PACS numbers: 37.10.Rs; 52.38.–r
DOI: 10.1070/QE2015v045n11ABEH015856

Laser cooling of electron–ion plasma in the case of optimal scanning of the laser frequency

A.P. Gavrilyuk, I.L. Isaev

Abstract. **Laser cooling of ions of electron–ion plasma is studied under the action of spontaneous radiation pressure forces. It is shown that the use of a constant detuning of the laser frequency from the quantum transition frequency ω_0 in ions significantly limits the conditions under which the ions are cooled. To extend the range of initial temperatures of possible cooling of ions and to increase the cooling efficiency we suggest scanning the laser frequency detuning so that the cooling rate remained maximal in the process of changing the temperature of ions. In the case of an optimal detuning, we have found an asymptotic expression for the cooling rate and identified intervals of electron concentrations and temperatures, where cooling of ions is possible.**

Keywords: *laser cooling, nonideal electron–ion plasma, frequency scanning.*

1. Introduction

Generation of plasma in a highly nonideal state and study of its properties have for a long time attracted the attention of many researchers [1, 2]. A promising object of research may be ultracold electron–ion plasma with a particle temperature $T \leqslant 100$ K and concentration $n \leqslant 10^{10}$ cm^{-3}. The possibility of generating such a plasma was first demonstrated in experimental work [3–5]. Earlier Gavrilyuk et al. [6, 7] proposed the use of laser cooling of ions under the action of the spontaneous radiation pressure forces to produce strongly nonideal plasma. They also showed that in the quasi-stationary plasma it is possible to achieve conditions of Wigner crystallisation of the ionic subsystem. Kilian et al. [3–5] stimulated a next series of investigations [8–16] of the properties of ultracold plasma produced using a near threshold photoionisation of 'cold' ($T \approx 10^{-5}$–10^{-4} K) atoms. In particular, these studies have shown that the fast relaxation of electrons and ions to an equilibrium energy distribution and the subsequent three-body recombination do not make it possible to reach a high degree of nonideality that is necessary to trigger crystallisation of an electronic or ionic subsystem. To solve this problem, Pohl et al. [14, 15] also considered the possibility of applying the methods of laser cooling of ions under the action of spontaneous radiation pressure forces. At the same time, as in [6, 7], so in [14, 15] the authors used a simplified (linear approximation) expression for the force, which is valid only in the case of slow ions:

$$k\langle|v_{ix}|\rangle \ll \gamma/2, \quad k = \omega/c,$$

where k is the wave number of laser radiation; v_{ix} is the projection of the ion velocity on the x axis; γ is the rate of spontaneous decay of the excited state of the ion; ω is the frequency of laser radiation; and c is the velocity of light. Already at temperatures of ions $T_{\rm i} \geqslant 0.01$ K, the use of the simplified expression leads to significant errors in the description of the dynamics of ion cooling [17]. Therefore, to study the process of plasma cooling at very low temperatures of ions, it is needed to use a more general, velocity nonlinear, expression for the spontaneous radiation pressure force. Moreover, the implementation of cooling of ions in a wide range of initial temperatures at a constant detuning of the laser frequency is not possible, because in this case, cooling occurs only in a limited range of ion velocities.

We can expect that effective cooling (until a nonideal state) of initially not very cold plasma ions is possible by scanning the laser frequency during a change in their temperature. This paper examines this possibility based on a model that takes into account the process of laser cooling of ions and the basic mechanisms of their heating.

2. Model of cooling of ions

In a weak ($|V_0| \ll \gamma$) standing light wave with frequency ω and wave vector along the x axis, the force of the spontaneous radiation pressure [18], acting on the ion along the x axis, is:

$$F_{\rm las}(v_{\rm ix}) = -\chi(v_{\rm ix})v_{\rm ix},$$

$$\chi(v_{ix}) = \frac{\hbar k^2\gamma|V_0|^2|\Delta|}{[(\Delta - kv_{\rm ix})^2 + g^2][(\Delta + kv_{\rm ix})^2 + g^2]}, \qquad (1)$$

$$g^2 = 0.5|V_0|^2 + 0.25\gamma^2.$$

where $\hbar$ is Planck's constant; $|V_0|$ is the Rabi frequency of the standing wave; ω_0 is the quantum transition frequency; and $\Delta = \omega - \omega_0$ is the frequency detuning ($\Delta < 0$, $|\Delta| \ll \omega_0$). Accordingly, the loss of the kinetic energy of ions per unit time due to laser cooling is described by

