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Numerical investigation of multichannel laser beam phase locking in turbulent atmosphere

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Abstract. **The efficiency of coherent multichannel beam combining under focusing through a turbulent medium on a target in the cases of phase conjugation and target irradiation in the feedback loop is investigated numerically in various approximations. The conditions of efficient focusing of multichannel radiation on the target are found. It is shown that the coherent beam combining with target irradiation in the feedback loop, which does not require a reference beam and wavefront measurements, is as good as the phase conjugation approach in the efficiency of focusing. It is found that the main effect of focusing is provided by properly chosen phase shifts in the channels, whereas taking into account local wavefront tip tilts weakly affects the result.**

Keywords: *coherent beam combining of multichannel cw laser radiation, stochastic parallel gradient algorithm.*

1. Introduction

Coherent phase locking of parallel laser channels is a promising method for increasing power and brightness of cw laser radiation (see, for example, [1]). Phase matching of multichannel laser radiation at the system output was demonstrated by an example of fibre-optic lasers (see [2–5]). A more complicated problem is the phase matching (actually focusing) of multichannel radiation in an optically heterogeneous medium, for example, turbulent atmosphere, which distorts the wavefront of radiation in the process of its propagation.

One possible approach to solving this problem is the phase conjugation method. It requires forming the reference beam, propagating from a target to the input aperture of the laser system and carrying information about optical heterogeneities of the path. In the case of a single channel, the wavefront of the initial beam formed by an adaptive mirror that is conjugate to the wavefront of the reference beam gives a possibility to compensate for the influence of an optically heterogeneous medium and obtain a diffraction-limited beam for the emitting aperture on the target [6]. In a multichannel laser system, the efficiency of initial beam focusing on the target will, seemingly, depend on the degree of filling the total aperture. The adaptive mirror can be controlled both by measuring the wavefront of the reference beam and by iterative methods without using a wavefront sensor.

There is another way for phase matching of multichannel laser radiation in an optically heterogeneous medium, namely, target irradiation in the feedback loop (see, for example, [7–9]). This approach does not require a reference beam. Compensation for optical heterogeneities of the path and focusing of the initial multichannel beam on the target are realised by an iterative search for the extremum of a certain parameter of the initial radiation that has reached the target (or reflected from it). Ideally, the phase of a multichannel beam can be controlled by using flexible adaptive mirrors in each channel or, in the first approximation, by introducing only phase shifts and tilts into the channels by means of electro-optical and piezoelectric elements.

The present work is aimed at a numerical comparison of the efficiency of target irradiation by a multichannel laser beam through an optically heterogeneous medium with the phase conjugation and in the feedback loop in various approximations (with and without making allowance for local tilts of the wavefront on sub-apertures). The phase control in the channels of initial radiation in the case of target feedback irradiation is realised by the stochastic parallel gradient (SPG) algorithm [9].

2. Problem statement

The output of a cw laser system comprising seven channels is in the plane $z = 0$ of an optically heterogeneous (turbulent) medium. The near-field intensity distribution of the output beam is shown in Fig. 1a, radiation phases in each channel being assumed plane. The distance between the sub-aperture centres is $R = 1.11d$, where d is the diameter of a single channel. A lens with a focal distance $F = L$ is also placed in the plane $z = 0$ for focusing the radiation through the optically heterogeneous medium to a target in the plane $z = L$ (L is the path length). Propagation of laser radiation in turbulent atmosphere is calculated by using the parabolic equation

$$2\mathrm{i}k\frac{\partial u}{\partial z} + \Delta u + k^2\tilde{\varepsilon}u = 0, \qquad (1)$$

where $k = 2\pi/\lambda$ is the wavenumber; λ is the wavelength; $u(z,x,y)$ is the slowly varying complex amplitude of the electric field of radiation propagating along the z axis; $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian with respect to transversal coordinates x, y; and $\tilde{\varepsilon}$ is a random function describing fluctuations of the dielectric constant in the turbulent atmosphere.

