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CONTROL OF LASER RADIATION PARAMETERS

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Comparative analysis of frequency and noise characteristics of Fabry–Perot and distributed feedback laser diodes with external optical injection locking

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Abstract. **Using a system of coupled travelling wave equations, in the small-signal regime we analyse frequency and noise characteristics of index- or absorption-coupled distributed feedback laser diodes, as well as of Fabry–Perot (FP) laser diodes. It is shown that the weakest dependence of the direct modulation efficiency on the locking frequency in the regime of strong external optical injection locking is exhibited by a FP laser diode formed by highly reflective and antireflective coatings on the end faces of a laser structure. A reduction in the dependence of output characteristics of the laser diode on the locking frequency can be attained by decreasing the reflection coefficient of the antireflective FP mirror.**

Keywords: *external optical injection locking, coupled travelling wave equations, frequency response, relative intensity noise.*

1. Introduction

Theoretical and experimental studies have shown that an increase in the limiting frequency of direct modulation and a decrease in the noise intensity occur in the regime of external optical injection locking of a laser diode in fibre-optic communication systems [1–3]. In particular, the limiting frequencies of direct current modulation, exceeding 100 GHz [4], have been obtained in the regime of external optical injection locking of InGaAsP distributed feedback (DFB) laser diodes and vertical-cavity surface-emitting lasers, which allows avoiding the use of expensive external optical modulators even in communication systems with a data transmission rate of 100 Gbit s^{-1}. The regime of optical injection locking of a laser diode is attained by injection of the master light into its resonator, which makes the laser diode generate at the master frequency, the reverse effect on the master being ruled out by means of a nonreciprocal optical element, e.g., an optical circulator (Fig. 1).

One of the technical challenges that impede a widespread practical use of optical injection-locked laser diodes is a critical dependence of its output characteristics on the difference between the free-running frequencies of master and slave lasers, which is called frequency detuning. The aim of this study is to determine the type of a laser diode, the output characteristics of which, in the regime of external optical injection locking, minimally depend on the frequency detuning. To this end, this paper analyses frequency and noise characteristics of four types of laser diodes operating in the regime of external optical injection locking: 1) an index-coupled distributed feedback laser diode (DFB), 2) an absorption-coupled distributed feedback laser diode (DFB-A), 3) a laser diode with a Fabry–Perot resonator formed by mirrors due to Fresnel reflection at the ‘semiconductor–air’ interface (FP), and 4) a laser diode with a Fabry–Perot resonator formed by highly reflective and antireflective coatings on the end faces (FP HR/AR).

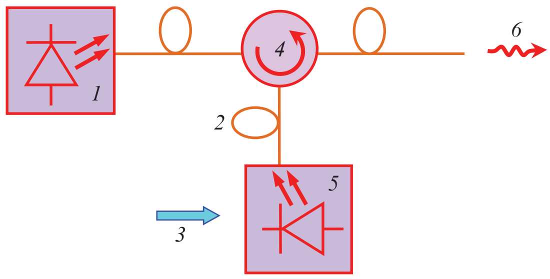

Figure 1. Scheme of external optical injection locking of a laser diode based on fibre-optic components:
(*1*) master laser diode; (*2*) polarisation-maintaining fibre; (*3*) microwave signal; (*4*) optical circulator; (*5*) slave laser diode; (*6*) intensity-modulated optical signal.

Typically, frequency and noise characteristics of laser diodes are analysed in the small-signal regime [1–3]. The use of a system of rate equations for the analysis of frequency characteristics is justified in the conditions of weak injection, when the direct modulation frequencies are much smaller than the intermode frequency separation of a resonator [5]. The features of laser resonators can also be taken into account in the framework of a distributed model [6–8]. In this work, a distributed model of the resonator is developed with regard to longitudinal inhomogeneity of the field and carrier concentration in the active region as applied to the regime of external optical injection locking.

2. A system of basic equations

We represent the electric field intensity at each point of the laser resonator as a sum of intensities of two electromagnetic waves propagating in opposite directions:

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$$E(x,t) = [a(x,t)\exp(\mathrm{i}k_\mathrm{b}x) + b(x,t)\exp(-\mathrm{i}k_\mathrm{b}x)] \times \exp(-\mathrm{i}\omega t)/\sqrt{2} + \mathrm{c.c.}, \quad (1)$$

where k_b is the basic propagation constant; ω is the circular frequency of the electromagnetic field in the laser resonator; and $a(x,t)$ and $b(x,t)$ are the slowly varying complex amplitudes of the electromagnetic wave in the laser resonator. Normalisation of the amplitudes of the electromagnetic waves can be conveniently chosen so that the photon density in a laser resonator, after averaging over spatial and temporal periods, acquires a simple form:

$$S(x,t) = \langle E^2(x,t)\rangle = |a(x,t)|^2 + |b(x,t)|^2. \quad (2)$$

In laser DFB-structures, it is convenient to choose the basic propagation constant k_b in accordance with a spatial dependence of the refractive index

$$\Delta n(x) = \Delta n_\mathrm{b}\sin|2k_\mathrm{b}(x - x_0)|. \quad (3)$$

Here, Δn_b is a variable component of the complex refractive index, and the point x_0 sets the position of a quarter-wave shift of the Bragg grating (if available).