A.P. Gavrilyuk Institute of Computational Modelling, Siberian Branch, Russian Academy of Sciences, Akademgorodok 50, str. 44, 660036 Krasnoyarsk, Russia; Siberian Federal University, prosp. Svobodnyi 79, 660041 Krasnoyarsk, Russia;
e-mail: glyukonat@icm.krasn.ru;
I.L. Isaev Institute of Computational Modelling, Siberian Branch, Russian Academy of Sciences, Akademgorodok 50, str. 44, 660036 Krasnoyarsk, Russia; e-mail: 1@crownet.ru

Received 8 May 2015; revision received 25 June 2015
Kvantovaya Elektronika **45** (11) 1037–1042 (2015)
Translated by I.A. Ulitkin

$$Q_{\mathrm{las}} = \int_{-\infty}^{\infty} F_{\mathrm{las}}(v_{\mathrm{ix}}) v_{\mathrm{ix}} f(v_{\mathrm{ix}}, T_{\mathrm{i}}) \mathrm{d}v_{\mathrm{ix}}, \quad (2)$$

where $f(v_{ix}, T_i)$ is the function of the ion velocity distribution, which is assumed close to the Maxwellian (see the Appendix); T_i is the temperature of the ions determined by their average kinetic energy $\varepsilon_i = 3k_BT_i/2$; and k_B is the Boltzmann constant. In addition to cooling, the laser field is characterised by a fluctuation heating of ions due to the random nature of the spontaneous decay of the excited state and the absorption of laser radiation, which is described by the expression [18]

$$Q_{\mathrm{f}} = \frac{(\hbar k)^2 \gamma |V_0|^2}{2M(\Delta^2 + 0.25\gamma^2)}, \quad (3)$$

where M is the mass of the ion. Fluctuation heating plays a significant role at super-low temperatures of the ions ($T_i \leqslant 10^{-3}$ K).

One of the main causes of the heating of the cooled plasma ions is the energy exchange with electrons in elastic collisions:

$$Q_{\mathrm{ei}} = \frac{2m}{M} \nu_{\mathrm{ei}} (\varepsilon_{\mathrm{e}} - \varepsilon_{\mathrm{i}}), \quad (4)$$

where m is the mass of the electron; ν_{ei} is the rate of elastic electron–ion collisions; and ε_e is the average kinetic energy of the electrons.

Another reason for the heating of ions can be correlation heating (or disorder-induced heating) [8], which manifests itself in the initial random distribution of ions formed as a result of fast photoionisation of cooled atoms ($T_i \approx 10$ μK) [3, 4]. The relaxation of the ionic subsystem to a state with a lower potential energy of the Coulomb interparticle interaction leads to an increase in the kinetic energy of the ions, i.e. in their heating. To estimate this effect, we use the simplest expression [19]:

$$Q_{\mathrm{cor}} = 0.9 \frac{e^2}{a} \delta(t), \quad \frac{4}{3} \pi a^3 n = 1, \quad (5)$$

where e is the elementary charge and a is the radius of the Wigner – Zeits cell. The introduction of the Dirac delta function $\delta(t)$ means that the characteristic times of photoionisation of atoms and correlation heating are significantly less than the time of ion cooling.

Taking all the above processes into account, we can write the equation for the ion temperature:

$$\frac{\mathrm{d}T_{\mathrm{i}}}{\mathrm{d}t} = W_{\mathrm{ei}} + W_{\mathrm{cor}} + W_{\mathrm{las}}, \quad W_{\mathrm{ei}} = \frac{2}{3k_{\mathrm{B}}} Q_{\mathrm{ei}},$$
$$W_{\mathrm{cor}} = \frac{2}{3k_{\mathrm{B}}} Q_{\mathrm{cor}}, \quad W_{\mathrm{las}} = \frac{2}{3k_{\mathrm{B}}} (Q_{\mathrm{f}} + Q_{\mathrm{las}}). \quad (6)$$

Obviously, the cooling of the system will be observed given that $|W_{las}| > W_{ei}$.

3. Effect of correlation heating on laser cooling of ions

Immediately following the rapid formation of ultracold plasma as a result of its photoionisation [3–5], the major role in heating of the ions will be played by correlation heating, which in our approximation (5) is considered as a momentary jump in their temperature. Figure 1 shows the dependence of the rate of laser cooling $|W_{las}|$ on the temperature of Be ions. One can see that this dependence has a maximum associated with a nonlinear dependence of the coefficient χ on the ion velocity v_{ix}. The horizontal lines in Fig. 1 demonstrate the dependence of the rate W_{ei} of heating of ions by the electrons at their different concentrations. Arc arrows show schematically the jump of the ion temperature to a value T_{cor}, resulting from correlation heating [T_{cor} values for different concentrations are given by curve (*4*)].

