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# Quantum fluctuations of radiation in a ring Nd:YAG chip laser

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***Abstract.*** **We report theoretical and experimental investigation of intensity fluctuations in a travelling-wave ring Nd:YAG chip laser, caused by the noise of spontaneous emission. In accordance with theory and experiment, quantum intensity fluctuations in the laser under study decrease dramatically with increasing pump over the threshold. As a result of the research performed, the factor $\beta$ is found, which determines the ratio of the rate of spontaneous emission into the generated mode to the total rate of spontaneous emission into all modes. The effect of the relaxation rate from the lower laser level on quantum fluctuations of the radiation intensity is found.**

***Keywords:*** *solid-state ring laser, spontaneous emission, quantum fluctuations, relaxation oscillations.*

## 1. Introduction

Quantum fluctuations of laser radiation are caused by the noise of spontaneous emission and other quantum noises occurring in an active medium with a population inversion on the laser transition. Radiation intensity fluctuations arising under the action of quantum noises are, as a rule, drastically reduced with increasing excess of pumping over the threshold. However, in the lasers with slow relaxation of the population inversion and small cavity length, quantum fluctuations can remain strong even at a large excess over the lasing threshold [1–6]. These fluctuations are referred to as thresholdless fluctuations because of an unusual behaviour of their intensity with the pump power increase.

Experimental studies of quantum fluctuations require the use of lasers with a low level of technical noises. In this regard, monolithic solid-state lasers (chip lasers) are more convenient and stable sources of radiation compared to lasers composed of discrete elements (a resonator formed by outer mirrors, an active crystal, and other intracavity devices). As far as we know, quantum radiation fluctuations in solid-state ring lasers have not previously been studied experimentally. This paper presents theoretical and experimental studies on quantum fluctuations of radiation in a ring Nd:YAG chip laser operating in the stationary regime of unidirectional lasing.

In ring chip lasers exposed to the action of an external inhomogeneous magnetic field, it is possible to effectively control the dynamics of laser radiation and implement a large number of lasing regimes differing in temporal and spectral characteristics of radiation under the conditions of single-mode lasing for each of counterpropagating waves [7]. Such laser systems are of great interest for fundamental studies of the peculiarities of quantum fluctuations of radiation in lasers with slow relaxation of the population inversion and small cavity length.

## 2. Theory

One of the methods of theoretical study on quantum fluctuations in lasers is based on the description of intracavity field dynamics using the Heisenberg–Langevin equations [8]. In the frame of this approach, the system of Heisenberg's quantum equations is transformed into a system of equations of semiclassical theory of lasing with Langevin noise sources. In a ring chip laser operating in the regime of single-mode unidirectional lasing (travelling-wave regime), quantum fluctuations of radiation intensity can be considered on the basis of rate equations [1]:

$$\begin{aligned} \dot{n} &= (\beta\gamma_1 N - \Gamma_{\rm c})n + R_{\rm sp} + f_n, \\ \dot{N} &= S - \gamma_1 N - \beta\gamma_1 N n + f_N, \end{aligned} \tag{1}$$

where

$$R_{\rm sp} = \beta\gamma_1 N \tag{2}$$

is the rate of spontaneous emission into the generated mode; $n$ is the number of photons inside the cavity in the generated mode; $N$ is the number of active atoms excited to the upper level of the laser transition; $\Gamma_{\rm c}$ is the intracavity field relaxation rate; $\gamma_1$ is the relaxation rate of the upper-level population; $S$ is the rate of pumping; and $\beta$ is the factor determining the ratio of the rate of spontaneous emission into the generated mode to the total rate of spontaneous emission into all modes. The Langevin sources of noises $f_n$ and $f_N$ have the correlation functions

$$\langle f_n(t) f_n(t')\rangle = 2R_{\rm sp} n \delta(t - t'), \tag{3}$$

$$\langle f_N(t) f_N(t')\rangle = 2\gamma_1 N \delta(t - t'). \tag{4}$$

Theoretical analysis performed in [1] has shown that the noise source $f_N$ weakly affects amplitude and phase fluctuations of radiation in solid-state lasers and therefore may be neglected. Equations (1) imply that the average number of photons in the stationary lasing regime appears as

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$$\langle n\rangle = \frac{\eta + \sqrt{\eta^2 + 4\beta(1+\eta)}}{2\beta}, \tag{5}$$

where $\eta$ is the excess of pumping over the lasing threshold ($S/S_{\rm th} = 1 + \eta$, $S_{\rm th}$ is the threshold rate of pumping). Fluctuations in the number of photons $\delta n = n - \langle n\rangle$ can be conveniently expressed through the relative variance $D_n$, which, as shown in [1], is determined by the formula

$$D_n = \frac{\langle(\delta n)^2\rangle}{\langle n\rangle^2} = \frac{\gamma_n}{\gamma_n + \gamma_N}\left(1 + \frac{\gamma_N^2}{\omega_{\rm r}^2 + \gamma_n\gamma_N}\right), \tag{6}$$

