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Controlling the interaction between optical solitons using periodic dispersion variations in an optical fibre

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Abstract. **This paper considers interaction between two fundamental optical solitons in an optical fibre with a periodically varying dispersion. Numerical simulation results indicate that, by properly adjusting the modulation period, one can change the type of interaction between solitons. We consider three particular cases: the fission of a soliton pair into two separate pulses, the generation of an intense pulse as a result of the fusion of two solitons and the formation of a coupled state of two solitons (soliton molecule). The present findings demonstrate the possibility of controlling the number and group velocity of solitons using passive single-mode optical fibres.**

Keywords: *optical solitons, inelastic interaction, nonlinear Schrödinger equation with variable coefficients.*

1. Introduction

A resonance effect of a periodic perturbation on the dynamics of soliton solutions to the nonlinear Schrödinger equation (NLSE) was first described by Hasegawa and Kodama [1]. Their results indicate that, if the perturbation modulation period is comparable to the oscillation period of a multisoliton pulse, it breaks up into several fundamental solitons. Experimental demonstration of this effect has become possible due to advances in the fabrication of optical fibres with a diameter varying along their length. More than 15 years after the report by Hasegawa and Kodama [1], Sysoliatin et al. [2] provided experimental evidence for a resonance effect of perturbations on soliton dynamics. They used an optical fibre of periodically varying diameter to break up a two-soliton pulse into two fundamental solitons. Varying the modulation period, one can control the group velocity, centre frequency and peak power of such solitons.

Optical solitons propagating in fibres with a diameter varying along their length [2] satisfy a nonautonomous NLSE with variable dispersion and nonlinearity coefficients. At a certain relationship between the coefficients, one can obtain analytical expressions for single-soliton and multisoliton solutions to the nonautonomous NLSE. A review of such studies was given by Maimistov [3], who pointed out that a collision between solitons does not cause them to decay. This type of interaction is, however, observed at modulation frequencies far from resonance. As shown below, if the modulation period of the dispersion coefficient and/or nonlinearity coefficient in the NLSE is comparable to the oscillation period of a soliton, interaction between solitons is essentially inelastic. Such interaction between NLS solitons in a nonlinear medium with varying dispersion was reported by Liu et al. [4]. Using analytical solutions, interaction between solitons was shown to be accompanied by the formation of periodic field structures [4]. Varying the modulation parameters, one can change the nature of interaction between solitons from attraction to repulsion.

Yan and Dai [5] considered a generalised NLSE with variable coefficients. Dispersion, nonlinearity and gain coefficients varying in a certain way may lead to the formation of a rogue wave, with a periodic potential as a trigger mechanism [6].

In an NLSE model with a harmonic potential [7], a periodic variation in the potential leads to decay of coupled soliton states. At the same time, decay of a soliton pair can be followed by its recovery. Sysoliatin et al. [8] examined the fission of an optical two-soliton breather in the case of periodic variations in dispersion and nonlinearity and spontaneous Raman scattering.

A number of approaches for finding analytical solutions to the NLSE with periodically varying coefficients (see e.g. Refs [3–5]) do not take into account the resonance nature of soliton propagation in a medium with oscillating dispersion (and/or nonlinearity). At resonance, the amplitudes of solitons, their velocities and even their number may change. In this study, the main tool is numerical calculation. We examine propagation modes corresponding to fission of a soliton pair, fusion of two solitons into a single intense pulse and formation of a coupled state of two solitons. These effects were described previously in systems obeying the complex Ginzburg–Landau equation ([9], ch. 13). The use of spectral filtration and a special type of loss, amplification, nonlinearity or dispersion was proposed for soliton pair management. Special conditions imposed on a medium or pulses are often difficult to realise in practice. In this paper, we discuss the feasibility of controlling soliton pulses using passive single-mode optical fibres with dispersion varying along their length, which is ensured by varying the fibre diameter. There are mature processes for the fabrication of such fibres [10], which allows the effects considered below to be brought about experimentally.