In numerical calculations we used the finite-difference scheme for integrating Eqn (1) and utilised splitting of physi-

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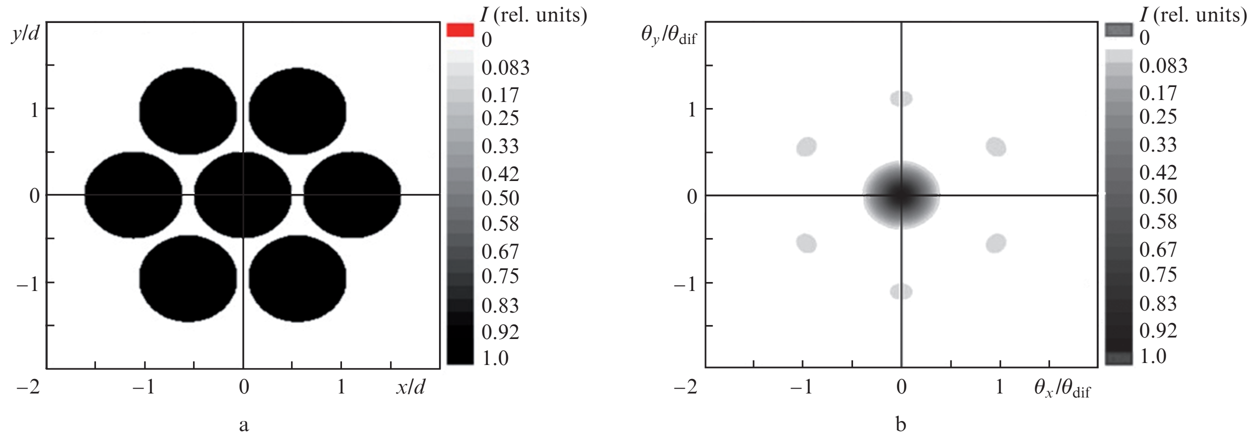


Figure 1. Intensity distributions I of the initial beam (a) at the system output in the near-field region and (b) in the focal plane of lens.

cal processes (diffraction and refraction) and directions x and y. The amplitude error in calculating the transverse differential operator in (1) is zero, and the phase error can be reduced to the fourth-sixth order [10]. Hence, the calculation accuracy is noticeably higher than that of ordinary spectral methods [11], in which the error rapidly increases with the path length L. The number of grid steps in the x and y directions was 1024, the number of steps in the z direction was determined by the path length and the Courant number equal to 0.1.

Influence of an optically heterogeneous medium on the radiation propagation process in the present work is characterised by the coherence radius r_0 of a spherical wave, which for the beam with a spherical wavefront (at $\lambda = 1$ μm) has the form [12]:

$$r_0 = (0.159 C_n^2 k^2 L)^{3/5}, \tag{2}$$

where C_n^2 is the structural constant of the medium refractive index, which is assumed constant across the path. The path length L is measured in diffraction length units for a single channel, $L_{\text{dif}} = kd^2$.

Optical heterogeneities of the medium are modelled by using the phase-screen method. Phase distortions in the beam passing through a layer of a continuous turbulent medium are substituted for equivalent changes in phase on the screen and propagation of radiation is considered as a process of its successive transit through the layers of vacuum and phase screens. The distance between the screens was determined in the calculations by the condition that the dispersion of phase fluctuations in a turbulent layer is well below unity. For a spatial spectrum of refractive index fluctuations, we used the von Karman model, and random phase distortions were realised on the screen according to the spectral method [13]. In calculations, the distances between screens were chosen from the condition that the dispersion of phase fluctuations in a turbulent layer should be much less than unity. Applicability of the spectral method for the von Karman spectrum of atmospheric turbulence has the limitations: it is necessary that the external scale of turbulence did not exceed half the calculation region. In the present work, the external scale of turbulence is $2d$.

In the ideal case where all the beams have similar phases and optical path heterogeneities are absent, the intensity distribution of radiation in the plane $z = L$, focused by a lens with a focal length $F = L$, normalised to the maximal value is shown in Fig. 1b.

In real conditions, the far-field pattern of radiation is distorted due to a combined influence of the initial phase mismatching in laser channels and optical heterogeneities along the propagation path. For reducing the influence of these factors and obtaining the maximal effect of phase matching of multichannel laser radiation we compare target illumination in the feedback loop and the regime of phase conjugation in various approximations (by taking into account or neglecting the tilt of the wavefront on sub-apertures). The efficiency of phase matching is estimated by the part of radiation fitting the diffraction-size diaphragm of the emitting seven-channel aperture.

