

Detection of a coherent population trapping resonance in a beam of ^{87}Rb atoms by the Ramsey method

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Abstract. Formation of a coherent population trapping (CPT) resonance is studied in the interaction of a beam of ^{87}Rb atoms with two spatially separated domains of the dichromatic field. Various resonance excitation schemes are compared depending on the choice of operation transitions and type of the polarisation scheme. In the case of a single-velocity atomic beam, the dependence of the CPT resonance profile is studied as a function of principal parameters of the system: beam velocity, distance between optical fields, laser beam dimensions and intensities, and applied permanent magnetic field. Influence of the atomic beam angular divergence and residual beam velocity spread on the resonance quality parameter is estimated.

Keywords: coherent population trapping, Ramsey method.

1. Introduction

Interaction of multi-level atomic systems with multimode coherent radiation yields a series of interesting and practically important effects. One such effect is coherent population trapping (CPT) [1, 2]. In the simplest case, CPT may be observed when a dichromatic field interacts with a three-level system having Λ -configuration of energy levels. If the frequencies of two modes satisfy the two-photon resonance condition then the atomic system transfers to the so-called dark state, which does not interact with the applied two-mode radiation. The result is that the absorption spectra exhibit a narrow dip (CPT resonance) with the width substantially less than the natural width of the excited state which gives a chance to employ CPT in many applications. In particular, one of the most promising applications is the employment of CPT for frequency stabilisation [3–5].

Even a more narrow resonance can be observed if the CPT effect is detected by the method of Ramsey spaced fields [6]. Thomas et al. [7] have for the first time demonstrated experimentally the formation of Ramsey resonances in a thermal beam of sodium atoms under the CPT conditions. In a series of later works, the experiments have been performed in which both the formation and detection of the dark state occurred at the same place in space but at different time

instants [8, 9]. In [10, 11] such experiments have been carried out with cold atoms in traps. The obtained resonance had the width of several tens of hertz. Spectral narrowing of the CPT resonance in the case of spatial-zone pumping in an atomic cell, which is an analogue of the Ramsey detection scheme, has been demonstrated in [12–14].

Cold atoms in traps can be considered as promising objects for constructing microwave standards in laboratory conditions. For mobile standards, in particular, onboard standards in small satellites they are too cumbersome. A scheme of an onboard standard on a beam of slow atoms was suggested in [15]. This scheme utilises a technologically simple principle of obtaining a slow beam without atomic traps. Initially, a thermal beam of atoms is formed in a standard atomic source; then, the beam is decelerated in a Zeeman slower [16]. Employment of the CPT effect in the scheme for forming and detecting the signal of a quantum discriminator by the Ramsey method will abandon cumbersome microwave resonators, and reduce dimensions of the unit and its energy consumption.

The present work is aimed at analysing the formation of a CPT signal in this standard. The CPT resonance profile has been analysed depending on the choice of the polarisation scheme for observations and the scheme of working transitions. An influence of the main setup parameters such as light intensity, magnetic field intensity, thickness of laser beams and the like on the amplitude and width of the resonance has been considered. Also the case of a single-velocity narrow atomic beam has been investigated and the influence has been estimated of the dimensions of a real atomic beam, its angular divergence, and the final width of the atomic velocity distribution.

2. Basic approximations. System of equations for the density matrix

The value of the discriminator signal is determined by the sum of populations of all excited atomic states in the domain of interaction with light regardless of the method of the detecting CPT signal – by absorption of radiation or by atomic fluorescence. In the present work, the change in this value under variation of the difference frequency of dichromatic field components is considered as an indicator of the CPT signal. For calculating the population and describing the interaction of multi-frequency coherent laser radiation with a beam of alkali metal atoms we use the density matrix formalism. This approach makes allowance for all the principal factors affecting formation of the CPT signal.