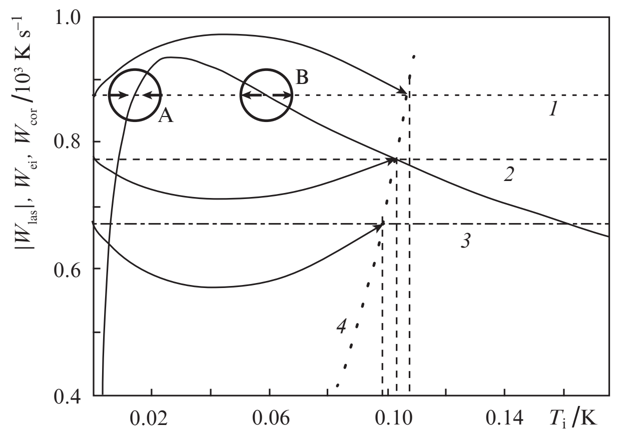

Figure 1. Dependences of the rates of laser cooling of ions $|W_{las}|$ (solid curves), their heating by electrons W_{ei} ($T_e = 100$ K) (*1–3*) and correlation heating (*4*) on the temperature of the ions at the plasma concentrations of (*1*) 2.25×10^6, (*2*) 1.98×10^6 and (*3*) 1.70×10^6 cm^{-3}.

In the region lying between the points of intersection of the dependences $|W_{las}|$ and W_{ei} (e.g., points A and B), the rate of laser cooling of ions exceeds the rate of their heating by electrons. If the correlation heating temperature of the ions falls into this area, they will be cooled to a temperature corresponding to point A of intersection of the above-mentioned dependences. One can see that in the case of the straight line (*1*), correlation heating does not allow the subsequent cooling of the ionic subsystem. The critical concentration below which cooling can be achieved with the given parameters of laser radiation ($|\Delta| = 0.5\gamma$, $|V_0| = 0.25\gamma$), is $n_0 = 1.98\times10^6$ cm^{-3}. At lower concentrations, despite the initial correlation heating, the plasma will be cooled. Note that for heavy ions (Sr^+, etc.) their temperature, reached by correlation heating, will not fall into the area where the heating of electrons prevails. In this case, curve (*4*) intersects curve $|W_{las}(T_i)|$ so that at $T_i \geqslant T_{cor}$ the cooling rate continues to increase. An increase in the detuning $|\Delta|$ also leads to the same result for all types of ions (including Be^+). However, the minimum temperature (point A in Fig. 1 is shifted to the right), reached during cooling, also increases. Thus, as noted above, the constant detuning of the laser frequency does not always make it possible to effectively cool the ionic subsystem, until it reaches a highly nonideal state. Especially it concerns the case when the ionic subsystem is initially not very cold ($T_{i0} \geqslant 1-10$ K). In our opinion, effective cooling in this case is possible by scanning the laser frequency during changes in the temperature of the ions. Thus, the detuning changes so that at a given temperature, the rate of laser cooling $|W_{las}|$ is maximal.

4. Determination of the optimal detuning

For convenience, we will use below the dimensionless Rabi frequency α, detuning β and velocity v:

$$\alpha = \frac{|V_0|}{\gamma}, \quad \beta = \frac{|\Delta|}{\gamma}, \quad v = \frac{kv_{ix}}{\gamma}. \tag{7}$$

The optimum value of $\beta_{opt}(T_i)$ is found from the condition of maximising the rate of laser cooling of ions at a given temperature:

$$W_{las}(\beta_{opt}, T_i) \to \max.$$

Figure 2 shows the calculated values of β_{opt} for Be and Sr, as well as the curves approximating these values by the formulas

$$\beta_{opt} = 5.35T_i^{1/2} + 0.25 \quad \text{for Be},$$

$$\beta_{opt} = T_i^{1/2} + 0.25 \quad \text{for Sr}. \tag{8}$$

One can see a good agreement of approximating dependences with the numerical results at both low (see the inset) and high temperatures.