3. Target irradiation in the case of phase conjugation

The phase matching problem for multichannel laser radiation passing through an optically heterogeneous medium in the case of phase conjugation is considered in the following way. The radiation of a reference source with the Gaussian amplitude distribution on a target is described by the formula

$$E(x, y; z = L) = A \exp\left[-\frac{x^2 + y^2}{w_0^2} - \frac{\mathrm{i}k(x^2 + y^2)}{2F}\right],$$

where A is a constant. The beam radius w_0 is chosen so that the angular dimension of the reference beam was constant regardless of the path length and equal to 0.22 of the diffraction divergence for the seven-channel aperture. The radiation beam propagates through a turbulent medium to the transmitting aperture of the laser and passes through a lens with a focal length $F = L$ set in the plane $z = 0$. Then, in the laser, the exit multichannel beam is formed of unit amplitude with the phase front $\psi(x, y; z = 0)$ complex-conjugate to the wavefront of reference radiation on each sub-aperture.

In calculations we consider both the exact phase conjugation and the approximate one, where the conjugate wavefront of the output beam on a sub-aperture $\psi(x, y)$ is approximated by a plane of the type $\alpha x + \beta y + \gamma$. In one case $\alpha = \beta = 0$, and the value of γ is determined as the average value of the phase on a sub-aperture, i.e., the wavefront is approximated by a plane wavefront without local tilts. In another case, the coefficients α, β and γ are determined by the least square method,

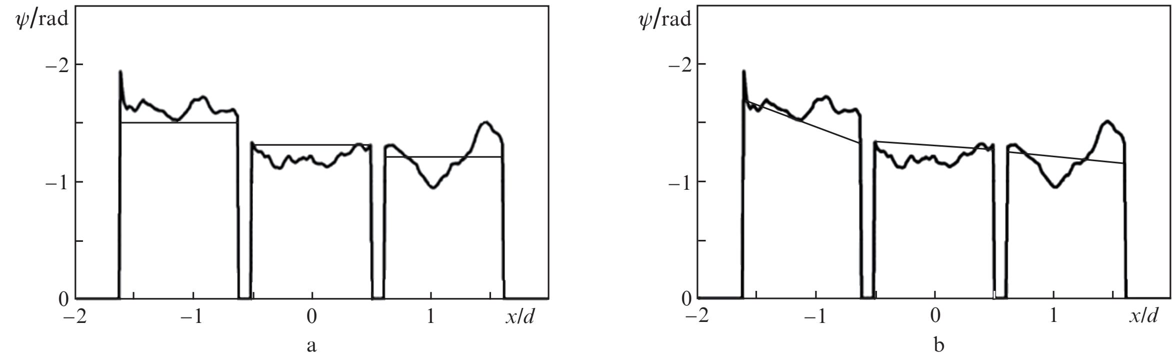


Figure 2. Approximation (thin lines) of the complex-conjugate wavefront (thick curves) (a) without and (b) with local tilts of the wavefront along one axis.

i.e., the wavefront of the output beam is approximated by the plane taking into account the average wavefront tilt within the limits of each sub-aperture. The approximation of a complex-conjugate wavefront with the allowance for the average tilt is more accurate but difficult in experimental realisation. An example approximation of the complex-conjugate wavefront for the reference beam is shown in Fig. 2 for the ratio of the coherence radius of the spherical wave to the sub-aperture size $r_0/d = 0.8$. The output beam with the complex-conjugate wavefront formed in this way is focused by a lens through an optically heterogeneous medium onto the target. For comparison, a similar system without the lens is also considered.

Actually, the complex-conjugate wavefront can be realised in the approximations mentioned above, for example, by measuring the wavefront of the reference beam by a Shack–Hartmann wave sensor, calculating the average value and local tilt of the wavefront within the limits of emitting sub-apertures and the following deformation of the total adaptive mirror or by organising phase shifts and inclinations on the sub-apertures.