The amplitudes $a(x,t)$ and $b(x,t)$ are found from the system of equations for the coupled waves [7]:

$$\begin{cases} \left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{v_\mathrm{g}\partial t}\right)a(x,t) = \mathrm{i}[k(\omega,x,t) - k_\mathrm{b}]a(x,t) \\ \qquad \pm\kappa_\mathrm{b}\exp(-2\mathrm{i}k_\mathrm{b}x_0)b(x,t), \\ \left(\dfrac{\partial}{\partial x} - \dfrac{\partial}{v_\mathrm{g}\partial t}\right)b(x,t) = -\mathrm{i}[k(\omega,x,t) - k_\mathrm{b}]b(x,t) \\ \qquad \pm\kappa_\mathrm{b}\exp(2\mathrm{i}k_\mathrm{b}x_0)a(x,t), \end{cases} \quad (4)$$

where $k(\omega,x,t)$ and v_g are, respectively, the complex propagation constant and the group velocity of the electromagnetic wave in the laser resonator; and $\kappa_\mathrm{b} = \Delta n_\mathrm{b}k_\mathrm{b}/2$ is the DFB coupling coefficient. The sign '+' is taken for the region $x > x_0$, and the sign '–' for the region $x < x_0$. For FP laser diodes, the coefficient κ_b is zero. With account for the internal loss coefficient ρ and gain $G(x,t)$, the complex propagation constant can be written as

$$k(\omega,x,t) = k_\mathrm{b} + (n_\mathrm{g} - n_\mathrm{b})(\omega - \omega_\mathrm{b})/c + [\mathrm{i}\rho + (\alpha + \mathrm{i})\,G(x,t)]/2. \quad (5)$$

Here, α is the linewidth enhancement factor; and n_g and n_b are, respectively, the group and phase refractive indices at the base frequency ω_b. The coordinate dependence of the gain is defined by the spatial distribution of charge carriers $N(x,t)$ and photon density $S(x,t)$:

$$G(x,t) = \frac{1}{1+\varepsilon S(x,t)}\begin{cases} gN_\mathrm{inv}\ln\left(\dfrac{N(x,t)}{N_\mathrm{inv}}\right), & N(x,t) \geqslant N_\mathrm{inv}, \\ g[N(x,t) - N_\mathrm{inv}], & N(x,t) < N_\mathrm{inv}, \end{cases} \quad (6)$$

where g is the differential gain; N_inv is the transparency carrier concentration; and ε is the nonlinear gain parameter. To determine the spatial distribution of charge carriers, a standard balance equation is used:

$$\frac{\partial N(x,t)}{\partial t} = \frac{j(t)}{e} - R(x,t) - v_\mathrm{g}G(x,t)S(x,t), \quad (7)$$

where j is the pump current density; $R(x,t) = N(x,t)/\tau$ is the recombination rate of nonequilibrium charge carriers, which is expressed through the effective charge carrier lifetime τ; and e is the electron charge.

Under the assumption that external injection locking is performed through mirror (*2*) located at the coordinate origin $x = 0$, and the laser resonator length is L (Fig. 2), the boundary conditions for the slowly varying complex amplitudes have the form

$$\begin{bmatrix} a_\mathrm{out}(t) \\ b_\mathrm{inj}(t) \end{bmatrix} = T_\mathrm{fr}\begin{bmatrix} a(0,t) \\ b(0,t) \end{bmatrix}, \quad \begin{bmatrix} b_\mathrm{out}(t) \\ 0 \end{bmatrix} = T_\mathrm{b}\begin{bmatrix} b(-L,t) \\ a(-L,t) \end{bmatrix}. \quad (8)$$

Here, $a_\mathrm{out}(t)$ and $b_\mathrm{out}(t)$ are the slowly varying complex amplitudes of the waves coming from the front mirror (*2*) and the rear mirror (*1*) of the laser diode; $b_\mathrm{inj}(t)$ is the slowly varying complex amplitude of injection locking; and T_fr and T_b is the matrix of transformation of the complex amplitudes of intracavity waves into the complex amplitudes of outgoing and incoming waves. In particular, for a reflector at the interface between a semiconductor with a refractive index n_b and air, the transform matrix has the form

$$T = \frac{1}{2n_\mathrm{b}}\begin{bmatrix} 1+n_\mathrm{b} & 1-n_\mathrm{b} \\ 1-n_\mathrm{b} & 1+n_\mathrm{b} \end{bmatrix}. \quad (9)$$

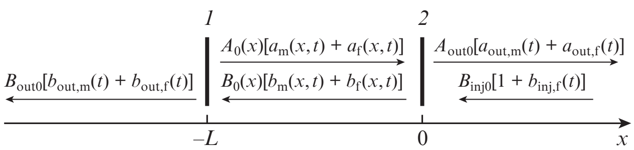

Figure 2. Scheme of a laser resonator with the coordinate system used in the paper, and notations of the stationary, modulation and noise amplitudes; (*1*, *2*) mirrors, L is resonator length.

