superluminal, as in [1, 2]). It is known [39] that a short pulse propagating through such a medium leaves behind induced polarisation of the medium, which will emit an electromagnetic field (coherent optical ringing of the medium) after the excitation pulse. Since the medium is optically dense, cooperative effects may occur; i.e., a complex time-dependent periodic exchange in energy between the field and medium (occurring for times shorter than the relaxation time of the polarisation of the medium T_2) is possible [40–44]. This should give rise to new frequencies in the emission spectrum of the medium.

Our consideration [2] for a medium with linear geometry showed that a frequency determined by expression (1) may also arise in the emission spectrum of the medium in this case.

5. Conclusions

Thus, we showed that, upon superluminal excitation of a resonant medium, both in the absence of nonlinear effects and with allowance for them, new frequencies can be generated in the case of different geometries with spatially modulated parameters. In the case of a medium with linear geometry, the new frequency is determined by the excitation propagation velocity, the period of oscillator spatial distribution, and the observation angle. For a medium with ring geometry, the relation determining the new frequency depends strongly on the spatial region where radiation from the medium is detected.

The calculations show that in the linear case, where the pump pulse power is low, the intensity of radiation at resonant frequency ω_0 always exceeds the radiation intensity at new frequency Ω_2. However, in the nonlinear case (high-power pump pulse), the situation is opposite: the radiation intensity at new frequency Ω_2 may exceed the intensity at resonant frequency ω_0. We observed a similar situation for a medium with linear geometry in [2].

The effect considered in this study can be used to detect superluminal motions, perform frequency conversion in resonant systems, and determine the spatial structure of a scattering system from the spectrum of scattered signal. The transient process observed by us can also be used to form light pulses shaped in time. The analytical technique described here can be applied to study time-dependent diffraction from photonic crystals.

Appendix. Derivation of formulas (5) and (7)

In the case of ring geometry, when radiation is detected at the centre of a circle, the expression for the transient process [integral (4)] has the form ($V > c$, $\gamma = 0$, $R/c < t < 2\pi R/V + R/c$)

$$
\begin{aligned}
E(t) &= \int_{V(t-R/c)/R}^{2\pi} N_\varphi(\varphi)\cos\Big[\omega_0\Big(t-\frac{R}{V}\varphi-\frac{R}{c}\Big)\Big]\mathrm{d}\varphi \\
&= -\frac{1}{\kappa}\frac{\Omega_2}{\omega_0}\sin\Big[\omega_0\Big(t-\frac{R}{c}-\frac{2\pi R}{V}\Big)\Big]+A\sin(2\pi\kappa) \\
&\times\cos\Big[\omega_0\Big(t-\frac{R}{c}-\frac{2\pi R}{V}\Big)\Big]+A\cos(2\pi\kappa) \\
&\times\sin\Big[\omega_0\Big(t-\frac{R}{c}-\frac{2\pi R}{V}\Big)\Big]-A\sin\Big[\Omega_2\Big(t-\frac{R}{c}\Big)\Big],
\end{aligned} \tag{A1}
$$

where

$$
A \equiv \frac{1}{\kappa}\frac{\Omega_2^2}{\Omega_2^2-\omega_0^2}.
$$

This formula contains terms oscillating at frequencies ω_0 and Ω_2.

When the transient process is over, i.e., at $t > 2\pi R/V + R/c$, the expression for integral (4) can be written as

$$
\begin{aligned}
E(t) &= \int_0^{2\pi} N_\varphi(\varphi)\cos\Big[\omega_0\Big(t-\frac{R}{V}\varphi-\frac{R}{c}\Big)\Big]\mathrm{d}\varphi = \frac{1}{\kappa}\frac{\Omega_2}{\omega_0} \\
&\times\Big\{\cos\Big[\omega_0\Big(t-\frac{R}{c}-\frac{2\pi R}{V}\Big)\Big]-\sin\Big[\omega_0\Big(t-\frac{R}{c}-\frac{2\pi R}{V}\Big)\Big]\Big\} \\
&+A\sin(2\pi\kappa)\cos\Big[\omega_0\Big(t-\frac{R}{c}-\frac{2\pi R}{V}\Big)\Big] \\
&+A\cos(2\pi\kappa)\cos\Big[\omega_0\Big(t-\frac{R}{c}-\frac{2\pi R}{V}\Big)\Big] \\
&+B\Big\{\cos(2\pi\kappa)\sin\Big[\omega_0\Big(t-\frac{R}{c}-\frac{2\pi R}{V}\Big)\Big]-\sin\Big[\omega_0\Big(t-\frac{R}{c}\Big)\Big]\Big\},
\end{aligned} \tag{A2}
$$

where

$$
B \equiv \frac{1}{\kappa}\frac{\Omega_2\omega_0}{\Omega_2^2-\omega_0^2}.
$$

The latter equation contains only the terms oscillating at the resonant frequency of the medium, ω_0.