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2. Calculational approach

Soliton dynamics in a fibre of periodically varying diameter obey the NLSE with variable coefficients [2, 3]:

$$\frac{\partial A}{\partial z} + \mathrm{i}\frac{\beta_2(z)}{2}\frac{\partial^2 A}{\partial \eta^2} = \mathrm{i}\gamma(z)\,|A|^2 A(z,\eta), \qquad (1)$$

where $A(z,\eta) = (cn\varepsilon_0 S_{\mathrm{eff}}/2)^{1/2}E(z,\eta)$; c is the speed of light in vacuum; n is the refractive index; ε_0 is the electric constant; S_{eff} is the effective area of the fundamental mode of the fibre [11, 12]; $E(z,\eta)$ is the complex electric field amplitude; z is the propagation distance; η is the retarded time ($z = z$, $\eta = t - z/u$) [11]; u is the group velocity of the pulse; and the parameter η determines the time interval between a pulse propagating at a velocity u and the pulse under investigation. In Eqn (1), the parameter $\gamma(z)$ is the Kerr nonlinearity coefficient and $\beta_2(z)$ is the second-order dispersion coefficient. The optical fibre used for soliton fission [2, 8] had the following parameters: $\gamma(z) = \langle\gamma\rangle[1 - 0.028\sin(2\pi z/z_{\mathrm{m}})]$ (where z_{m} is the modulation period), $\langle\gamma\rangle = 8.2$ W^{-1} km^{-1}, $\beta_2(z) = \langle\beta_2\rangle[1 + 0.2\sin(2\pi z/z_{\mathrm{m}})]$ and $\langle\beta_2\rangle = -12.76$ ps^2 km^{-1}. These values were used in our calculations. Since the dispersion modulation amplitude $\langle\beta_2\rangle$ far exceeds the nonlinearity modulation amplitude $\langle\gamma\rangle$, in what follows we address soliton dynamics in the case of a periodically varying dispersion.

In the numerical scheme, we used the split-step Fourier method [13]. Relative uncertainty did not exceed 10^{-9}. To suppress the waves reflected from the boundaries of the calculation window, we used absorbing boundary conditions. The results of previous calculations [2, 8] by a similar procedure were in good agreement with experimental data.

The initial field can be represented as a superposition of two single-soliton pulses,

$$E(0,\eta) = A_0\,\mathrm{sech}(\eta/\eta_0 - T) + A_0\,\mathrm{sech}(\eta/\eta_0 + T), \qquad (2)$$

where $\eta_0 = 1.13$ ps is the initial pulse duration; $A_0 = \eta_0^{-1}\times\sqrt{|\langle\beta_2\rangle|/\langle\gamma\rangle}$ is the initial single-soliton pulse amplitude [9, 11]; and the dimensionless parameter $T = 6$ determines the separation between the peaks of the initial pulses.

To analyse the numerical solution to the NLSE with variable coefficients (1), we use the inverse scattering method ([12], Sect. 5.8). The algorithm for evaluating parameters of solitons comprises three steps:

(1) We find a solution to Eqn (1) in the z_{s} plane: $E(\eta) = E(z_{\mathrm{s}},\eta)$.

(2) For the function $E(\eta)$, we calculate the scattering matrix of the NLSE with fixed dispersion and nonlinearity coefficients: $\beta_2 = \beta_2(z_{\mathrm{s}})$ and $\gamma = \gamma(z_{\mathrm{s}})$.

(3) Using Newton's method, we find complex numbers (spectral parameters) λ_j that correspond to zero coefficient of the scattering matrix: $a^*(\lambda_j) = 0$.