For the HR- and AR-coated faces, the transform matrix can be calculated by means of the method of transfer matrices [9]. Solving jointly the system of equations (4) and (7) with the boundary conditions (8) in the case of injection of monochromatic light with the frequency ω_inj and amplitude $b_\mathrm{inj}(t) = B_\mathrm{inj0}$, one can find stationary distributions of the charge carrier concentration $N_0(x)$, gain $G_0(x)$, recombination rate $R_0(x)$, photon density $S_0(x)$ and slowly varying complex amplitudes $A_0(x)$ and $B_0(x)$ in the laser resonator. The output power is calculated using the expression:

$$P_\mathrm{out} = \hbar\omega_\mathrm{inj}v_\mathrm{g}W|A_\mathrm{out0}|^2/n_\mathrm{b}, \quad (10)$$

where W is the active region width of the laser diode. The laser diode parameters used in the simulation are given below.

Injection wavelength λ_inj/μm	1.55
Laser diode resonator length L/μm	300
Laser diode resonator width W/μm	2.5
Resonator background refractive index n_0	3.3
Resonator group refractive index n_g	3.8
Bragg refractive index Δn_b for DFB	0.0025
for DFB-A	–0.0015i

Effective lifetime of charge carriers τ/ns 1
Differential gain g/cm . . . . 5×10^{-11}
Transparency carrier concentration $N_{\rm inv}$/cm^{-2} . . . . . 5×10^{11}
Nonlinear gain ε/cm^2 . . . . 10^{-12}
Linewidth enhancement factor α 3
Contribution of spontaneous transitions to the laser mode $\beta_{\rm sp}$ . . . . 3×10^{-5}

The locking condition indicates that, firstly, a stationary solution of (4) and (7) exists for a given frequency of external injection $\omega_{\rm inj}$, and, secondly, the excitation level of the active region is not sufficient to generate self-radiation, i.e. the slave operates as an amplifier of external light. To test the second condition in FP laser diodes in the framework of the distributed model, one can compare the amplification averaged over the active region with the loss coefficient. However, this method is not suitable for DBF laser diodes. To determine whether the self-excitation of the resonator eigenmodes occurs, let us calculate them for a given distribution of the carrier concentration, obtained in the regime of external injection locking, with the oscillation frequency acting as a complex eigenvalue $\omega = \omega' + {\rm i}\omega''$ [9]. Using the imaginary part ω'' of the complex frequency, which determines damping of the electromagnetic mode in the resonator, it is possible to express the difference between the effective coefficient $G_{\rm eff}$ of the mode gain and the loss coefficient:

$$\Delta G_{\rm eff} = -2\omega''/v_{\rm g}. \qquad (11)$$

In the regime of external optical injection locking, this value should remain negative for all modes of the laser resonator.

The coefficient $\kappa_{\rm inj}$ of coupling with external injection is determined by a change in the resonator effective gain when injecting low-power external light with the frequency $\omega_{\rm inj}$ equal to a frequency of a free-running laser diode:

$$\kappa_{\rm inj} = \frac{\sqrt{1+\alpha^2}}{2}\frac{{\rm d}G_{\rm eff}}{{\rm d}B_{\rm inj0}}\sqrt{S_0}. \qquad (12)$$

To find dynamic solutions, it is convenient to present the field amplitudes in the form of a product of stationary and relative dynamic amplitudes

$$a(x,t) = A_0(x)a_{\rm m}(x,t), \quad b(x,t) = B_0(x)b_{\rm m}(x,t), \qquad (13)$$

then the relative dynamic amplitudes $a_{\rm m}(x,t)$ and $b_{\rm m}(x,t)$ can be derived from the system of equations:

$$\begin{cases} \left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{v_{\rm g}\partial t}\right)a_{\rm m}(x,t) = \dfrac{1-{\rm i}\alpha}{2}[G(x,t)-G_0(x)]a_{{\rm m}(x,t)} \\ \quad \pm\kappa_{\rm b}\dfrac{B_0(x)}{A_0(x)}\exp(-2{\rm i}k_b x_0)[b_{\rm m}(x,t)-a_{\rm m}(x,t)], \\ \left(\dfrac{\partial}{\partial x} - \dfrac{\partial}{v_{\rm g}\partial t}\right)b_{\rm m}(x,t) = -\dfrac{1-{\rm i}\alpha}{2}[G(x,t)-G_0(x)]b_{\rm m}(x,t) \\ \quad \pm\kappa_{\rm b}\dfrac{A_0(x)}{B_0(x)}\exp(2{\rm i}k_b x_0)[a_{\rm m}(x,t)-b_{\rm m}(x,t)], \end{cases} \qquad (14)$$

which is equivalent to equations [7] written in terms of the amplitude and phase of light.

3. Frequency response in the small-signal regime

To calculate the frequency response of a laser diode with external optical injection locking, consider the injection current modulation in the small-signal regime:

$$j(t) = j_0 + j_{\rm m}\exp({\rm i}\Omega t) + j_{\rm m}^*\exp(-{\rm i}\Omega t), \qquad (15)$$

where Ω is the modulation frequency; j_0 is the average pump current; and $j_{\rm m}$ is the modulation component of the pump current ($|j_{\rm m}| \ll j_0$). In this case, a linearised equation for the carrier concentration balance appears as

$$\left({\rm i}\Omega + \frac{\partial R_0(x)}{\partial n}\right)N_{\rm m}(x,\Omega) = \frac{j_{\rm m}}{e} - v_{\rm g}S_0(x)G_{\rm m}(x,\Omega) - v_{\rm g}G_0(x)S_{\rm m}(x,\Omega), \qquad (16)$$