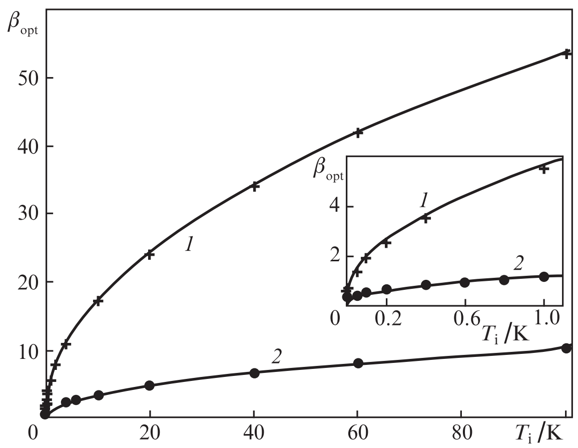

Figure 2. Dependences of the optimal detuning $\beta_{opt}(T_i)$ on the temperature of the ions for (*1*) Be and (*2*) Sr. Points show numerical calculations, and solid curves are their approximation.

Figure 3 presents the rate of cooling of Sr and Be ions at optimal and constant ($\beta = 0.5$) detunings. One can see that even at low temperatures (about 0.04 K for Be and 0.8 K for Sr) there is a significant difference in the cooling rates. The dashed lines in Fig. 3 indicate asymptotic values $|W_{las}| = |W_{las}|_{max}$, which are found below.

In addition to the numerical calculation of the optimal detuning β_{opt}, of interest is the fact how it is defined and affects the rate of cooling of the ions, as well as changes the cooling rate with increasing temperature. To do this, we will analyse the process of laser cooling.

When using notations (7), the expressions for the friction coefficient χ take the form:

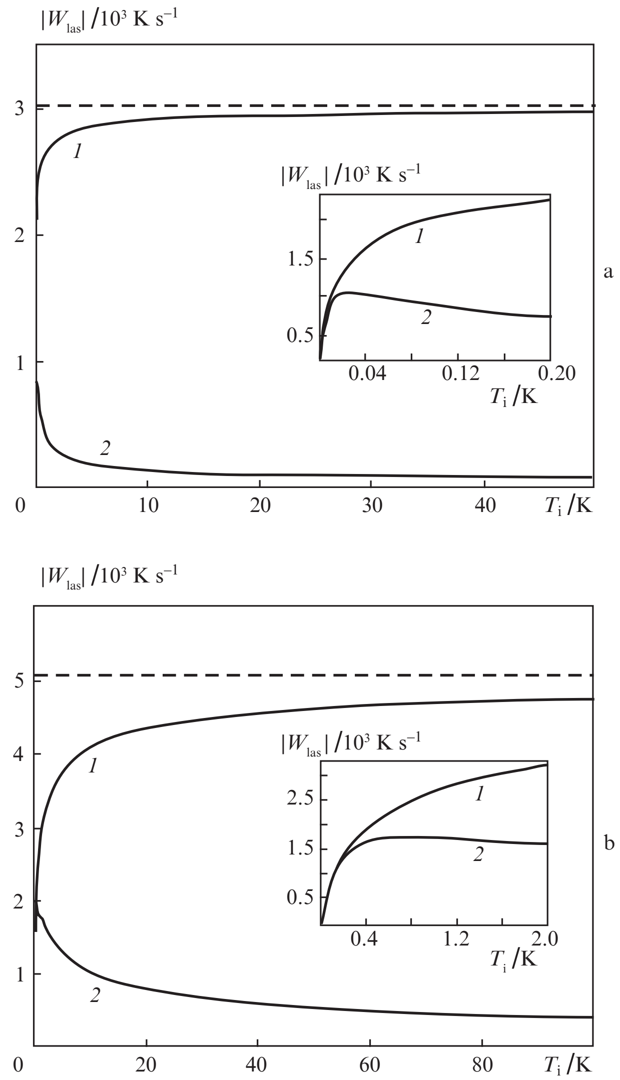

Figure 3. Cooling rates $|W_{las}|$ of (a) Be and (b) Sr ions at (*1*) an optimum detuning and (*2*) a constant detuning $\beta = 0.5$ at $\alpha = 0.25$. The insets show the same dependences at low temperatures.

$$\chi(v) = \frac{\hbar k^2\alpha^2\beta}{[(\beta - v)^2 + \tilde{g}^2][(\beta + v)^2 + \tilde{g}^2]}, \quad \tilde{g}^2 = 0.5\alpha^2 + 0.25. \tag{9}$$