The steps are then repeated at a new value $z = z_{\mathrm{s}}$. As a result, we obtain spectral parameters of solitons as functions of their path length: $\lambda_j = \lambda_j(z)$. To compare the parameters λ_j to physical parameters, note that, if all solitons have different velocities, the soliton component A_{s} of the function $A(\eta)$ for $z \gg 1$ is determined by the superposition [11]

$$A_{\mathrm{s}}(z,\eta) = \sum_{j=1}^{N} R_j\,\mathrm{sech}\,(\varkappa_j\eta - \tau_j - v_j z)\exp[\mathrm{i}\varphi_j(z,\eta)], \qquad (3)$$

where N is the number of solitons; R_j is the soliton amplitude; $\varkappa_j$ is the inverse soliton duration; τ_j and $\varphi_j(z,\eta)$ are the coordinate and phase of the maximum; and v_j determines the change in the group velocity of the soliton: $v_{\mathrm{g}} = (1/u + v_j)^{-1}$. The amplitude, duration and group velocity of the soliton can be expressed through the spectral parameters λ_j:

$$R_j = \tau_0^{-1}\sqrt{|\beta_2|/\gamma}\,2\,\mathrm{Im}\lambda_j,\ \ \varkappa_j = \tau_0^{-1}2\mathrm{Im}\lambda_j,$$
$$v_j = \beta_2\tau_0^{-1}2\mathrm{Re}\lambda_j, \qquad (4)$$

where $\tau_0 = \eta_0(|\beta_2|/\gamma)(|\langle\beta_2\rangle|/\langle\gamma\rangle)^{-1}$ corresponds to the single-soliton pulse duration in a waveguide with adiabatic ($z_{\mathrm{m}} \gg 1$) variations in the parameters $\beta_2(z)$ and $\gamma(z)$. With this choice of the parameter τ_0, the energy of an individual soliton is given by

$$J = \int_{-\infty}^{\infty}|R_j\mathrm{sech}(\varkappa_j\eta - \tau_j - v_j z)|^2\mathrm{d}\eta = 2R_j^2\varkappa_j^{-1} = J_0\,2\,\mathrm{Im}\lambda_j, \qquad (5)$$

where $J_0 = 2A_0^2\eta_0 = 2\eta_0^{-1}(|\langle\beta_2\rangle|/\langle\gamma\rangle)$ is the initial single-soliton pulse energy in (2). If we consider a pulse with a carrier frequency shift $\Delta\Omega$, it can be noticed that substitution of the expression $A(z,\eta) = \tilde{A}(z,\eta)\exp(-\mathrm{i}\Delta\Omega\eta)$ into Eqn (1) allows us to find a relationship between the pulse carrier frequency shift and the change in the group velocity of the pulse in the form $v_j = \beta_2\Delta\Omega$. According to (4), the soliton carrier frequency shift is given by

$$\Delta\Omega = \tau_0^{-1}\,2\mathrm{Re}\lambda_j. \qquad (6)$$

Strictly speaking, (3) and (4) are not a soliton solution to the NLSE (1) with variable coefficients. Parameters (4) were found under the assumption that, after passing through a fibre with a periodically varying dispersion, the light propagates in a fibre with constant dispersion and nonlinearity coefficients. In fact, this means that, at each z step, the numerical solution $A(z,\eta)$ is analysed using data of an inverse scattering problem formulated for the NLSE with fixed dispersion (β_2) and nonlinearity (γ) coefficients. This approach was used previously to analyse soliton dynamics in the case of two-soliton breather fission [8].

The pulse amplitude, group velocity and duration can be directly calculated for a given $A(\eta)$ field. However, these parameters can only be calculated for pulses whose fields do not overlap in time. The main advantage of the above method is the possibility of gaining insight into the dynamics of interacting solitons. The use of data of an inverse scattering problem makes it possible to clearly illustrate the effect of perturbing factors on the parameters of breathers and individual solitons [8]. For solitons that are sufficiently well separated in time, estimates of the amplitude, group velocity and duration directly from the $A(\eta)$ distribution and from data of the inverse scattering problem (3) and (4) yield identical results.