where $N_{\rm m}(x,\Omega)$, $G_{\rm m}(x,\Omega)$ and $S_{\rm m}(x,\Omega)$ are the distributions of the complex modulation components of the charge carrier concentration, gain and photon density, respectively;

$$\begin{aligned} N(x,t) &= N_0(x) + N_{\rm m}(x,\Omega)\exp({\rm i}\Omega t) + N_{\rm m}^*(x,\Omega)\exp(-{\rm i}\Omega t), \\ S(x,t) &= S_0(x) + S_{\rm m}(x,\Omega)\exp({\rm i}\Omega t) + S_{\rm m}^*(x,\Omega)\exp(-{\rm i}\Omega t), \\ G_{\rm m}(x,\Omega) &= \frac{\partial G_0(x)}{\partial N}N_{\rm m}(x,\Omega) + \frac{\partial G_0(x)}{\partial S}S_{\rm m}(x,\Omega). \end{aligned} \qquad (17)$$

In this case, the relative dynamic amplitudes can be sought for in the form

$$\begin{aligned} a_{\rm m}(x,t) &= 1 + A_{\rm m1}(x,\Omega)\exp({\rm i}\Omega t) + A_{\rm m2}^*(x,\Omega)\exp(-{\rm i}\Omega t), \\ b_{\rm m}(x,t) &= 1 + B_{\rm m1}(x,\Omega)\exp({\rm i}\Omega t) + B_{\rm m2}^*(x,\Omega)\exp(-{\rm i}\Omega t), \end{aligned} \qquad (18)$$

where $A_{\rm m1,2}$ and $B_{\rm m1,2}$ are the modulation components of relative dynamic amplitudes of electromagnetic waves in the laser resonator. The modulation component of the photon density can be expressed through the values $A_{\rm m1,2}$ and $B_{\rm m1,2}$ in the following way:

$$S_{\rm m}(x,\Omega) = |A_0(x)|^2[A_{\rm m1}(x,\Omega) + A_{\rm m2}(x,\Omega)] + |B_0(x)|^2[B_{\rm m1}(x,\Omega) + B_{\rm m2}(x,\Omega)]. \qquad (19)$$

The efficiency of the laser diode modulation by the pump current in the small-signal regime is defined as

$$\eta(\Omega) = \frac{\hbar\omega_{\rm inj}v_{\rm g}}{n_{\rm b}L}\frac{|A_{\rm out0}|^2|A_{\rm out,m1}(\Omega) + A_{\rm out,m2}(\Omega)|}{|j_{\rm m}|}. \qquad (20)$$

It should be clarified that equation (16) for the balance of carriers does not account for peculiarities of the carrier transfer and capturing carriers into the quantum wells. In the first approximation, such processes can be taken into account in the modulation characteristics by means of replacement of j_m with $j_m/(1 + \mathrm{i}\Omega\tau_c)$, where τ_c is effective time of capturing carriers into the quantum wells. If we assume that the processes of current injection in all the laser structures under consideration are identical, the presence of an additional factor does not affect the comparison results.

Stability of the solutions to the system of equations (14), (16) was tested in accordance with the Lyapunov criterion. In equations (16)–(18), $\mathrm{i}\Omega$ was replaced by z, and the current modulation density j_m was assumed equal to zero. A determinant of the linear system (14), (16) was used to form a characteristic equation for deriving the characteristic numbers z. A solution was considered unstable if at least one root with a positive real part was present.

4. Noise characteristics

Amplitude and phase fluctuations of light at the slave laser output are determined by the fluctuations in the most active area of the slave laser (intrinsic noises), and also by the amplitude and phase fluctuations of injected light of the master (amplified noises of external injection locking).

Intrinsic noises. The sources of amplitude fluctuations of an electromagnetic wave in the laser resonator appear in the wave equation as polarisation fluctuations of the active medium. The size of quantum oscillators interacting with the electromagnetic field in the resonator is determined by the mean free path of charge carriers (less than 1 μm), which is much less than the resonator length; therefore, a coordinate dependence of noise sources in the form of the delta-function can be used in calculations. If the quantum noise function $f(t)$ is taken the same as in the rate equation for the field amplitude, it can be incorporated as $2(k_bL/v_g)f(t)\delta(x - x')$ into the original wave equation for the amplitude. Next, the noise characteristics should be averaged over all possible positions x' of the quantum oscillator inside the resonator. A solution of the wave equation with the delta function in the right-hand side may be represented in the form of a 'sewing' of two solutions of the homogeneous wave equation, which maintains the continuity of the noise field amplitude $E_f(x,t)$, whilst its spatial derivative has a discontinuity:

$$\left.\frac{\partial E_f(x,t)}{\partial x}\right|_{x\to x'+0} - \left.\frac{\partial E_f(x,t)}{\partial x}\right|_{x\to x'-0} = 2\frac{k_bL}{v_g}f(x',t). \quad (21)$$

In the approximation of the slowly varying amplitude of the noise, similar to (1), (18), we may assume that

$$\frac{\partial E_f(x,t)}{\partial x} = \frac{\mathrm{i}k_b}{\sqrt{2}}[a_f(x,t)A_0(x)\exp(\mathrm{i}k_bx)$$
$$- b_f(x,t)B_0(x)\exp(-\mathrm{i}k_bx)]\exp(-\mathrm{i}\omega t) + \text{c.c.} \quad (22)$$