Instead of the distribution function $f(v_{ix}, T_i)$, we introduce the function

$$F(v, T_i) = v_{ix}^2 f(v_{ix}, T_i) = \left(\frac{M}{2\pi k_B T_i}\right)^{1/2}\left(\frac{\gamma}{k}\right)^2 v^2$$

$$\times \exp\left[-\frac{M(\gamma/k)^2 v^2}{2k_B T_i}\right], \tag{10}$$

whose qualitative form is similar to the Maxwellian distribution function in the absolute velocity of the ions. At significant temperatures T_i (when $\langle v^2\rangle \gg \tilde{g}^2$), the function $\chi(v)$ in the vicinity of its maximum value, which is achieved at $v_m = \beta$, can be written as

$$\chi(v) \approx \frac{\hbar k^2\alpha^2}{\left[(\beta - v)^2 + \tilde{g}^2\right]4\beta}. \tag{11}$$

The characteristic width Δv_L of this function (FWHM) can be defined as follows:

$$\beta - (v_m + \Delta v_L/2) = \tilde{g}, \quad \Delta v_L = 2\tilde{g}. \tag{12}$$

For comparison, we also define the characteristic width Δv_F of the function $F(v,T_i)$:

$$\Delta v_F \approx v_F = \frac{k}{\gamma}\sqrt{\frac{2k_B T_i}{M}}, \tag{13}$$

where v_F is the velocity at which the maximum $F(v,T_i)$ is reached.

The found widths determine the characteristic regions of changes in these functions. For typical parameters $\alpha \leqslant 0.3$, $\gamma \approx 10^8$ s^{-1} and $k \approx 10^5$ cm^{-1}, we obtain $\Delta v_L \approx 0.5$ and $\Delta v_F \approx 13\sqrt{T_i/A_i}$, where A_i is the atomic mass of the element. Accordingly, their ratio must satisfy the condition

$$\frac{\Delta v_F}{\Delta v_L} \geqslant 25\sqrt{\frac{T_i}{A_i}}. \tag{14}$$

From (14) it is clear that even for heavy ions ($A_i \sim 100$) already at $T_i \geqslant 1$ K this ratio is much greater than unity. In this case, there is a reason to believe that in the neighbourhood of $v_m - \Delta v_L \leqslant v \leqslant v_m + \Delta v_L$, the function $F(v,T_i)$ varies little; then, for Q_{las} instead of (2) we can write the expression

$$Q_{las} \approx -2F(v_m, T)\int_0^\infty \frac{\gamma}{k}\frac{\hbar k^2\alpha^2}{\left[(\beta - v)^2 + \tilde{g}^2\right]4\beta}\mathrm{d}v. \tag{15}$$

Through the integration, we find

$$Q_{las} \approx -F(v_m, T_i)\frac{\hbar k\alpha^2\gamma}{2\beta\tilde{g}}\left[\left(\arctan\frac{\beta}{\tilde{g}}\right) + \frac{\pi}{2}\right]. \tag{16}$$

For $\beta \gg \tilde{g}$ we finally obtain

$$Q_{las} \approx -\pi\hbar\left(\frac{M}{2\pi k_B T_i}\right)^{1/2}\frac{\alpha^2\beta\gamma^3}{2\tilde{g}k}\exp\left(-\frac{M(\gamma\beta/k)^2}{2k_B T_i}\right). \tag{17}$$

Differentiating this expression with respect to β, we find $\beta_{opt}(T_i)$, at which the cooling rate is maximal:

$$\beta_{opt} = \frac{k}{\gamma}\sqrt{\frac{k_B T_i}{M}}. \tag{18}$$

The asymptotic values of β_{opt} for Be and Sr will be equal to $5.3T_i^{1/2}$ and $1.0T_i^{1/2}$, respectively. They are in good agreement with approximating dependences (8) at $T_i \geqslant 1$ K.

Thus, by varying the detuning β according to (18) as a function of the ion temperature, we can ensure a maximum cooling rate, which is determined by the expression

$$|Q_{las}|_{max} = \hbar(0.5\pi)^{1/2}\frac{\alpha^2\gamma^2}{2\tilde{g}}\exp(-0.5). \tag{19}$$

It is noteworthy that it is not dependent on the ion temperature T_i. The corresponding values of $|W_{las}|_{max}$ are shown by dashed lines in Fig. 3.