In deriving the equation for density matrix one should take into account all main mechanisms influencing the evolu-

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tion of the atomic subsystem. One such mechanism in the general case is inter-atomic interaction. In the beam conditions realised in [15] where the atom velocity is $\sim 300 \text{ cm s}^{-1}$ and the flux density is at most $10^9 \text{ cm}^{-2} \text{ s}^{-1}$, the atomic concentration is less than 10^7 cm^{-3} . At such densities and typical relative velocities of atoms one may neglect atomic collisions in the beam. One can also ignore a long-range resonance dipole–dipole interaction which may shift and distort the spectrum of the atomic transition in ensembles of cold atoms [17–19]. This interaction becomes noticeable at atomic densities of $\sim 10^{12} \text{ cm}^{-3}$.

Thus, describing the dynamics of atoms in a beam of the standard under consideration one may surely neglect the inter-atomic interaction, which gives a possibility of analysing the evolution of single atoms in the beam and consider it in the system of coordinates related with the atom. Motion of atoms at typical velocities in the beam can be considered classical. The momentum of a resonance-frequency photon for alkali atoms, for example, rubidium, at such velocities is several hundred times less than the momentum of the atom. Transversal velocities of atoms are substantially less than longitudinal. However, the recoil velocity for the rubidium atom is $5.8\text{--}5.9 \text{ mm s}^{-1}$ depending on the chosen transition. In the considered standard it is suggested to employ lasers with a power on the order of several microwatts and the Gaussian beam width of $\sim 0.5 \text{ cm}$. At such parameters, the momentum transferred during the interaction of the atom with light is substantially less than the transversal momentum of the atom. This is the reason to neglect the recoil effect.

In a concomitant system of coordinates, an atom is stationary. Taking into account the approximations made above, its dynamics in external electromagnetic fields is described by the equation

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H_0 + V, \rho] + \Gamma \rho, \quad (1)$$

where Γ is the operator of relaxation, which makes allowance for a spontaneous decay of excited states; H_0 is the Hamiltonian of free atoms; and V is the operator of their interaction with the external fields E_{01} and E_{02} which is expressed in the dipole approximation as

$$\begin{aligned} V = & -dE = -d[e_1 E_{01}(t) \exp(-i\omega_1 t) \\ & + e_2 E_{02}(t) \exp(-i\omega_2 t) + \text{c.c.}]/2. \end{aligned} \quad (2)$$

Here, the allowance is made for the fact that the atomic ensemble interacts with two fields, which in the general case have different frequencies (ω_1 and ω_2) and polarisations (e_1 and e_2). Note that in the considered system of coordinates, the fields acting on the atom are essentially nonstationary, which is connected with entry and exit processes of the atom in spatially nonuniform laser beams. This circumstance is explicitly taken into account in the time dependence of amplitudes of both E_{01} and E_{02} fields that form the CPT signal. In the present work we will assume that the light beams have transverse Gaussian profiles. We also assume that each of the fields induces transitions from only one of the hyperfine sublevels of the ground state of the alkali atom: $F_1 = I - 1/2$ and $F_2 = I + 1/2$, respectively (here I is the nuclear angular moment).

In a matrix form Eqn (1) is expressed as

$$\begin{aligned} \frac{\partial \rho_{ij}}{\partial t} = & \sum_k \left[-\frac{i}{\hbar}(H_{0ik} + V_{ik})\rho_{kj} + \frac{i}{\hbar}\rho_{ik}(H_{0kj} + V_{kj}) \right] + (\Gamma \rho)_{ij} \\ = & i\omega_{ji}\rho_{ij} + \sum_k \left[-\frac{i}{\hbar}V_{ik}\rho_{kj} + \frac{i}{\hbar}\rho_{ik}V_{kj} \right] + (\Gamma \rho)_{ij}, \end{aligned} \quad (3)$$

where $\omega_{ji} = (E_j - E_i)/\hbar$. In (3) one can separate the subsystems that describe evolution of lower ($\rho_{gg'}$) and upper ($\rho_{ee'}$) atomic states and of optical coherences (ρ_{eg} and ρ_{ge}). In the approximation of a rotating wave for slow envelopes we obtain the system of differential equations with variable coefficients