The time dependence of noise sources at the frequency $\omega - \Omega$ in the narrow spectral range $\Delta\Omega$ can be represented as

$$f(x,t) = \frac{\mathrm{i}}{\sqrt{2}}[F(x,\Omega)\exp(-\mathrm{i}(\omega-\Omega)t)$$
$$- F^*(x,\Omega)\exp(\mathrm{i}(\omega-\Omega)t)]\sqrt{\Delta\Omega}. \quad (23)$$

To find frequency components of the slowly varying noise amplitudes $A_f(x,\Omega)$ and $B_f(x,\Omega)$, a system of equations and the substitutions similar to (14), (16) can be used, with replacing the subscript 'm' by 'f'. Then, given the continuity of $E_f(x,t)$, condition (21) can be written as

$$A_0(x)[A_{f1}(x,\Omega)|_{x\to x'+0} - A_{f1}(x,\Omega)|_{x\to x'-0}]$$
$$= \frac{L}{v_g}F^{(1)}(x',\Omega)\exp(-\mathrm{i}k_bx')\sqrt{\Delta\Omega}, \quad (24)$$

$$B_0(x)[B_{f1}(x,\Omega)|_{x\to x'+0} - B_{f1}(x,\Omega)|_{x\to x'-0}]$$
$$= -\frac{L}{v_g}F^{(1)}(x',\Omega)\exp(\mathrm{i}k_bx')\sqrt{\Delta\Omega}. \quad (25)$$

Here, $F^{(1)}(x,\Omega) = F(x,\Omega)$. For calculating the amplitudes A_{f2} and B_{f2}, the complex-conjugate conditions (24) and (25) along with the independent source of noise $F^{(2)}(x,\Omega) = F^*(x,-\Omega)$ should be used. To account for quantum noises in the balance of charge carriers, we may take advantage of the linearised equation (16), with replacing the modulation component of the pump current j_m/e by the noise component $F^{(N)}(x,\Omega)L\delta(x-x')$.

Given the three considered noise sources $F^{(1)}(x,\Omega)$, $F^{(2)}(x,\Omega)$ and $F^{(N)}(x,\Omega)$ and their relevant slowly varying noise amplitudes, the amplitude and phase fluctuations of output light can be represented as

$$\frac{S_{\text{out,f}}(t)}{S_{\text{out0}}\sqrt{\Delta\Omega}} = \sum_{j=1,2,\text{N}}[A^{(j)}_{\text{out,f1}}(\Omega) + A^{(j)}_{\text{out,f2}}(\Omega)]\exp(\mathrm{i}\Omega t) + \text{c.c.}, \quad (26)$$

$$\frac{\varphi_{\text{out,f}}(t)}{\sqrt{\Delta\Omega}} = \frac{1}{2\mathrm{i}}\sum_{j=1,2,\text{N}}[A^{(j)}_{\text{out,f1}}(\Omega) - A^{(j)}_{\text{out,f2}}(\Omega)]\exp(\mathrm{i}\Omega t) + \text{c.c.} \quad (27)$$

Consequently, the spectral power of the amplitude noise, the phase noise and their correlations take the form

$$\mathrm{RIN}(\Omega) = 2\frac{\langle S^2_{\text{out,f}}(t)\rangle}{S^2_{\text{out0}}\Delta\Omega} = 4\sum_{j,j'=1,2,\text{N}}\Big\langle[A^{(j')*}_{\text{out,f1}}(\Omega) + A^{(j')*}_{\text{out,f2}}(\Omega)]$$
$$\times[A^{(j)}_{\text{out,f1}}(\Omega) + A^{(j)}_{\text{out,f2}}(\Omega)]\Big\rangle, \quad (28)$$

$$L_\varphi(\Omega) = 2\frac{\langle\varphi^2_{\text{out,f}}(t)\rangle}{\Delta\Omega} = \sum_{j,j'=1,2,\text{N}}\Big\langle[A^{(j')*}_{\text{out,f1}}(\Omega) - A^{(j')*}_{\text{out,f2}}(\Omega)]$$
$$\times[A^{(j)}_{\text{out,f1}}(\Omega) - A^{(j)}_{\text{out,f2}}(\Omega)]\Big\rangle, \quad (29)$$

$$K(\Omega) = 2\frac{\langle S_{\text{out,f}}(t)\varphi_{\text{out,f}}(t)\rangle}{S_{\text{out0}}\Delta\Omega}$$
$$= -2\mathrm{i}\sum_{j,j'=1,2,\text{N}}\Big\langle[A^{(j')*}_{\text{out,f1}}(\Omega) + A^{(j')*}_{\text{out,f2}}(\Omega)]$$
$$\times[A^{(j)}_{\text{out,f1}}(\Omega) - A^{(j)}_{\text{out,f2}}(\Omega)]\Big\rangle. \quad (30)$$