On the basis of expression (19) from equation (6) for the temperature of the ions we can estimate the maximum plasma concentration n_{0m} at various temperatures of the electrons at which laser cooling of ions is still possible. In this case, fluctuation and correlation heating can be neglected. Comparing the heating of ions by electrons with a subsequent cooling of ions, described by expression (19), we obtain the cooling condition

$$\frac{2m}{M}\nu_{ei}(T_e - T_i) < \hbar(0.5\pi)^{1/2}\frac{\alpha^2\gamma^2}{3k_B\tilde{g}}\exp(-0.5), \tag{20}$$

which allows one to assess the feasibility of cooling the ions in specific circumstances by using a specified plasma concentration, electron temperature and frequency α. For example, for ions Be^+ at $n = 10^7$ cm^{-3}, $T_e = 100$ K and $\alpha = 0.25$, we obtain $36.7(T_e - T_i)$ and 3.07×10^3 in the left- and right-hand side of this inequality, respectively. Then, for condition (20) to be fulfilled, the temperature of ions should exceed 16.6 K: only in this case they will be cooled under predetermined conditions.

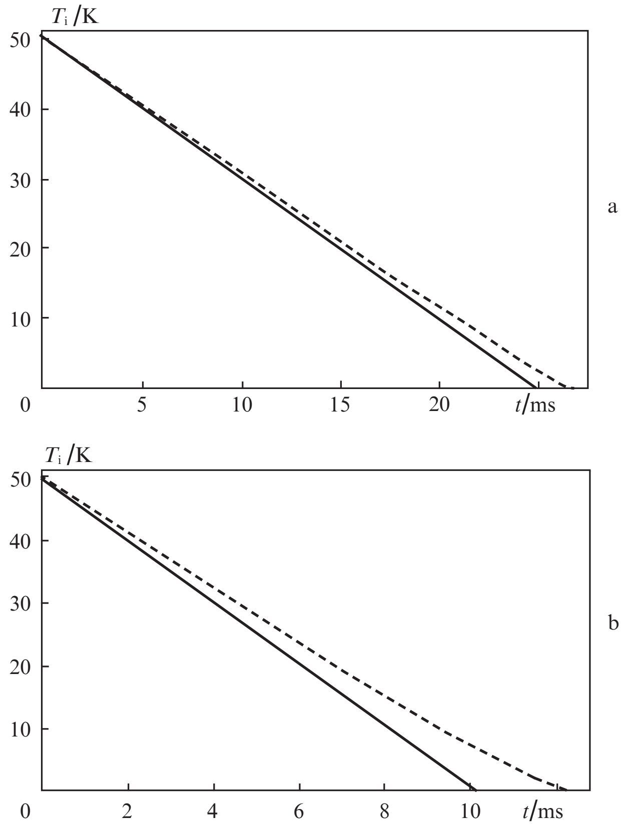

Figure 4. Dynamics of cooling of (a) Be and (b) Sr ions at $T_e = 500$ K, $n_0 = 5\times10^6$ ^{3}cm^{-3}. The dashed lines show the calculation using the exact expression (2), and solid lines – using the asymptotic expression (19) for the cooling rate of the ions.

Using expressions (19) in the equation for the temperature of the ions (6) greatly simplifies the description of the cooling dynamics. In this case (at $T_i > 1$ K), the temperature decreases linearly with time, making it easy to estimate the required cooling time. Figure 4 shows the time dependence of the temperature found numerically using expressions (2) and (8), as well as analytical solutions of the equations for the temperature of the ions (6) in which Q_{las} is determined by its asymptotic expression (19). It is seen that a change in temperature in the region $T_i > 1$ K at an optimum detuning is linear, and the asymptotic expression for Q_{las} allows one to accurately describe this change.

Note that the linear dependence of the ion temperature on time at $T_i > 1$ K allows one to easily pass from $\beta_{opt}(T_i)$ to $\beta_{opt}(t)$, which facilitates the implementation of the scanning of the frequency detuning in this temperature range.

As follows from the above, the optimal scanning of the laser frequency detuning can significantly extend the temperature range of possible cooling of plasma ions. Also of interest is to determine both the temperature range and the concentration range in which cooling of the plasma ions is theoretically possible. To address this issue, we use the solution to equation (6) at β_{opt} specified by expressions (8).

Figure 5 shows the areas (not shaded) where cooling of the ions is possible. They are determined by the electron temperature and plasma density. It is assumed that the electron temperature is considerably higher than the initial temperature of the ions. In these areas, presented are numbered contour lines corresponding to different values of the nonideality parameter $\Gamma_i = e^2/(ak_B T_i)$ of the ionic subsystem. At the bottom of the shaded areas, the electrons are not weakly nonideal and therefore the approximation of the pair electron – ion interaction used in this work is not correct.