When radiation is recorded on a circumference at a point with angular coordinate ψ, the expression for integral (6) for the transient process (at $V > c$, $\gamma = 0$, $R\psi/V < t < R\psi/V + R\psi/c$) takes the form

$$
\begin{aligned}
E(t) &= \int_{W(t-(R/c)\psi)/R}^{2\pi} N_\varphi(\varphi)\cos\Big\{\omega_0\Big[t-\frac{R\psi}{c}-2\pi R\Big(\frac{1}{V}-\frac{1}{c}\Big)\Big]\Big\}\mathrm{d}\varphi \\
&= -C\sin\Big[\Omega_3\Big(t-\frac{R\psi}{c}\Big)\Big]+C\sin(2\pi\kappa) \\
&\times\cos\Big[\omega_0\Big(t-\frac{R\psi}{c}-\frac{2\pi R}{V}\Big)\Big]+D\cos(2\pi\kappa) \\
&\times\sin\Big[\omega_0\Big(t-\frac{R\psi}{c}-\frac{2\pi R}{W}\Big)\Big] \\
&-\frac{W}{\omega_0 R}\sin\Big[\omega_0\Big(t-\frac{R\psi}{c}-\frac{2\pi R}{W}\Big)\Big],
\end{aligned} \tag{A3}
$$

where

$$W \equiv \frac{1}{V} - \frac{1}{c},\ C \equiv \frac{1}{\kappa}\frac{\Omega_3^2}{\Omega_3^2 - \omega_0^2},\ D \equiv \frac{1}{\kappa}\frac{\Omega_3\omega_0}{\Omega_3^2 - \omega_0^2}.$$

This formula contains terms oscillating are frequencies ω_0 and Ω_3.

When the transient process is over, (A3) contains only terms oscillating at frequency ω_0. This expression is omitted here.