$$\begin{aligned} \frac{\partial \rho_{gg'}}{\partial t} = & i[-\omega_{gg'} - (\omega_g - \omega_{g'})]\rho_{gg'} \\ & + \sum_c \left[-\frac{i}{\hbar}\tilde{V}_{gc}\rho_{cg'} + \frac{i}{\hbar}\rho_{gc}\tilde{V}_{c'g'} \right] + (\Gamma \rho)_{gg'}, \\ \frac{\partial \rho_{eg}}{\partial t} = & i(-\omega_{eg} + \omega_g)\rho_{eg} - \sum_g \frac{i}{\hbar}\tilde{V}_{eg'}\rho_{g'g} + (\Gamma \rho)_{eg}, \\ \frac{\partial \rho_{ee'}}{\partial t} = & -i\omega_{ee'}\rho_{ee'} + \sum_g \left[-\frac{i}{\hbar}\tilde{V}_{eg}\rho_{ge'} + \frac{i}{\hbar}\rho_{eg}\tilde{V}_{g'e'} \right] + (\Gamma \rho)_{ee'}. \end{aligned} \quad (4)$$

Here

$$\tilde{V}_{gc} = \langle F_g M_g | d_{\mu} | F_c M_c \rangle e_g^{*\mu} E_{0g}(t), \quad (5)$$

$$\tilde{V}_{eg} = \langle F_c M_c | d_{\mu} | F_g M_g \rangle e_g^{\mu} E_{0g}(t) \quad (6)$$

are time-dependent matrix elements of the operator of interaction of the atom with radiation. The frequency with a single index in (4) denotes the frequency of light, which induces transitions from this sublevel of the ground state.

The system of equations (4) takes into account the existence of a series of hyperfine and Zeeman sublevels of the excited state. In the calculation we will restrict ourselves to the levels that only belong to the corresponding resonance D-line. In the present work, system (4) was solved numerically for the beams having different velocities and, hence, passing through spatially inhomogeneous laser beams in different time instants. The velocity difference is taken into account by the dependences $E_{01}(t)$ and $E_{02}(t)$. A solution to the system of equations yields information about the state of the considered atom at any instant. In passing to a laboratory system of coordinates we obtain the information about the atomic state in this spatial domain. Particular calculations have been performed for ^{87}Rb atoms.

3. Calculation results

In constructing the detection scheme based on the CPT effect a key problem is the choice of the working transition and polarisation scheme for observations. In the standard suggested, which employs a slower, the beam is generated in which all atoms populate one of the hyperfine sublevels of the ground state F_g (as a rule, it is $F_g = 2$; however, it is possible to obtain atoms in the state $F_g = 1$ as well). At a small angular divergence of the atomic beam and small spectral width of laser radiation the hyperfine structure of the excited

state is well resolved, which gives a chance to choose a particular hyperfine transition, that is, the transition to a certain hyperfine sublevel of the excited state F_c . This hyperfine sublevel can be chosen as in the D_1 - so and in the D_2 -multiplete. As far as the polarisation scheme is concerned, for observing the CPT effect one may use light with circular or linear polarisation [5, 8, 20–22]. Investigations performed earlier showed that circular polarisations are less promising, because due to existing selection rules, there always remain so-called pockets, that is, such Zeeman sublevels on which atoms are accumulated due to optical pumping. These pockets reduce the amplitude of the CPT resonance. By this reason we will not consider circular polarisations. Considering only linearly polarised radiation and taking into account various initial atomic states and various possible transitions, in the case of ^{87}Rb we have eight variants for lines D_1 and D_2 and for each such variant there are two polarisation schemes: $\text{lin} \parallel \text{lin}$ and $\text{lin} \perp \text{lin}$, which correspond to parallel and crossed linear polarisations.