where

$$\omega_{\rm r}^2 = \beta\gamma_1\Gamma_{\rm c}\langle n\rangle \tag{7}$$

is the square of frequency of the relaxation oscillations of radiation. The parameters $\gamma_n$ and $\gamma_N$ in (6) are determined by the expressions

$$\gamma_n = \frac{\Gamma_{\rm c}}{\langle n\rangle + 1}, \tag{8}$$

$$\gamma_N = \gamma_1(1 + \beta\langle n\rangle). \tag{9}$$

Formulas (6)–(9) are valid in the case of fast relaxation from the lower level that is assumed unpopulated. As shown in [1], given the finiteness of the relaxation rate $\gamma_2$ of the low-level population, an approximate formula

$$D_n = \frac{\langle(\delta n)^2\rangle}{\langle n\rangle^2} = \frac{\gamma_n}{\gamma_n + \gamma_N + \gamma_{\rm NL}}\left[1 + \frac{\gamma_N^2}{\omega_{\rm r}^2 + (\gamma_n + \gamma_{\rm NL})\gamma_N}\right] \tag{10}$$

can be used, where $\gamma_{\rm NL} = \omega_{\rm r}^2/\gamma_2$ is the additional relaxation constant related to the relaxation from the low level.

## 3. Experimental setup

The chip laser under investigation represents a monoblock in the form of a prism with one spherical face (curvature radius of 50 mm) and three planar faces of total internal reflection. The geometric perimeter of the cavity is 2.8 cm. The angle of the cavity nonplanarity is 80°. In paper [7], a simple and efficient way of controlling the radiation dynamics of the ring Nd : YAG chip laser is proposed for the case when the laser is exposed to the action of an external constant magnetic field producing both frequency nonreciprocity and considerable amplitude nonreciprocity, which can be varied by moving the magnet relative to the active element of the chip laser. We have used that technique in the present work to obtain the regime of stationary unidirectional lasing.

The laser was pumped by a laser diode having a radiation wavelength of 0.810 μm. The characteristics of radiation were registered using a TektronixTDS 2014 digital oscilloscope.

## 4. Experimental results

The main aim of this work is to study experimentally the radiation intensity fluctuations (relative variance $D_n$) as functions of the pump rate in the vicinity of the lasing threshold. Figure 1 shows the oscillograms of the output radiation intensity for two values of the excess of pumping over the threshold $\eta$. These oscillograms show the relaxation oscillations of radiation intensity at the relaxation frequency, excited by the noise of spontaneous radiation. In Figs 1a and 1b, these fluctuations are presented for the case of two time intervals. One can see that the intensity fluctuations in the laser under investigation decrease with increasing pump power near the lasing threshold. Figure 2 presents the spectrum of radiation intensity fluctuations at $\eta = 0.0022$, in which a peak at the relaxation oscillation frequency $f_{\rm r} = \omega_{\rm r}/(2\pi) = 10$ kHz is visible.

In processing the oscillograms obtained for different values of $\eta$, the relative variance