Here, $\langle\ldots\rangle$ means averaging, while the additional factor of 2 arises when using the physical scale of frequencies Ω in the frequency range $[0,+\infty)$. In averaging, we use the spectral power of the quantum noise in the semiclassical representation

$$\langle F^{(j)*}(x,\Omega)F^{(j')}(x,\Omega)\rangle = \beta_{sp}R_0(x)\delta_{jj'}, \quad j,j' = 1,2, \quad (31)$$

$$\langle F^{(\text{N})*}(x,\Omega)F^{(\text{N})}(x,\Omega)\rangle = \frac{1}{LW}\left[\frac{j_0}{e} + R_0(x)\right]$$
$$+ 2\beta_{sp}R_0(x)S_0(x), \quad (32)$$

$$\langle F^{(N)*}(x,\Omega)\,F^{(1)}(x,\Omega)\rangle = -\beta_{sp}R_0(x)$$
$$\times[A_0(x)\exp(ik_bx) + B_0(x)\exp(-ik_bx)], \qquad (33)$$
$$\langle F^{(N)*}(x,\Omega)\,F^{(2)}(x,\Omega)\rangle = -\beta_{sp}R_0(x)$$
$$\times[A_0^*(x)\exp(-ik_bx) + B_0^*(x)\exp(ik_bx)], \qquad (34)$$

where β_{sp} is the contribution of spontaneous transitions to the laser mode. After averaging the output characteristics over the coordinate, due to the presence of rapidly oscillating spatial factors in the right-hand sides of equations (26), (27), (33) and (34), we come to the same result as if the noise sources for the forward and backward waves were regarded as independent.

Amplified noises of external injection locking. The formation of the relative noise power at the frequency Ω involves the use of spectral components of injected light at the frequencies $\omega\pm\Omega$, which correspond to the relative noise amplitudes $B_{inj,f1}(\Omega)$ and $B_{inj,f2}(\Omega)$. Let us introduce a matrix of the relative amplitude coefficients of the laser diode amplification in the small-signal injection regime:

$$M_{ij}(\Omega) = \partial A_{out,mi}/\partial B_{inj,fj}, \quad i,j = 1, 2. \qquad (35)$$

The nondiagonal coefficients M_{ij} are due to the nonlinearity of the original system of equations and describe a conversion of the laser frequency $\omega-\Omega$ into the frequency $\omega+\Omega$. The relative power of the amplified noise of external injection locking can be written in the form

$$\mathrm{RIN_{inj}}(\Omega) = 4\langle|[M_{11}(\Omega) + M_{21}(\Omega)]B_{inj,f1}(\Omega)$$
$$+[M_{12}(\Omega) + M_{22}(\Omega)]B_{inj,f2}(\Omega)|^2\rangle. \qquad (36)$$

Considering expressions (28)–(30) as applied to injected light, we can see that, in the general case, the relative power of amplified noises represents a linear combination of the relative power of noise, the mean-square phase fluctuation and a correlation of the amplitude and phase noises of injected light. This is due to nonequivalence of injection amplification at the frequencies $\omega\pm\Omega$.

5. Results and discussion

A comparison of the laser diodes in the optical injection locking regime is performed at identical injection currents. The characteristics of optical injection locking depend substantially on an internal injection ratio equal to the ratio of photon densities being injected and accumulated in the free-running resonator; therefore, the coefficients of internal losses of different structures are chosen in such a way that the threshold currents approximately coincide, and, at equal injection currents inside the resonator, equal average photon densities are generated inside the resonator. In practice, it is more convenient to use an external injection ratio equal to the ratio of injected and output laser power in the free-running regime.

Estimates have shown that the diode of FP HR/AR type (Table 1) possesses the biggest coupling factor with respect to external light. In this type of laser diode, in contrast to a simple FP, external light penetrates easily into the resonator cavity through the antireflective coating and interacts with the laser mode. In the DFB and DFB-A laser diodes with antireflection coatings, Bragg reflection prevents penetration of external light into the resonator centre, where main energy of the laser mode is concentrated. A larger value of the coefficient κ_{inj} that characterises coupling with external light should provide a greater range of frequency detuning for which optical injection locking can be implemented. Numerical solution of the systems of equations (4) and (7) and the calculations of injection locking region versus the frequency detuning of injected light and injection power confirm these estimates (Fig. 3), which is consistent with the results of analysis [10]

Table 1. Calculated parameters of lasers with different types of structures.

Type of structure	$\lvert r_1\rvert$	$\lvert r_2\rvert$	Threshold current I_{th}/mA	ρ/cm^{-1}	κ_{inj}/cm^{-1}	Low-frequency modulation efficiency η_0 in the free-running regime/W A^{-1}
DFB	0.22	0.22	5.8	15	16	0.27
DFB-A	0.96	0.22	5.5	55	22	0.42
FP	0.53	0.53	5.8	15	15	0.29
FP HR/AR	0.96	0.22	5.8	4	32	0.69

Note: $\lvert r_1\rvert$ and $\lvert r_2\rvert$ are the amplitude reflection coefficients of the 1st and 2nd mirrors.