In order to determine the best variant we compared numerically these sixteen cases. The calculations were performed for typical values of parameters of the standard chosen. The velocity of the atomic beam was taken equal to 300 cm s^{-1} , the intensities of both spectral components were $10 \mu\text{W cm}^{-2}$, the radii of Gaussian laser beams were 0.25 cm , the distance between optical fields was 30 cm , and the induction of longitudinal, parallel to light beams, permanent magnetic field was $B = 10 \text{ mGs}$. The atomic beam was considered single-velocity and narrow one, which crosses the axis of symmetry of the light beams.

Calculation results for one case in a wide range of two-photon detunings are presented in Fig. 1. The calculation was performed under the assumption that atoms fly from the source in the $F_g = 2$ state, the upper working level is $F_c = 2$ of the D_1 -line, and the polarisation scheme is $\text{lin} \perp \text{lin}$. In frequency scanning it was assumed that the frequency of the radiation resonant with the $F_g \leftrightarrow F_c$ transition exactly matches up the resonance and the two-photon detuning is provided by a frequency variation in one of the frequency components. The signal shown in Fig. 1 corresponds to the number of fluorescent photons at the unity total flux of atoms in the beam. In the figure, one can see specific Ramsey oscillations. The period of these oscillations is determined

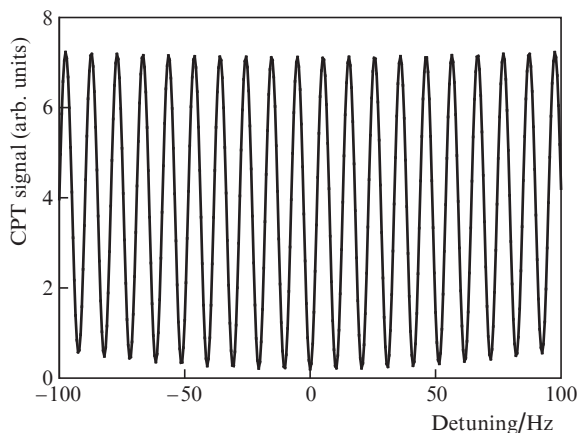


Figure 1. Calculated dependence of the CPT signal on the two-photon resonance detuning. Beam parameters correspond to the standard chosen. The polarisation scheme is $\text{lin} \perp \text{lin}$.

by the time between crossing the two domains of atom interaction with electromagnetic radiation and reduces as this time becomes longer.

Calculations performed confirm some results known for the CPT signal obtained under stationary conditions in gas cells. The highest contrast is specific for the D_1 -line in the polarisation scheme with crossed linear polarisations. In Fig. 2 one can see two resonances corresponding to this case. The calculations were performed for a narrow spectral range which only includes one of the resonances shown in Fig. 1.

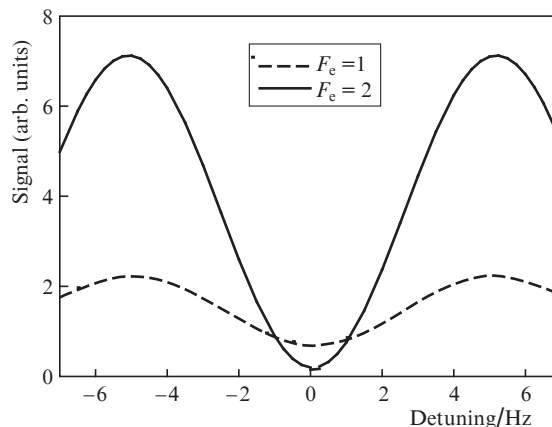


Figure 2. Dependence of the CPT signal on the two-photon resonance detuning for various working transitions in the D_1 -line. The rest parameters are the same as in Fig. 1.

Two curves corresponding to the case where all atoms in the beam initially populate the sublevel $F_g = 2$ of the ground state are shown in Fig. 2. For the other initial state ($F_g = 1$), the calculation yields actually the same results. However, it is difficult to obtain this state in experiments and we will not consider this case in what follows.

In order to understand how one or other parameters of the atomic beam affect the efficiency of the CPT process we may first consider the beam of atoms which have similar velocities and directions and issue from one infinitesimal domain in the source plane. The aim is to determine the parameters which provide not only a maximal signal but also its maximal slope, that is, a maximal increase rate of the CPT signal under a greater detuning.

