*Quantum Electronics* **45** (12) 1151 – 1152 (2015) 

***OPTICS OF METAMATERIALS***

PACS numbers: 81.05.Xj; 78.67.Pt
DOI: 10.1070/QE2015v045n12ABEH015840

# Transient processes in the parametric interaction of counter-propagating waves

V.V. Slabko, A.K. Popov, S.A. Myslivets, E.V. Rasskazova, V.A. Tkachenko, A.K. Moskalev

***Abstract.*** **We present a comparative analysis of transient processes in media with a negative refractive index for the parametric interaction of co- and counter-propagating waves. The transient time for the interaction of counter-propagating waves is shown to considerably exceed that for the interaction of co-propagating waves. In the case of counter-propagating waves, we present fitting results for the generated wave amplitude as a function of time and for the transient time vs. the amplitude of the pump wave and the length of the medium.**

***Keywords:*** *transient parametric processes, counter-propagating waves, negative refractive index.*

Recent advances in the development and fabrication of media with a negative refractive index [referred to as negative index metamaterials (NIMs)] in the microwave [1] and optical [2, 3] ranges of electromagnetic radiation frequencies have revived interest in the parametric interaction of counter-propagating waves. Predicted by Bobroff [4], the anomalously high gain in a parametric pump wave decay process involving counter-propagating waves opens up wide possibilities for practical application of this phenomenon in creating a cavity-free optical parametric oscillator [5, 6] and amplifiers and oscillators in the optical and microwave ranges of electromagnetic radiation. These possibilities, however, have not yet been implemented experimentally in positive-dispersion media, because two conditions should be met simultaneously: phase matching and counter-propagation of interacting waves. The parametric interaction of counter-propagating waves can be realised most naturally in NIMs if a nonlinear medium for one of the waves being amplified is an NIM in which the wave vector and Poynting vector have opposite directions. This allows the phase matching condition to be met and, at the same time, a counter-propagating wave to be present [7, 8]. Shalaev et al. [9] and Popov et al. [10, 11] examined Raman scattering in crystals, which is an analogue of parametric processes in NIMs. Under pulsed pumping, significant distinctions between time-dependent amplification of counter- and co-propagating waves were detected. The objectives of this study are to perform a numerical comparative analysis of transient parametric processes for co- and counter-propagating waves and find quantitative relationships for these processes.

Consider the interaction of three waves propagating along the $z$ axis in a quadratically nonlinear medium of length $L$. The electric field of the waves is $E_j(z,t) = A_j(z,t)\exp[\mathrm{i}(\omega_j t - k_j z)]$, where $A_j(z,t)$ are their complex amplitudes; $\omega_j$ are their frequencies; $k_j$ are their wavenumbers; and $t$ is time. The frequencies and wavenumbers meet the conditions $\omega_3 = \omega_1 + \omega_2$ and $k_3 = k_1 + k_2$. In the case of slowly varying amplitudes and a given pump ($A_3 \gg A_1, A_2$), the system of equations has the form [8, 9, 12]

$$\begin{aligned} &\frac{\partial a_1}{\partial z} + \frac{1}{v_1}\frac{\partial a_1}{\partial t} = -\mathrm{i} g a_2^*, \\ &\frac{\partial a_2}{\partial z} + \frac{1}{v_2}\frac{\partial a_2}{\partial t} = -\mathrm{i} g a_1^*, \end{aligned} \tag{1}$$

where $a_j$ is a normalised amplitude related to the field amplitude $A_j$ by $a_j = \sqrt[4]{\varepsilon_j/(\mu_j\omega_j^2)}\,A_j(z,t)$; $v_j$ are the group velocities of the interacting waves; $g = \chi^{(2)}(8\pi/c)a_3\sqrt[4]{\mu_1\mu_2/(\varepsilon_1\varepsilon_2\omega_1^2\omega_2^2)}$ is the gain parameter; $\chi^{(2)}$ is the second-order nonlinear susceptibility of the medium; and $\varepsilon_1$, $\varepsilon_2$, $\mu_1$ and $\mu_2$ are the dielectric permittivities and magnetic permeabilities of the medium at frequencies $\omega_1$ and $\omega_2$, respectively.