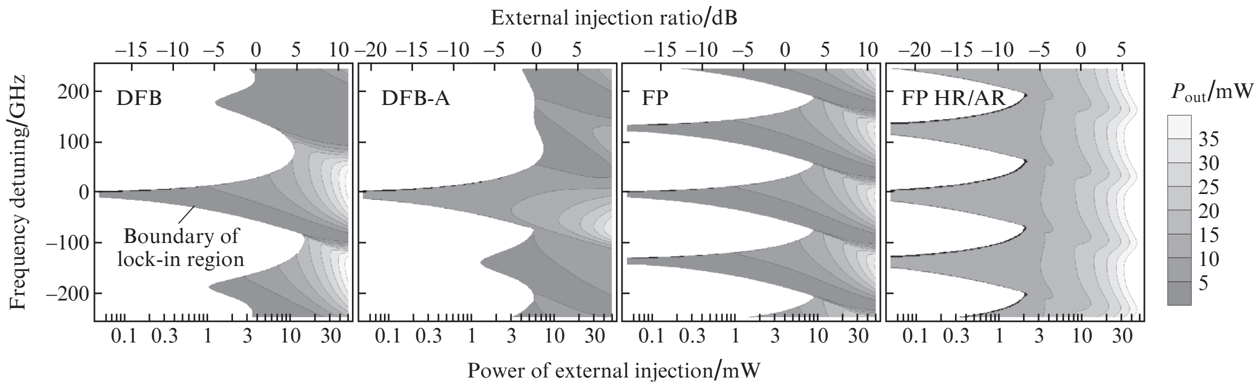

Figure 3. Dependence of the laser diode output power P_{out} at the pump current of 20 mA in the regime of external optical injection locking on the power of optical injection and frequency detuning of external injection from the eigenfrequency. The region of unstable generation is marked by black.

showing that the locking band is significantly increased with a decrease in the DFB coefficient.

In FP laser diodes, the structure of the boundary of injection locking region represents a pattern that is repeated with a period equal to the intermode frequency separation, i.e. all the modes are virtually equivalent. The eigenmodes in the DFB laser diodes are not equidistant so that the fundamental mode is easier to lock-in (with a lesser optical injection power) than the adjacent modes. A minimal laser power (we call it critical), at which the laser diode is locked-in at any frequency detuning of external injection, for the diode of FP HR/AR type turns out several times less than in other structures. The output power of stationary oscillation in the DFB, DFB-A and FP laser diodes, in consequence of interference effects of reflected and generated laser output, can be reduced to almost zero, herewith the entire light exits through the opposite face of the resonator. Complete interferential damping of the output power does not occur in the laser diode of FP HR/AR type (Fig. 3). Spontaneous pulsations of light in the injection locking regime are realised if the power value is less than a critical one, with the gain verging towards its threshold. When the injection power exceeds significantly its critical value, external injection lowers the inversion level so that the gain derived from the slave turns out significantly below the threshold, which contributes to stable generation.

The efficiency of direct modulation and the relative intensity noise of the slave laser diode possess a resonance at the modulation frequencies corresponding to the frequency detuning of external injection locking from the nearest eigenmode of the slave (Figs 4a, 4b). The near-resonance region is peculiar of a significant increase in the noises of injected light (Fig. 4c). Because of the outrunning growth of the relative noise intensity compared to the modulation efficiency, the