Let us start with the analysis of the magnetic field influence. The Ramsey signal (Fig. 1) has many local minima, each of them can be chosen for frequency discrimination. Real frequency standards operate on the resonance mostly close to zero detuning. In the considered case, there is a shift due to the Zeeman effect. A reduction of the magnetic induction of field B to zero, surely, reduces the value of the shift to zero; however, in most schemes utilising the CPT effect the magnetic field is still used. A specially produced longitudinal magnetic field makes it possible to select the transitions weakly affected by the magnetic field. In our case those are $F_g = 1, m = 0 \leftrightarrow F_g = 2, m = 0$, that is, the so-called $0-0$ transition. The shift of this transition is proportional to B^2 . The corresponding constant is $575.15 \text{ Hz Gs}^{-2}$. A reduction of the magnetic induction to zero is unsuitable on another reason as well. At exactly zero magnetic induction, there is a series of CPT resonances, which are observed for different sublevel

pairs. These resonances affect each other and the total influence may be negative, which is illustrated in Fig. 3 where the central CPT resonance is shown for various values of B .

In Fig. 3 one can clearly see that the contrast of the resonance at a zero magnetic field is noticeably lower. Switching on the permanent magnetic field improves such a characteristic of the discriminator as the slope. At the same time, the magnetic field leads to shifts. Thus, from the results presented in Fig. 3 one may find the limiting value of the magnetic field, starting from which a further increase in B has little sense because it does not lead to an increase in the slope of the discrimination characteristic: it only results in shifts. All further calculations are performed for $B = 10$ mGs. At this value of induction, the Zeeman frequency shift becomes equal to ~ 0.06 Hz. A reduction of B below 10 mGs is not reasonable mainly due to a reduced amplitude of the CPT signal at low values of induction.

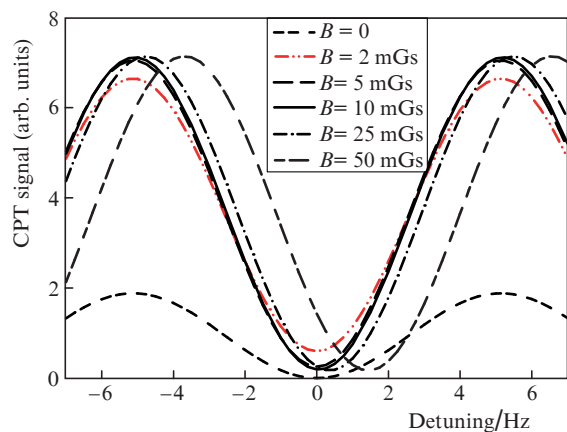


Figure 3. Dependence of the CPT signal on magnetic field intensity. The rest parameters are the same as in Fig. 1.

Among the rest parameters of the installation affecting the quality parameter of the standard under consideration, there are parameters with easily predicted influence. These are the distance between the optical fields and the velocity of atoms in the beam. Both these parameters may increase the time interval between instants of dark state generation and detection, which proportionally reduces the resonance width that at other parameters being reasonable does not reduce its amplitude. These facts are confirmed by direct calculations. Limitations imposed on the increase in the system dimensions and reduction of the beam velocity are, generally, of technical character.

The dependences of the CPT signal on the radiation intensity and on dimensions of the atomic beam are less trivial. The corresponding calculation results are presented in Fig. 4.

An increase in the radiation intensity to $\sim 10 \mu\text{W cm}^{-2}$ leads to a higher resonance amplitude (its width being constant), that is, enhances the contrast. If, at a prescribed beam dimensions, the intensity still increases, then the saturation occurs and the parameters of resonance become actually constant. Note that in this case the light shifts are very small.