We will examine a time-dependent solution to system (1), $a_2(z = L, t)$, at the output of the medium in the 'on' regime (semi-infinite pump pulse whose leading edge travels at a velocity $v_3$). Let $a_3 = a_{30}\{1 - \tanh[(z/v_3 - t)/t_{\mathrm{f}}]\}/2$, where $a_{30}$ is the maximum normalised pump wave amplitude and $t_{\mathrm{f}}$ is the wave front slope, which is taken to be $0.05L/v_3$ in our calculations. In the case of co-propagating waves (Fig. 1a), the boundary conditions then have the form $a_1(z = 0) = 0$ and $a_2(z = 0) = u$. Here, $u$ is a small quantity taken to be $10^{-4}a_{30}$ in our calculations. In the case of counter-propagating waves (Fig. 1b), phase matching conditions are satisfied because the wave vector $\boldsymbol{k}_1$ and Poynting vector $\boldsymbol{S}_1$ have opposite direc-

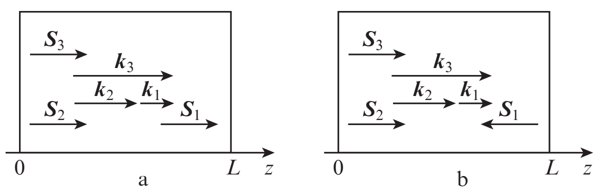


**Figure 1.** Mutual orientations of the wave vectors $\boldsymbol{k}_j$ and Poynting vectors $\boldsymbol{S}_j$ for (a) co-propagating and (b) counter-propagating interacting waves.

**V.V. Slabko, E.V. Rasskazova, V.A. Tkachenko, A.K. Moskalev** Siberian Federal University, Svobodnyi prosp. 79, 660041 Krasnoyarsk, Russia; e-mail: slabko49@mail.ru;
**A.K. Popov** Purdue University, 1205 W State St, West Lafayette, IN 47907 USA; e-mail: popov@purdue.edu;
**S.A. Myslivets** L.V. Kirensky Institute of Physics, Siberian Branch, Russian Academy of Sciences, Akademgorodok 50/38, 660036 Krasnoyarsk, Russia; Siberian Federal University, Svobodnyi prosp. 79, 660041 Krasnoyarsk, Russia; e-mail: sam@iph.krasn.ru

Received 17 April 2015; revision received 10 August 2015
Kvantovaya Elektronika **45** (12) 1151 – 1152 (2015)
Translated by O.M. Tsarev

tions, as is characteristic of NIMs. This case can be described by system (1) after changing the sign of the right-hand side of the former equation [8, 9]. It should be taken into account that the group velocity appears in this equation with a minus sign. The boundary conditions for counter-propagating waves then have the form $a_1(z = L) = 0$ and $a_2(z = 0) = u$.

Steady-state solutions are well-known: $a_2(z = L) \propto \exp(gL)$ in the case of co-propagating waves and $a_2(z = L) \propto [\cos(gL)]^{-1}$ in the case of counter-propagating waves [4, 5]. As seen from these solutions, in the case of counter-propagating waves the amplitude rises faster than exponentially with increasing $gL$ and has a discontinuity for $gL \to \pi/2$.

System (1) was solved by numerical simulation techniques using the MATLAB software package. The inset in Fig. 2 shows the solution $a_2(z = L, t)$ to a time-dependent problem for co-propagating (dashed line) and counter-propagating (dotted line) waves at $|v_1| = |v_2| = |v_3| = v$ and $gL = 0.984\pi/2$. It is well seen that, in the case of co-propagating waves, the time needed for a steady state to be reached is approximately $L/v$ and does not exceed the time it takes for the leading edge of a pump pulse to pass through the medium, with allowance for its slope $t_f$. In the case of counter-propagating waves, the amplitude continues to rise for $t > L/v$. It is seen from Fig. 2 that the interaction of counter-propagating waves is characterised by an anomalously long transient time, which depends on $gL$. To demonstrate the advantages of the amplification of counter-propagating waves, we present the time dependence of $a_2$ (with the vertical scale expanded by a factor of 10) for co-propagating waves. Note that qualitatively similar results were obtained at various boundary conditions for $a_2$ and at $|v_1| \neq |v_2| \neq |v_3|$.

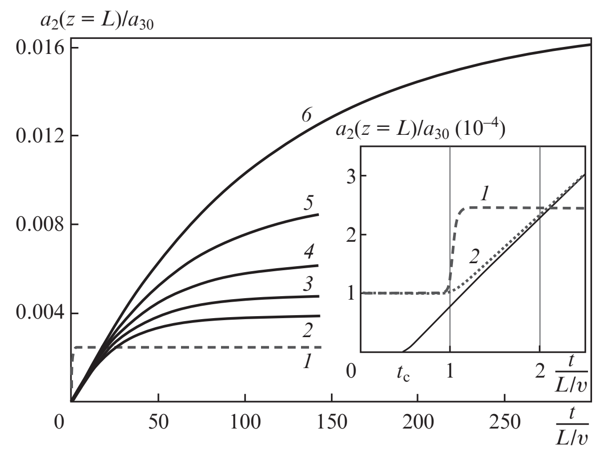


**Figure 2.** $a_2(z = L)/a_{30}$ as a function of normalised time $t/(L/v)$ for (*1*) co-propagating and (*2–6*) counter-propagating waves at $gL$ = (*1, 2*) $0.984\pi/2$, (*3*) $0.987\pi/2$, (*4*) $0.990\pi/2$, (*5*) $0.993\pi/2$ and (*6*) $0.996\pi/2$. The solid line in the inset represents a theoretical fit.