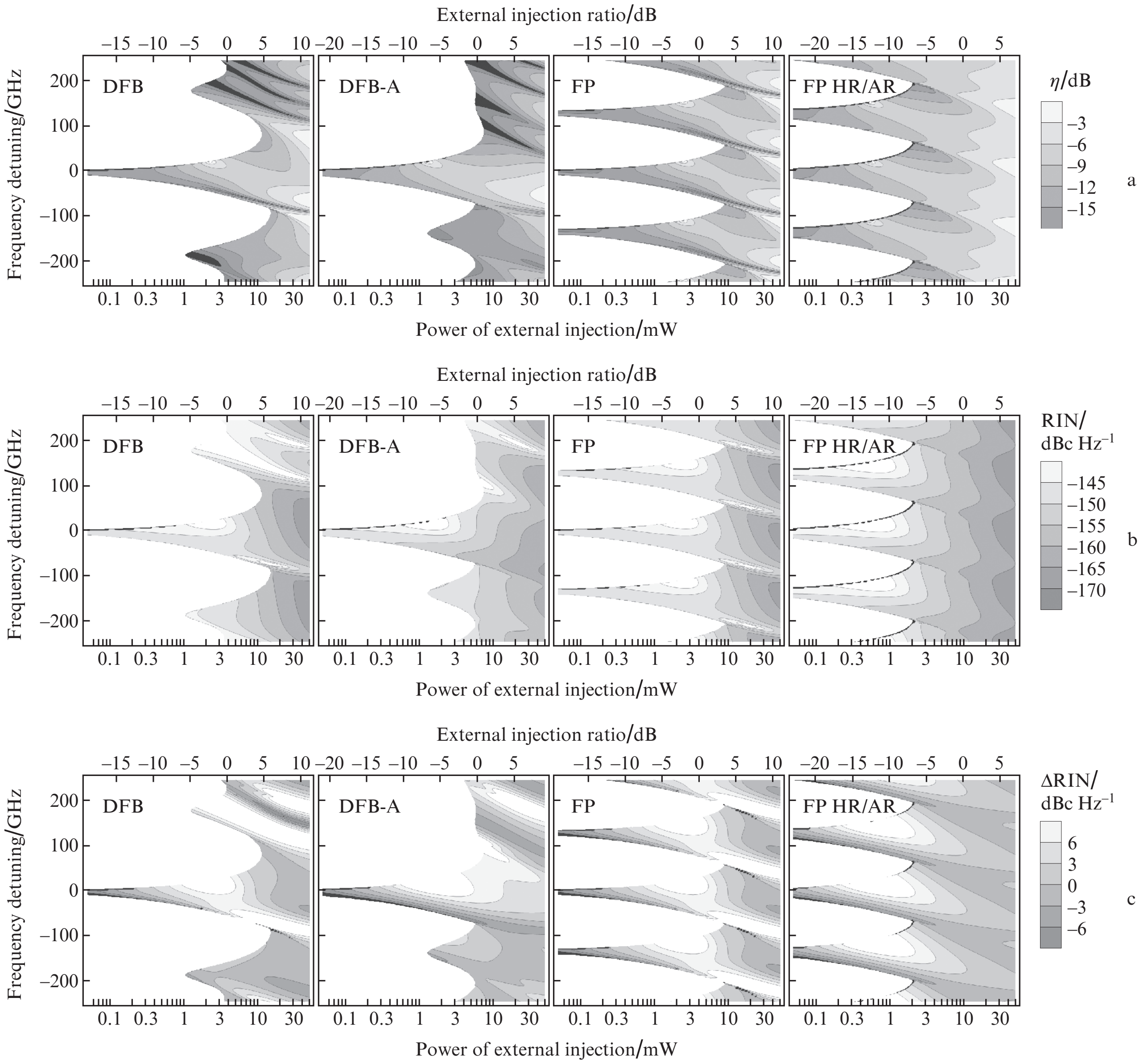

Figure 4. Efficiency η of direct modulation of the laser diode, normalised to the low-frequency modulation efficiency in the free-running regime (a), the relative intensity noise of the laser diode (b), and the amplification of external noise of the laser diode (c) at a frequency of 25 GHz with external optical injection locking vs. injected power and frequency detuning of external light from the laser eigenfrequency at the pump current of 20 mA.

signal-to-noise ratio is deteriorated when approaching the resonance.

When the power of external injection locking exceeds a critical value, the maximal efficiency of direct modulation of the laser diode at a given frequency increases with increasing power of the master laser diode, while the relative intensity noise is reduced, and thus the signal-to-noise ratio is enhanced. Herewith the characteristics are not resonant, and the requirement of maintaining a desired accuracy of the frequency difference of the master and the slave is reduced from a few to tens of gigahertz. In the DFB, DFB-A and FP laser diodes, the efficiency of direct modulation may decrease down to zero, while the relative intensity noise may significantly rise near the regions of interferential damping of the output power. The region of minimal relative noise power, depending on the frequency detuning, coincides with the region of maximal output power. The frequency bands of maximal modulation efficiency and maximal output power approximately coincide only in the DFB-A structure. Minimal intrinsic noises for various lasers with an injection power over 20 mW are contained in the range from –165 to –170 dBc Hz^{-1} (Fig. 4b). The noises in injected light are transmitted into the output, varying in the range of ±3 dBc Hz^{-1}; at typical noises of injected light in the range from –150 to –160 dBc Hz^{-1}, they provide a dominating contribution to the resultant relative noise power (Fig. 4c).

In the regime of strong optical injection, the modulation efficiency in the FP HR/AR laser diode, unlike other structures, is not reduced to zero at any frequency detuning. Herewith, the variation range of the relative modulation efficiency at the frequency of 25 GHz only constitutes about ±1.5 dB. Calculations show that if the reflection coefficient of a coated reflector is reduced, thus reducing the Q-factor and selective properties of the resonator, it is possible to further reduce the dependence of characteristics on the frequency difference of the master and slave lasers. In particular, when decreasing the reflection coefficient of an antireflective mirror from 5% to 1% or 0.01% and with other laser parameters being the same (including the current and power of injection), the variation range of the relative modulation efficiency at the frequency of 25 GHz is reduced down to ±0.7 or ±0.05 dB, respectively. Moreover, for most applications, the necessity in maintaining a predetermined frequency detuning may totally disappear. It should also be noted that a decrease in the reflection coefficient of an antireflective mirror leads to an increase in the threshold of the slave oscillation, which can eliminate the generation without optical injection.

The range of the modulation efficiency variation for the FP HR/AR laser diode also has a minimum dependence on the injection current (15 mA) and resonator length (320 μm). With an increase in the pump current, a weakly expressed decrease in the maximal modulation efficiency occurs, depending on the frequency detuning. This is due to the fact that, in the regime of strong optical injection, the photon density in the resonator is determined to a larger degree by the injected light rather than by the pump current.

6. Conclusions

We have simulated characteristics of different laser diodes in the optical injection locking regime, based on a distributed model of the resonator, with account for the longitudinal field inhomogeneity and carrier concentration in the active region. It is shown that the output power of stationary generation in the laser diodes with identical reflectors or DFB, depending on the frequency detuning determined by the interference effects of reflected and generated light, may be reduced to virtually zero. In the FP laser diode with different mirrors, complete interferential damping of the output power does not occur. A resonant increase in modulation efficiency is observed in the regime of low and moderate optical injection near the boundary of injection locking region, the noise power increasing significantly.

In the regime of strong optical injection, frequency characteristics are not resonant, and the noise intensity is practically limited only by the noise of external injection locking. The enhanced modulation efficiency and reduced relative noise power are peculiar to the laser diode with a Fabry– Perot resonator formed by HR and AR coatings on the end faces of the laser structure. Reducing the dependence of output characteristics on the frequency difference of the master and slave lasers can be attained by reducing the reflectance of the antireflective mirror.

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