3. Interaction between solitons

For modelling, the initial pulse separation was taken to be sufficiently large ($T = 6$). With this condition, for a soliton pair (2) propagating in a constant-diameter fibre ($z_{\mathrm{m}} = \infty$) there exists an analytical solution in the form of semi-infinite pulses [14]. For $T \to \infty$, we have $\lambda_1 = \lambda_2 = \mathrm{i}\cdot 0.5$, which corresponds to noninteracting solitons. At $T = 6$, $\lambda_1 = \mathrm{i}\cdot 0.49753$ and $\lambda_2 = \mathrm{i}\cdot 0.50249$. In-phase solitons (2) attract and, after collision, separate (Fig. 1). The distance after which the pulses collide is given by [14]

$$z_c = z_0(\lambda_1^2 - \lambda_2^2)^{-1}, \tag{7}$$

where $z_0 = (\pi/2)\eta_0^2|\langle\beta_2\rangle|^{-1}$ is the soliton period in a constant-diameter fibre ($z_m = \infty$). The oscillation period of the soliton pair is $2z_c$. In our case, $z_c = 31.7$ km (Fig. 1). In the case of the NLSE with constant coefficients ($z_m = \infty$), solitons interact elastically and their parameters λ_j remain unchanged. If there is dispersion modulation, interaction between solitons may have an inelastic nature, and the parameters λ_j vary under such conditions.

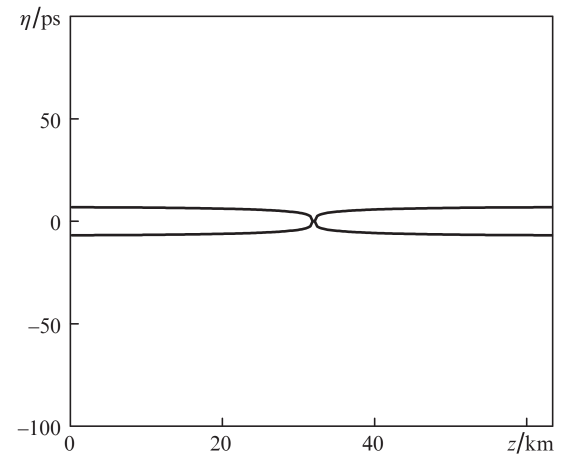

Figure 1. Propagation of two in-phase solitons in a constant-dispersion fibre ($z_m = \infty$). Shown in the (z,η) plane are peak intensity trajectories subject to the initial conditions (2).

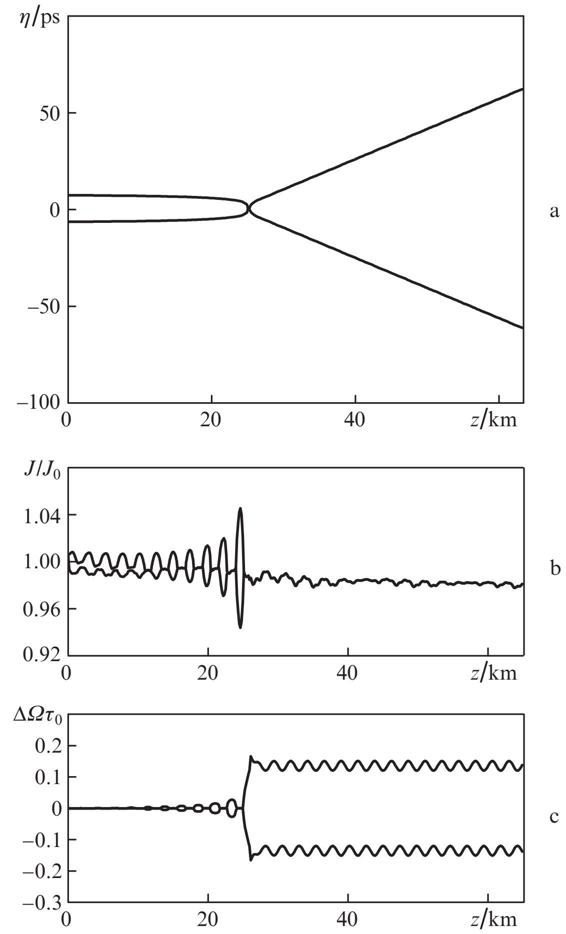

Figure 2. Fission of a soliton pair in a fibre with a periodically varying dispersion ($z_m = 2.4$ km): (a) peak intensity trajectories, (b) normalised soliton energies J/J_0 [calculated by formula (5)] and (c) normalised spectral shifts of the soliton carrier frequency [formula (6)] for both solitons. The other parameters are the same as in Fig. 1.