The dependences under consideration were fitted by a curve of the form $a_2(z = L, t)/a_{30} = (a_2/a_{30})_{\max}\{1 - \exp[(t - t_c)/\tau]\}$, which adequately represents the numerical simulation data (Fig. 2). Deviations from the best fit curve become insignificant by time $t = 2L/v$, which is the sum of the times needed for the fronts of the pump wave and counter-propagating wave to pass through the medium. It is seen from Fig. 2 that the time constant $\tau$ of the transient process increases as $gL$ approaches $\pi/2$, as does $a_2(z = L, t)$. The variation of the time constant $\tau$ with $gL$ is shown in Fig. 3, where the data points represent calculated $\tau$ at a number of $gL$ values and the solid line represents this dependence by a function proportional to $1/\cos(gL)$, with a discontinuity at $gL = \pi/2$.

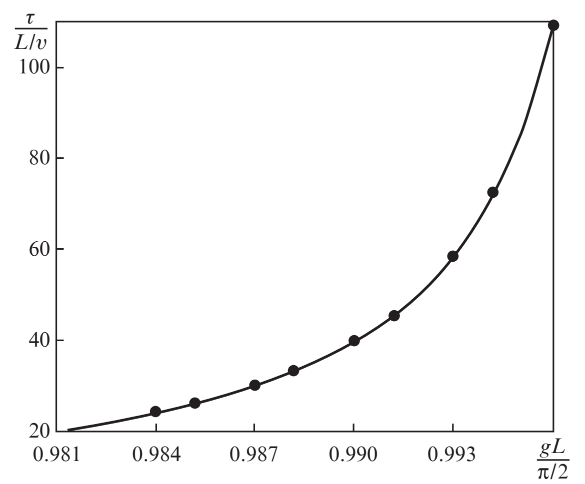


**Figure 3.** Calculated $\tau/(L/v)$ as a function of $gL$ (data points). The solid line represents a theoretical fit.

Thus, our results are the first to demonstrate that the transient processes involved in the parametric interaction of counter- and co-propagating waves differ drastically. The transient time for co-propagating waves is equal to the time needed for the pump wave front to pass through the medium. In the case of counter-propagating waves, the transient process can be described by the relation $a_2(z = L, t) \propto [1 - \exp(t/\tau)]$, well known in pulse engineering, and corresponds to a transient process in feedback systems [13] (for example, when an optical pulse having the resonance frequency of a resonator passes through it). The $gL$ product then plays the role of the resonance $Q$-factor, and the transient time considerably exceeds $L/v$. These distinctions are due to the existence of feedback and, hence, to spatiotemporal response nonlocality in the case of the parametric interaction of counter-propagating waves.

***Acknowledgements.*** This work was supported by the RF Ministry of Education and Science (State Research Task 2014–2016, Project Nos 2014/71, 2014/K and 3470) and the US Army Research Office (Grant No. W911NF-141-0619).

## References

1. Shelby R.A., Smith D.R., Schultz S. *Science*, **292**, 77 (2001).
2. Shalaev V.M., Cai W., et al. *Opt. Lett.*, **30**, 3356 (2005).
3. Drachev V.P., Cai W., et al. *Laser Phys. Lett.*, **3**, 49 (2006).
4. Bobroff D.L. *J. Appl. Phys.*, **36**, 1760 (1965).
5. Harris S.E. *Appl. Phys. Lett.*, **9**, 114 (1966).
6. Canalias C., Pasiskevicius V. *Nat. Photonics*, **1**, 459 (2007).
7. Popov A.K., Slabko V.V., Shalaev V.M. *Laser Phys. Lett.*, **3** (6), 293 (2006).
8. Popov A.K., Shalaev V.M. *Appl. Phys. B*, **84**, 131 (2006).
9. Shalaev M.I., Myslivets S.A., et al. *Opt. Lett.*, **36**, 3861 (2011).
10. Popov A.K., Shalaev M.I., et al. *Appl. Phys. A*, **115**, 523 (2013).
11. Popov A.K., Nefedov I.S., et al. *Proc. SPIE Int. Soc. Opt. Eng.*, **8725**, 87252E (2013).
12. Shadrivov I.V., Zharov A.A., Kivshar Y.S. *J. Opt. Soc. Am. B*, **23**, 529 (2006).
13. Gibbs H.M. *Optical Bistability: Controlling Light with Light* (Orlando: Academic, 1985; Moscow: Mir, 1988).