4. Target irradiation in the feedback loop

Phase combining of a multichannel laser beam through a turbulent medium with the optical control of target irradiation in the feedback loop, when the reference beam is absent, is considered in the following statement. The output radiation of a seven-channel laser system is focused from the plane $z = 0$ by a lens with a focal distance $F = L$ through an optically heterogeneous medium onto a target placed in the plane $z = L$. Phase matching is performed by iteratively controlling the phase shifts and wavefront tilts, which is aimed at a search for the extremum of a certain parameter (an objective function) of the radiation passed to the target. This may be, for example, the maximum part of the total radiation power in a small angle around the optical axis. Again, in this formulation, in addition to the main configuration of the system, the case is also considered where the focusing lens at the system output is absent. During laser operation, the phase of radiation in the channels is controlled by using the stochastic parallel gradient (SPG) algorithm [9]. The employed SPG algorithm includes two stages. At the first stage, a test phase variation is performed in channels (the phase shift or wavefront tilt), and then the corresponding change of the objective function on the target is calculated. At the second stage, the phase in channels is corrected based on the obtained value of the objective function until the latter reaches the limiting value corresponding to a certain criterion.

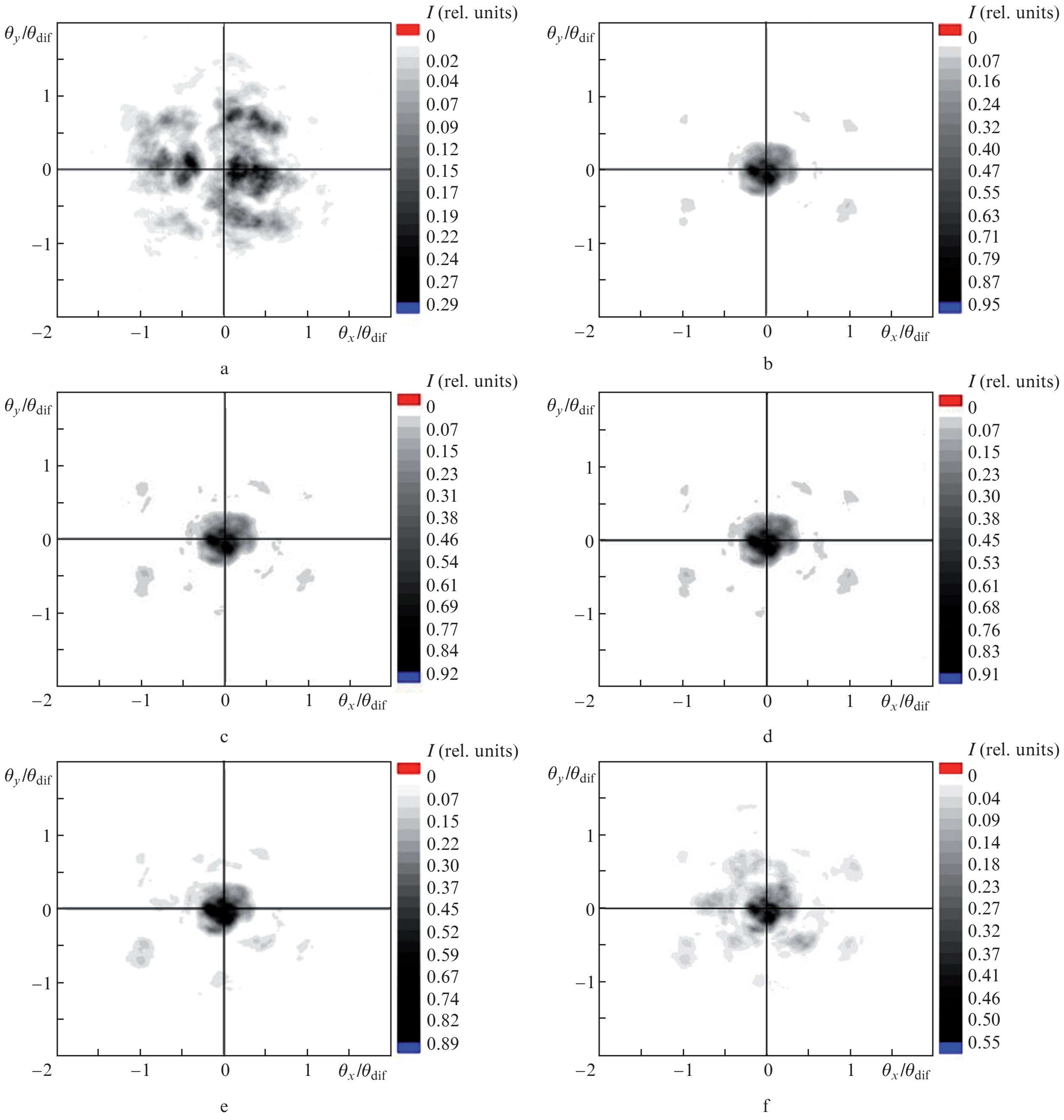
Initially the phase in channels is a random value uniformly distributed over the interval $[0, 2\pi)$. The whole SPG process of controlling the phase of the output beam can be divided into two parts. First, only phase shifts in channels are corrected; the control makes no allowance for local wavefront tilts on the sub-apertures. When the corresponding change in the objective function stops, i.e., an extremum is attained, the wavefront tilts in the channels are corrected by the SPG algorithm. In both cases, the objective function was part of the radiation power in the solid angle equal to half the diffraction angle for the emitting seven-channel aperture. Calculations show that for attaining the maximum of the objective function at optimal parameters of the SPG algorithm, 50--100 iterations will suffice as in the first so and in the second cases.