3.1. Fission of a soliton pair

Using a periodic variation in dispersion, a soliton pair (Fig. 1) can be separated into two individual pulses propagating with different group velocities (Fig. 2). The starting pulses gradually attract in the initial stage and then approach each other very rapidly (Fig. 2a). After the fusion ($z = 25.92$ km), the solitons repel and diverge with different group velocities. The change in the group velocity of the pulse is caused by the shift of its carrier frequency (6). It is convenient to assess the behaviour of individual solitons using the parameters $\mathrm{Re}\,\lambda$ and $\mathrm{Im}\,\lambda$. These parameters separately determine the energy (5) and frequency shift (6) of each soliton (Figs 2b, 2c). During the transient process ($0 < z < 25.92$ km), the energy and frequency shift of the solitons oscillate with a small amplitude. After the transient process ($z > 25.92$ km), the two solitons have the same energy J/J_0. In the range 27.6 km $< z <$ 63.6 km, the average frequency shift of one soliton is $\Delta\Omega\tau_0 = 0.137$ and that of the other is $\Delta\Omega\tau_0 = -0.137$. This symmetry of the shifts is dictated by the law of conservation of momentum [9]. The net frequency shift, related to the total momentum of the solitons, should remain zero. After the change in the group velocity of the pulses ($z > 25.92$ km), the soliton energy gradually decreases. The decrease in soliton energy and the irregular character of the function $J = J(z)$ are due to the generation of dispersive waves in the case of periodic variations in dispersion and nonlinearity [1].

The fission of a soliton pair into two solitons having different velocities is observed in a wide z_m range. A change in z_m leads to a change in the distance over which a collision between solitons occurs and their group velocities change. However, the behaviour of the spectral parameters $\lambda_{1,2}$ remains unchanged: after some transient process, the parameters $\mathrm{Im}\,\lambda_1$ and $\mathrm{Im}\,\lambda_2$ become equal. Both λ_1 and λ_2 acquire nonzero real parts such that $\mathrm{Re}\,\lambda_1 = -\mathrm{Re}\,\lambda_2$. Note that the fission of a second-order soliton into two fundamental solitons follows a similar scenario [8].

3.2. Fusion of two solitons

Figure 3 illustrates soliton dynamics at a fibre modulation period $z_m = 2$ km, which leads to soliton fusion a distance $z =$ 15.1 km from the input fibre end. Next, the pulse breaks up into a central, intense soliton and two associated solitons (Fig. 3a). At $z = 63.4$ km, the peak power of the central pulse is nine times that of the side pulses. The energy of the central pulse is a factor of 2.7 higher than that of each side pulse, which are equal to each other (Fig. 3b).

An inelastic collision may change the number of solitons. After the soliton fusion, an additional, third solution, λ_3, emerges at $z = 15.46$ km. This solution appears in the vicinity