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References

1. Arkhipov M.V., Arkhipov R.M., Tolmachev Yu.A. *Opt. Spectrosk.*, **112** (2), 268 (2012) [*Opt. Spectrosc.*, **112** (2), 243 (2012)].
2. Arkhipov R.M., Babushkin I., Lebedev M.K., Tolmachev Yu.A., Arkhipov M.V. *Phys. Rev. A*, **89**, 043811 (2014).
3. Malykin G.B., Romanets E.A. *Opt. Spectrosk.*, **112** (6), 993 (2012) [*Opt. Spectrosc.*, **112** (6), 920 (2012)].
4. Recami E. *Foun. Phys.*, **31** (7), 1119 (2001).
5. Recami E., Fontana F., Garavaglia R. *Int. J. Mod. Phys. A*, **15**, 2793 (2000).
6. Barashenkov V.S. *Usp. Fiz. Nauk*, **114** (1), 133 (1974) [*Sov. Phys. Usp.*, **18**, 774 (1975)].
7. Cherenkov P.A. *Dokl. Akad. Nauk SSSR*, **21** (8), 451 (1934).
8. Cerenkov P.A. *Phys. Rev.*, **52**, 378 (1937).
9. Askar'yan G.A. *Zh. Eksp. Teor. Fiz.*, **42** (5), 1360 (1962).
10. Tamm I.E., Frank I.M. *Usp. Fiz. Nauk*, **93**, 388 (1967).
11. Frank I.M. *Usp. Fiz. Nauk*, **143** (1), 111 (1982) [*Sov. Phys. Usp.*, **27**, 385 (1984)].
12. Bolotovskii B.M. *Usp. Fiz. Nauk*, **179** (11), 1161 (2009) [*Phys. Usp.*, **52**, 1099 (2009)].
13. Bolotovskii B.M., Ginzburg V.L. *Usp. Fiz. Nauk*, **106** (4), 577 (1972) [*Sov. Phys. Usp.*, **15**, 184 (1972)].
14. Ginzburg V.L. *Theoretical Physics and Astrophysics* (Oxford: Pergamon Press, 1979) Ch. VIII. p. 171.
15. Ginzburg V.L. *Usp. Fiz. Nauk*, **103**, 393 (1971) [*Sov. Phys. Usp.*, **14**, 83 (1971)].
16. Bolotovskii B.M., Serov A.V. *Usp. Fiz. Nauk*, **175** (9), 943 (2005) [*Phys. Usp.*, **48**, 903 (2005)].
17. Bolotovskii B.M., Bykov V.P. *Usp. Fiz. Nauk*, **160** (6), 141 (1990) [*Sov. Phys. Usp.*, **33** (6), 477 (1990)].
18. Ukleev T.A., Lazareva Yu.N., Shevchenko N.N., Kosobukin V.A., Sel'kin A.V. *Trudy Mezhdunar. konf. 'Fundamental'nye problemy optiki – 2014' (Proc. Int. Conf. Fundamental Problems of Optics – 2014), St. Petersburg, October 20–24, 2014* (St. Petersburg: Univ. ITMO, 2014) p. 45.
19. Fedorov S.V., Rozanov N.N., Shipulin A.V. *Trudy Mezhdunar. konf. 'Fundamental'nye problemy optiki – 2014' (Proc. Int. Conf. Fundamental Problems of Optics – 2014), St. Petersburg, October 20–24, 2014* (St. Petersburg: Univ. ITMO, 2014) p. 50.
20. Bolotovskii B.M., Voskresenskii G.V. *Usp. Fiz. Nauk*, **94**, 378 (1968) [*Sov. Phys. Usp.*, **11**, 143 (1968)].
21. Smith S.J., Purcell E.M. *Phys. Rev.*, **92**, 1069 (1953).
22. Basov N.G., Ambartsumyan R.V., Zuev V.S., Kryukov P.G., Letokhov V.S. *Zh. Eksp. Teor. Fiz.*, **50**, 23 (1966) [*Sov. Phys. JETP*, **23** (1), 16 (1966)].
23. Kryukov P.G., Letokhov V.S. *Usp. Fiz. Nauk*, **99** (2), 169 (1969) [*Sov. Phys. Usp.*, **12**, 641 (1970)].
24. Oraevskii A.N. *Usp. Fiz. Nauk*, **168** (12), 1311 (1998) [*Phys. Usp.*, **41**, 1199 (1998)].
25. Rozanov N.N. *Usp. Fiz. Nauk*, **175** (2), 181 (2005) [*Phys. Usp.*, **48**, 167 (2005)].
26. Kryabin D.V., Gorbach A.V. *Rev. Mod. Phys.*, **82**, 1287 (2010).
27. Tolmachev Yu.A. *Vestn. SPbGU, Ser. Fiz. Khim.*, Issue 1, 246 (2013).
28. Reimann K. *Rep. Prog. Phys.*, **70**, 1597 (2007).
29. Thomson M.D., Kress M., et al. *Laser Photon. Rev.*, **1**, 349 (2007).
30. Kim K.Y., Taylor A.J., Glownia J.H., et al. *Nat. Photon.*, **2**, 605 (2008).
31. Babushkin I., Skupin S., Herrmann J. *Opt. Express*, **18**, 9658 (2010).
32. Novotny L., van Hulst N. *Nat. Photon.*, **5**, 83 (2011).
33. Biagioni P., Huang J.-S., Hecht B. *Rep. Prog. Phys.*, **75**, 024402 (2012).
34. Mohan A., Felici M., Gallo P., Dwir B., Rudra A., Faist J., Kapon E. *Nat. Photon.*, **4**, 302 (2010).
35. Juska G., Dimastrodonato V., Mereni L.O., Gocalinska A., Pelucchi E. *Nat. Photon.*, **7**, 527 (2013).
36. Allen L., Eberly J.H. *Optical Resonance and Two-Level Atoms* (New York: Wiley, 1975).
37. Vahala K.J. *Nature*, **424** (6950), 839 (2003).
38. Oraevsky A.N. *Kvantovaya Elektron.*, **32** (5), 377 (2002) [*Quantum Electron.*, **32** (5), 377 (2002)].
39. Siddons P. *J. Phys. B: Atom., Mol., Opt. Phys.*, **47** (9), 3001 (2014).
40. Bagaev S.N., Egorov V.S., Moroshkin P.V., Fedorov A.N., Chekhonin I.A. *Opt. Spectrosk.*, **86** (6), 912 (1999) [*Opt. Spectrosc.*, **86**, 814 (1999)].
41. Bagaev S.N., Egorov V.S., Mekhov I.B., Moroshkin P.V., Chekhonin I.A., Davlyatshin E.M., Kindel E. *Opt. Spectrosk.*, **94** (1), 126 (2003) [*Opt. Spectrosc.*, **94** (1), 92 (2003)].
42. Bagayev S.N., Vasilev V.V., Egorov V.S., Lebedev V.N., Mekhov I.B., Moroshkin P.V., Fedorov A.N., Chekhonin I.A. *Laser Phys.*, **15** (7), 975 (2005).
43. Arkhipov M.V., Arkhipov R.M., Egorov V.S., Chekhonin I.A., Chekhonin M.A., Bagaev S.N. *Izv. Vyssh. Uchebn. Zaved., Ser. Priborostroenie*, **55** (7), 42 (2012).
44. Arkhipov M.V., Arkhipov P.M., Pulkin S.A. *Opt. Spectrosk.*, **114** (6), 912 (2013) [*Opt. Spectrosc.*, **114** (6), 831 (2013)].