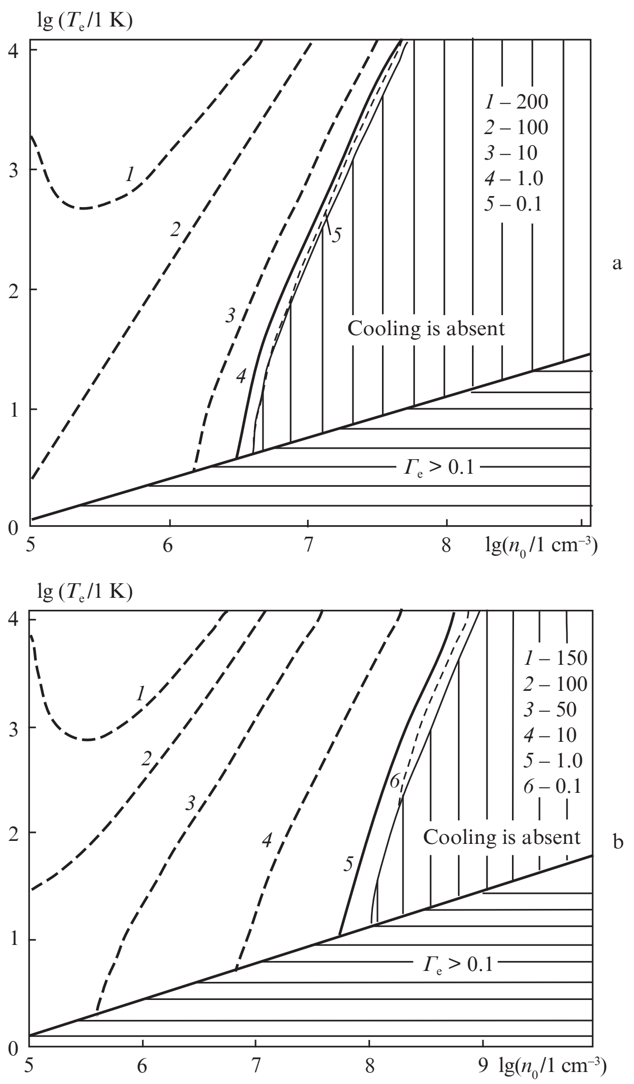

Figure 5. Areas of the parameters in which the laser cooling of the electron – ion (a) Be and (b) Sr plasma is possible at $\alpha = 0.25$ (Γ_e is the nonideality parameter for the electrons).

Using Fig. 5 we can determine the cooling feasibility and the nonideality parameter Γ_i, which can be achieved under these conditions, at a given concentration and temperature of the electrons.

5. Conclusions

We have shown that effective laser cooling (under the action of spontaneous radiation pressure forces) of plasma ions in a wide range of their initial temperatures is possible in the case of scanning of the laser frequency. For Be and Sr ions, we have obtained expressions for optimal frequency detunings, which provide a maximum cooling rate of the ions at their different temperatures. In the general case, we have also obtained asymptotic expressions for the optimal detunings and cooling rates. As it turns out, in an optimal case, the cooling rate does not depend on the temperature of the ions. Based on the solution of the equation for the temperature of Be and Sr ions we have determined the regions (in the coordinates T_e and n_0) of possible cooling (using optimal detunings) up to a highly nonideal state, which is particularly important for the preliminary estimates in experiments.

Acknowledgements. The authors express their sincere gratitude to N.Ya. Shaparev for useful discussions and valuable advice.

Appendix. Relaxation of the distribution function (DF)

The action of the spontaneous radiation pressure force distorts the distribution function of the ions. At the same time, the ion – ion interaction (collisions) interferes with this process and determines the relaxation of the distribution function to the equilibrium (Maxwellian) one. If the relaxation rate of the distribution function is much higher than the rate of its deformation due to cooling, the distribution function will be slightly different from the equilibrium one. Let us compare the rates of these two processes.

The maximum change in the momentum of the ion during its cooling can be estimated by the formula

$$\mathrm{d}p_{\mathrm{res}}^{\mathrm{las}} \approx \hbar\gamma k \mathrm{d}t,$$

where p_{res} is the momentum of the ion ('resonant' to radiation) in the region of the maximum action of the spontaneous radiation pressure force.