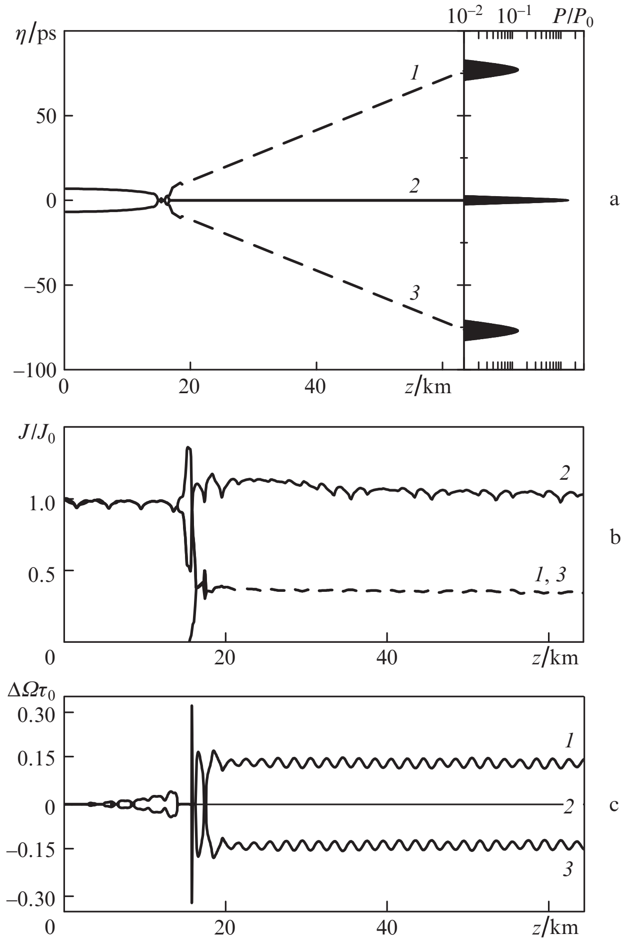

Figure 3. Fusion of two solitons ($z_m = 2$ km): (a) peak intensity trajectories [the inset shows the time variation of the instantaneous field power $P = P(\eta)$ at $z = 63.4$ km; P is normalised to the initial pulse peak power P_0 (2); the graph in the inset has a logarithmic scale]; (b) normalised energy distributions J/J_0 (5) and (c) normalised spectral shifts of the carrier frequency $\Delta\Omega\tau_0$ (6) for both solitons; (*1*–*3*) solitons obtained after interaction between the starting pulses. The other parameters are the same as in Figs 1 and 2.

of $\mathrm{Re}\,\lambda = 0$ and $\mathrm{Im}\,\lambda = 0$. The magnitude of $\mathrm{Im}\,\lambda_3$ rapidly increases, which corresponds to an increase in the energy of the additional soliton in the range 15.46 km $< z <$ 16.63 km (Fig. 3b). The transient process is followed by the formation of two weak pulses, with a shifted carrier frequency, and one intense pulse. During subsequent propagation, only a reduction in pulse energy due to the emission of a dispersive wave is possible. In the case of single-soliton pulses, the dispersive wave has the highest intensity when the dispersion variation period coincides with the soliton period [1]. We determine the soliton period as $\tilde{z}_0 = (\pi/2)\langle\eta_j\rangle^2|\langle\beta_2\rangle|^{-1}$, using the average soliton duration $\langle\eta_j\rangle = \eta_0(2\,\mathrm{Im}\,\lambda_j)^{-1}$. The variable soliton duration is given by (4). At $z = 100$ km, we have $\tilde{z}_0 = 0.134$ km for the central soliton and 1.38 km for the side solitons. The dispersion variation period, $z_m = 2$ km, does not coincide with the period of these solitons, so the energy loss due to the emission of a dispersive wave is a rather slow process.

The relationship between the peak power of the central soliton and that of the side solitons depends on the time separation between the starting pulses and the modulation period. In our simulations, we obtained a number of regimes in which side solitons were essentially missing. The effect considered above can be thought of as fusion of two solitons into a pulse with high peak power.

3.3. Formation of a coupled state of two solitons

In a constant-dispersion fibre, two solitons attract and repel cyclically (Fig. 1). In a fibre with a periodically varying dispersion, a regime can be obtained in which solitons propagate essentially without attraction. At a dispersion modulation period $z_m = 0.1$ km, the soliton separation remains unchanged (Fig. 4a), and there is a coupled state of two solitons: a soliton molecule. As a result of dispersive wave emission, the amplitude of the solitons gradually decreases and their duration increases. This effect leads to a reduction in the initial magnitude of the imaginary part of the spectral parameters $\mathrm{Im}\,\lambda_1$ and $\mathrm{Im}\,\lambda_2$, whereas the real parts $\mathrm{Re}\,\lambda_1$ and $\mathrm{Re}\,\lambda_2$ remain zero. It seems likely that an important role in the formation of a coupled state of solitons is played by their interaction through the field of dispersive waves [15]. The decrease in pulse peak power as a result of dispersive wave emission is illustrated in Fig. 4b. In the case of modulation ($0 < z < 100$ km), the pulse peak power decreases on average linearly with increasing path length z. If at a certain instant in time ($z = 100$ km) periodic