$$D_n = \frac{\langle(\delta I)^2\rangle}{\langle I\rangle^2},$$

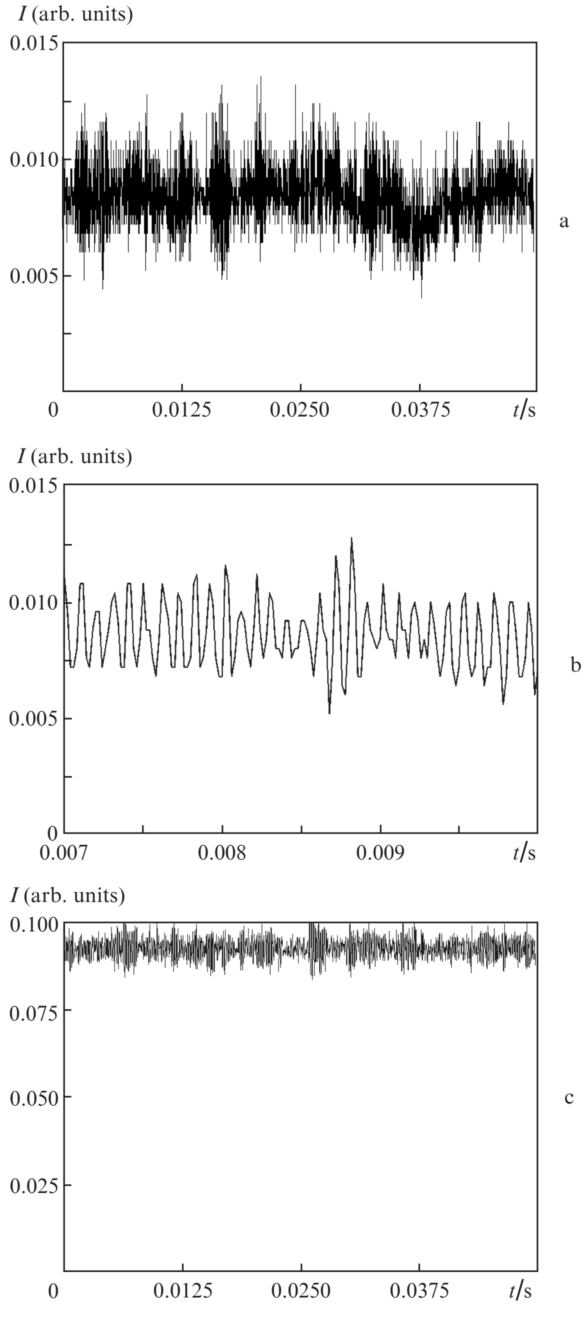

**Figure 1.** Oscillograms of the output radiation intensity in the stationary regime of unidirectional lasing at $\eta$ = (a, b) 0.0022 and (c) 0.045.

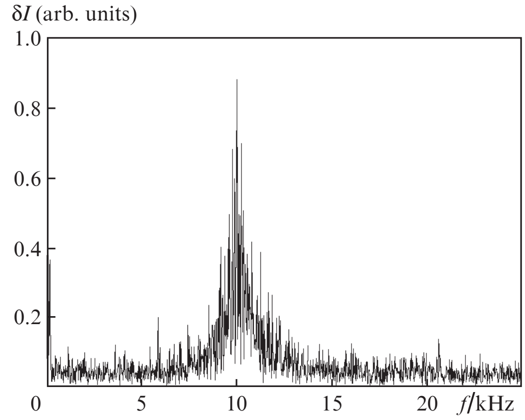

**Figure 2.** Spectrum of intensity fluctuations $\delta I(f)$ at $\eta = 0.0022$.

of intensity fluctuations was calculated, where $\delta I = I - \langle I \rangle$, and $\langle I \rangle$ is the average intensity value. The averaging was performed over the observation time interval equal to 50 ms. The experimental dependence of $D_n$ on the parameter $\eta$ is shown by dots in Fig. 3.

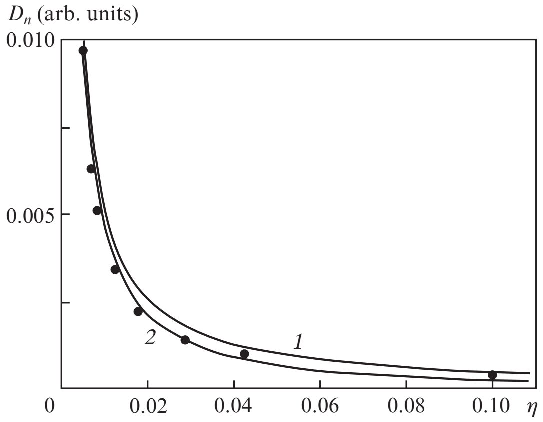

**Figure 3.** Experimental (dots) and calculated (solid lines) dependences of $D_n$ on the excess of pumping over the threshold $\eta$.

## 5. Comparison of theoretical and experimental results

The parameters $\varGamma_c$ and $\gamma_1$ of the laser under study are known from the previous experiments [7]: $\varGamma_c = 4.38 \times 10^8$ s$^{-1}$, $\gamma_1 = 1/T_1$ ($T_1 = 240 \times 10^{-6}$ s). The only unknown parameter $\beta$ is found using the following procedure. At the first experimental point $\eta = 0.0022$ (see Fig. 2), the variance $D_n = 0.028$ and the relaxation oscillation frequency $f_r = 10$ kHz are measured. Solving numerically equations (5)–(9) for these values of $D_n$ and $f_r$, we obtain $\beta = 5 \times 10^{-10}$.

Figure 3 shows curve (*1*) calculated by formulas (5)–(9) at $\beta = 5 \times 10^{-10}$ and curve (*2*) calculated at the same value of $\beta$ with taking into acoutn the finite relaxation rate from the lower laser level [see (10)] at $\gamma_2/\gamma_1 = 10^4$. It is seen that the theoretical dependence [curve (*2*)] is in good agreement with experimental results.

Thus, in accordance with theory and experiment, the radiation intensity fluctuations in the laser in question sharply decrease with increasing excess of pumping over the threshold: at $\eta = 0$ the relative variance $D_n = 0.85$, while at $\eta = 0.04$ it falls down to 0.001.

## 6. Conclusions

The results obtained in the course of experimental investigation of intensity fluctuations in the output radiation from a travelling-wave ring chip laser are in good agreement with theory under the conditions when the spontaneous emission noise $f_n$ is taken into account, whilst the noise $f_N$ in the equation for the population difference $N$ is neglected. These studies have produced a value of the spontaneous emission factor $\beta$ for the laser under study. Exposure of the ring chip laser to the action of an inhomogeneous constant magnetic field allows implementation of a number of self-modulation regimes of bidirectional lasing (periodic, quasi-periodic and chaotic) [7]. It is of interest to study the quantum fluctuations of radiation intensity in those regimes.

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