The CPT signal also weakly depends on the width of the optical beams when their dimensions are greater than a prescribed value. The width determines the spatial inhomogene-

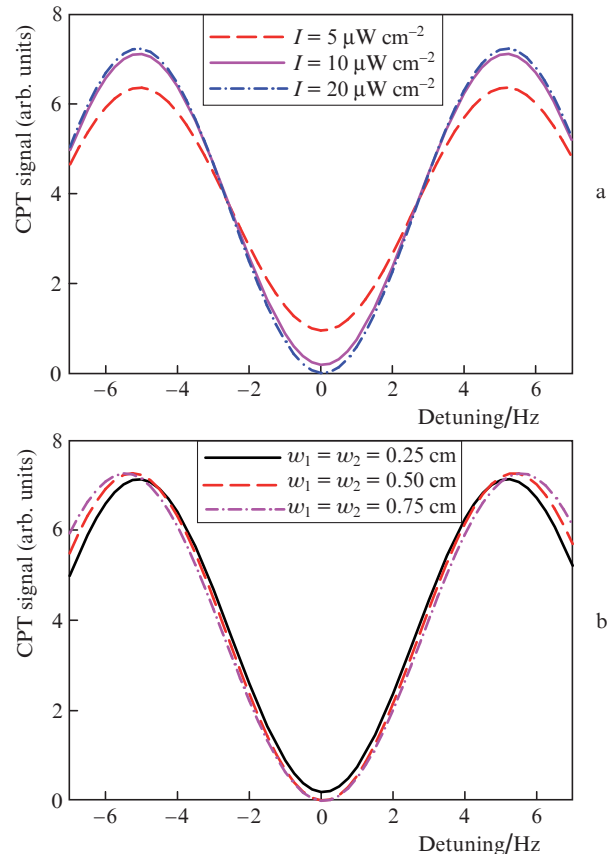


Figure 4. Dependence of the CPT signal on (a) the intensity of optical radiation (the radius of Gaussian laser beams is 0.25 cm) and (b) the width of light beams (the intensity of optical radiation is $10 \mu\text{W cm}^{-2}$). In both cases the velocity of the atomic beam is 300 cm s^{-1} , the intensities of both spectral components are equal, $B = 10$ mGs, and the distance between the optical fields is 30 cm.

ity of the light field and, if it is sufficiently high, all atoms transfer to the dark state. A further increase in the width is not reasonable – the saturation effect is also present here.

The results discussed above make it possible to analyse the influence of various parameters in the case of a single-velocity beam. Thus, we may better understand the formation mechanisms of the CPT signal. In real experimental installations, velocities of different atoms vary. The dwelling time of an atom in the laser beam depends on the velocity of the atom. The density of atoms possessing different velocities also varies. This circumstance affects the signal observed. The contribution of slower atoms into the signal is higher, which is a positive factor that provides a narrower resonance as compared to a beam having a velocity equal to the average velocity of a multi-velocity beam. Existence of atoms having different impact parameters and different directions of motion is the negative factor that reduces the signal amplitude. In addition, the contribution of fast atoms that are present in a nonmonochromatic beam is lower. On the whole, for a fixed flux of atoms, negative factors are stronger. It is seen in Fig. 5 where two curves are shown: one curve for a monochromatic beam and the second curve for real beams [15] with a radius of 0.2 cm and angular divergence of $1/100$ rad, having the velocity dispersion of $\sim 1 \text{ m s}^{-1}$. The velocity distribution of atoms was numerically simulated according to the results presented in [16]. The calculation has been performed by the Monte-Carlo method.