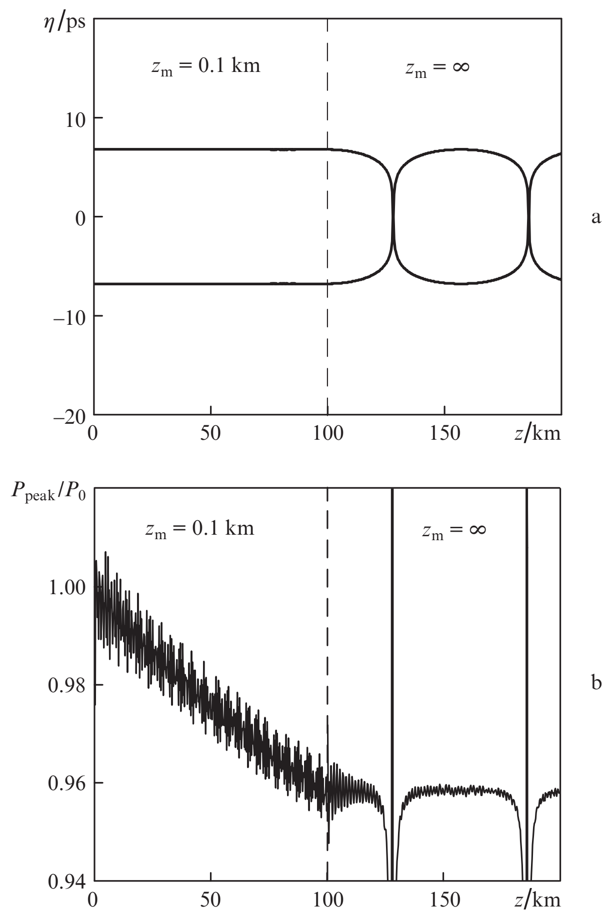

Figure 4. Coupled state of two solitons in the case of modulation (z_m = 0.1 km in the range $0 \leqslant z \leqslant 100$ km) and transition to a periodic solution after elimination of the modulation ($z_m = \infty$ for $z > 100$ km): (a) peak intensity trajectories and (b) ratio of the pulse peak power P_{peak} to the initial pulse peak power P_0 (2). The vertical dashed line separates the dispersion modulation region from the constant dispersion region. The other parameters are the same as in Fig. 1.

dispersion modulation is eliminated, the two pulses begin to propagate as a soliton pair whose shape experiences periodic changes (Fig. 4). The dispersive wave then disappears. The soliton pair period is given by (7).

Note that the modulation period z_m needed for a particular regime is not unique. There are several resonance frequencies $2\pi/z_m$ for each of the effects considered above. Additional work is needed to determine these frequencies.

4. Conclusions

Periodic fibre diameter modulation has been proposed as a means of controlling soliton interaction. A particular regime can be ensured by adjusting the modulation period. Three types of regimes have been considered, which can find practical application in controlling laser pulses and optical information processing. The fission of a soliton pair in a fibre with a periodically varying dispersion makes it possible to produce a sequence of picosecond pulses with two carrier frequencies. Such pulses can be used for creating terabit communication systems with frequency division multiplexing and in terahertz spectroscopy. The fusion of two solitons allows one to obtain a pulse with a relatively high peak power. This effect can be used for converting a sequence of closely spaced pulses (pulse train) into new pulses with increased peak power.

In the case of data transfer in optical communication systems, attraction between in-phase solitons may lead to information loss. For solitons propagating at a small distance from each other, a fibre with a periodically varying dispersion can be used to increase the distance over which a collision between solitons occurs. Coupled states of two solitons might prevent collisions between them.

Note that all the effects in question occur in a passive single-mode optical fibre without using additional controlling pulses or a special type of nonlinearity, dispersion or loss.

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