A change in the momentum of the ion as a result of the interaction (collision) with other ions can be estimated by the formula

$$\mathrm{d}p_{\mathrm{res}}^{\mathrm{ii}} \approx p_{\mathrm{res}}\Omega_{\mathrm{ii}}\mathrm{d}t,$$

where $\Omega_{ii} = \min(\omega_i, \nu_{ii})$ is the relaxation rate of the distribution function, which in the case of highly nonideal ions is proportional to the ion plasma frequency ω_i, and in the case of weakly nonideal ions – to the frequency of ion – ion collisions ν_{ii}.

We also use here the momentum of the particle in the region of a maximum force of the laser friction:

$$k\frac{p_{\mathrm{res}}}{M\gamma} \approx |\Delta|, \quad p_{\mathrm{res}} \approx \frac{M\gamma|\Delta|}{k}.$$

For the distribution function relaxation to occur in the process of cooling, the condition

$$p_{\mathrm{res}}\Omega_{\mathrm{ii}} \gg \hbar\gamma k$$

should be fulfilled, where

$$\frac{M\beta}{k}\Omega_{\mathrm{ii}} \gg \frac{\hbar k}{\gamma}, \quad \beta \gg \frac{\hbar k^2}{M\gamma\Omega_{\mathrm{ii}}}.$$

At characteristic parameters $k \approx 10^5$ cm^{-1}, $\gamma \approx 10^8$ s and $M \sim (10-100)\times10^{-24}$ g, we obtain

$$\beta \gg (10^{-2}-10^{-3})/\Omega_{\mathrm{ii}}.$$

This condition may be violated (even at low concentrations, $n \leqslant 10^6$ cm^{-3}) for reasonable values of the relative detuning ($\beta > 0.1$) only at high temperatures of the ions ($T_{\mathrm{i}} > 10^3$ K). Thus, we can assume that in the process of laser cooling of ions, their distribution function remains equilibrium.

References

1. Ichimaru S. *Rev. Mod. Phys.*, **54**, 10 (1982).
2. Fortov V.E., Khrapak A.G., Yakubov I.T. *Fizika neideal'noi plazmy* (Physics of Nonideal Plasma) (Moscow: Fizmatlit, 2004).
3. Kilian T.C., Kulin S., Bergeson S.D., Orozco L.A., Orzel C., Rolston S.L. *Phys. Rev. Lett.*, **83**, 4776 (1999).
4. Kulin S., Kilian T.C., Bergeson S.D., Rolston S.L. *Phys. Rev. Lett.*, **85**, 318 (2000).
5. Kilian T.C., Lim M.J., Kulin S., Dumke R., Bergeson S.D., Rolston S.L. *Phys. Rev. Lett.*, **86**, 3759 (2001).
6. Gavrilyuk A.P., Krasnov I.V., Shaparev N.Ya. *Pis'ma Zh. Tekh. Fiz.*, **23** (2), 28 (1997).
7. Gavrilyuk A.P., Krasnov I.V., Shaparev N.Ya. *Laser Phys.*, **8**, 653 (1998).
8. Murillo M.S. *Phys. Rev. Lett.*, **87**, 115003 (2001).
9. Tkachev A.N., Yakovlenko S.I. *Kvantovaya Elektron.*, **31**, 1084 (2001) [*Quantum Electron.*, **31**, 1084 (2001)].
10. Mazevet S., Colins L.A., Cress J.D. *Phys. Rev. Lett.*, **88**, 055001 (2002).
11. Robicheaux F., Hanson J.D. *Phys. Rev. Lett.*, **88**, 055002 (2002).
12. Kuzmin S.G., O'Neil T.M. *Phys. Plasmas*, **9**, 3743 (2002).
13. Killian T.C., Ashoka V.S., Gupta P., et al. *J. Phys. A: Math. Gen.*, **36**, 6077 (2003).
14. Pohl T., Pattard T., Rost J.M. *Phys. Rev. Lett.*, **92**, 155003 (2004).
15. Pohl T., Pattard T., Rost J.M. *J. Phys. B: At. Mol. Opt. Phys.*, **38**, 343 (2004).
16. Killian T.C., Pattard T., Pohl T., Rost J.M. *Phys. Rep.*, **449**, 77 (2007).
17. Gavrilyuk A.P., Isaev I.L., Karpov S.V., Krasnov I.V., Shaparev N.Ya. *Phys. Rev. E*, **80**, 056404 (2009).
18. Kazantsev A.P., Surdutovich G.I., Yakovlev V.P. *Mekhanicheskoe deistvie sveta na atomy* (Mechanical Action of Light On Atoms) (Moscow: Nauka, 1991).
19. Gavrilyuk A.P. 2011 arXiv: plasma physics/1107.5904.