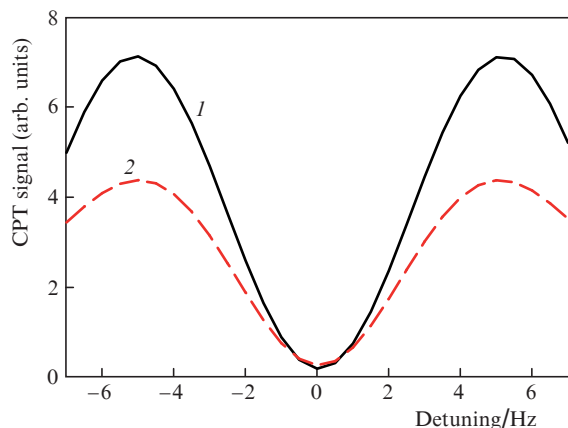


Figure 5. Comparison of resonances for a (1) monochromatic and (2) nonmonochromatic atomic beam with the same average velocity 300 cm s^{-1} . The radius of laser beams is 0.25 cm , the intensities of both spectral components are equal, $B = 10 \text{ mGs}$, the distance between the optical fields is 30 cm , the radius of the atomic beam is 0.2 cm and its angular divergence is $1/100 \text{ rad}$. The intensities of optical radiation are $10 \mu\text{W cm}^{-2}$.

In conclusion, it may be said that for all particular results given above (Figs 1–5) the parameters of electromagnetic radiation in two interaction domains were similar. The intensities of two spectral components were also taken equal. This is related to the fact that actually the Ramsey method is realised with a single laser, radiation of which is modulated in frequency for obtaining the second spectral component. The two-mode radiation obtained is split into two beams. One beam is used for producing and the second beam for detecting the resonance. Hence, the parameters of all radiation components are similar. However, one can change the beam parameters by using a system of collimators, lenses, spectral filters, etc. Keeping in mind these possibilities, we have studied the cases when the intensities of two modes, light intensities in different domains and diameters of the beams producing resonance and detecting it are different. We have not observed substantial signal amplification in this case because of the effect of the CPT signal saturation mentioned above.

4. Conclusions

In the present work, we have calculated the signal of coherent population trapping in a beam of rubidium atoms, which is excited and detected by the Ramsey method. Various possible schemes of working transitions with linearly polarised radiation are considered in the model case of a monochromatic atomic beam. Calculations show that, similarly to the case of observing a CPT signal in a gas cell, under stationary conditions the most contrast Ramsey resonances are obtained in the $\text{lin} \perp \text{lin}$ geometry by using the $F_c = 2$ level of the D_1 transition as the working one.

We have analysed the dependence of resonance parameters (its amplitude and width) on the main system parameters. Expected reduction of the resonance width was confirmed in the case where the time of the transit between the optical fields was increased by making the distance longer or the beam velocity lower. An influence of the magnetic field on the slope of the discrimination characteristic has been

analysed. The value of induction has been determined, starting from which a further increase in the magnetic field becomes unreasonable because it does not affect the slope but results in shifts. At a fixed transit time, the effect of signal saturation has been discovered. The resonance amplitude reaches a maximal values at relatively low intensities of electromagnetic fields ($10\text{--}20 \mu\text{W cm}^{-2}$ depending on the beam aperture) and at facile dimensions of light beams. This effect can be observed already at the beam radii of $\sim 0.25\text{--}0.5 \text{ cm}$ (depending on the intensity). A further increase in these parameters does not result in a higher intensity and weakly affects the resonance width. The calculation has been performed with allowance for a spatial inhomogeneity of the light beams.

In the case of a monochromatic beam, we succeeded in obtaining ‘optimistic’ numerical estimates for a reachable slope of the discrimination characteristic. Taking into account real geometry, in particular, the finite width of the atomic beam, its angular divergence and finite width of the atom velocity distribution in the beam resulted in a certain reduction of the slope. While optimising operation of the considered standard from the viewpoint of obtaining the best frequency stability one should take into account all these factors as well as fluctuations of the principal parameters determining the signal amplitude: the density of the atomic beam, laser intensity, its phase fluctuations and so on. One should also take into account many technical features of the variant of the suggested beam standard. One of the main features is low density of the beam of cold atoms. In this case, the CPT signal can be detected only by observing fluorescence, whereas the fluorescence signal itself is weak and has to be selected against the background of radiation scattered by elements of the standard construction. The second feature is a pulse-periodic character of the modulations of the slow atomic beam and of retarding laser radiation, which leads to strong fluorescence, when the flux of fast atoms passes, and to strong scattering of retarding radiation at the stage of preparing the beam of slow atoms. All these factors limit the design of the CPT signal detection scheme.

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