5. Calculation results and discussion

Calculations were performed for three path lengths: $L = 0.13L_{\text{dif}}$, $0.5L_{\text{dif}}$ and L_{dif}. The phase screens of the medium were the same in the numerical modelling of target irradiation by multichannel laser radiation with the phase conjugation and in the feedback loop. Interesting, how efficient is each of the approaches as compared to setting the exact phase matching within the emitting sub-apertures, which, seemingly, provides the best phase matching conditions.

As an example, Fig. 3 shows intensity distributions for combined radiation in the target plane normalised to the maximal value (without turbulence), obtained without phase conjugation (only simple focusing) and with the phase conjugation in various approximations. The length of the optically heterogeneous path is $L = 0.5L_{\text{dif}}$ and the ratio of the coherence radius of spherical wave to the sub-aperture size is $r_0/d = 0.8$. The intensity distributions of radiation in a target plane normalised to the maximal value (without turbulence), obtained under target illumination in the feedback loop in the same conditions are presented in Fig. 4. In both cases, results for the system configuration without a focusing lens are also presented.

One can see from Fig. 3 that in the case of phase matching of seven-beam radiation through an optically heterogeneous medium, the maximal radiation flux density on the target is actually provided by the exact phase conjugation (Fig. 3b). However, even in this case the picture differs from the ideal one (see Fig. 1b).



a

b

c

d

e

f

Figure 3. Intensity distributions in the target plane: (a) without phase matching, (b) exact phase conjugation, phase conjugation with local WF tilts taken into account (c, e) and neglected (d, f); (a, b, c, d) with the lens at the system output and (e, f) without the lens; $L = 0.5L_{\rm dif}$, $r_0/d = 0.8$.

In addition, one can see that intensity distributions under target irradiation with phase conjugation and in the feedback loop do not differ noticeably. Allowance made for local wavefront tilts on sub-apertures relatively weakly affects the result in both approaches, but only if there is the lens at the system output. Without the lens, the allowance made for the wavefront tilts introduces a more pronounced contribution into the phase matching efficiency.

Figure 5 presents a typical dependence of the part of the radiation power in a diffraction angle ΔP normalised to the same part without turbulence $\Delta P_{\rm max}$ on the iteration number in the process of the SPG algorithm operation at $L = 0.5L_{\rm dif}$ and $r_0/d = 0.8$.

Figures 3–5 illustrate operation of the phase matching system in similar conditions. Figure 6 presents generalised results: the normalised part of the power versus the ratio of the coherence radius of a spherical wave to the sub-aperture dimension at three characteristic path lengths, obtained with phase conjugation in various approximations. The dependences are averaged over three realisations of random phase screens of the path. Figure 7 shows similar results under target irradiation in the feedback loop.

The efficiency of focusing at perfect phase conjugation (Figs 6a–c) and the same r_0 reduces at longer path lengths. The main reason, similarly, is appearance of strong intensity fluctuations ('scintillations') of the reference beam [14, 15]. Indeed, the Rytov number σ^2 [16], which characterises the regular regime of beam propagation at $\sigma^2 \ll 1$ and generation of singularities (speckles) at $\sigma^2 \geqslant 1$, for Kolmogorov spectrum, taking into account (1), can be written as

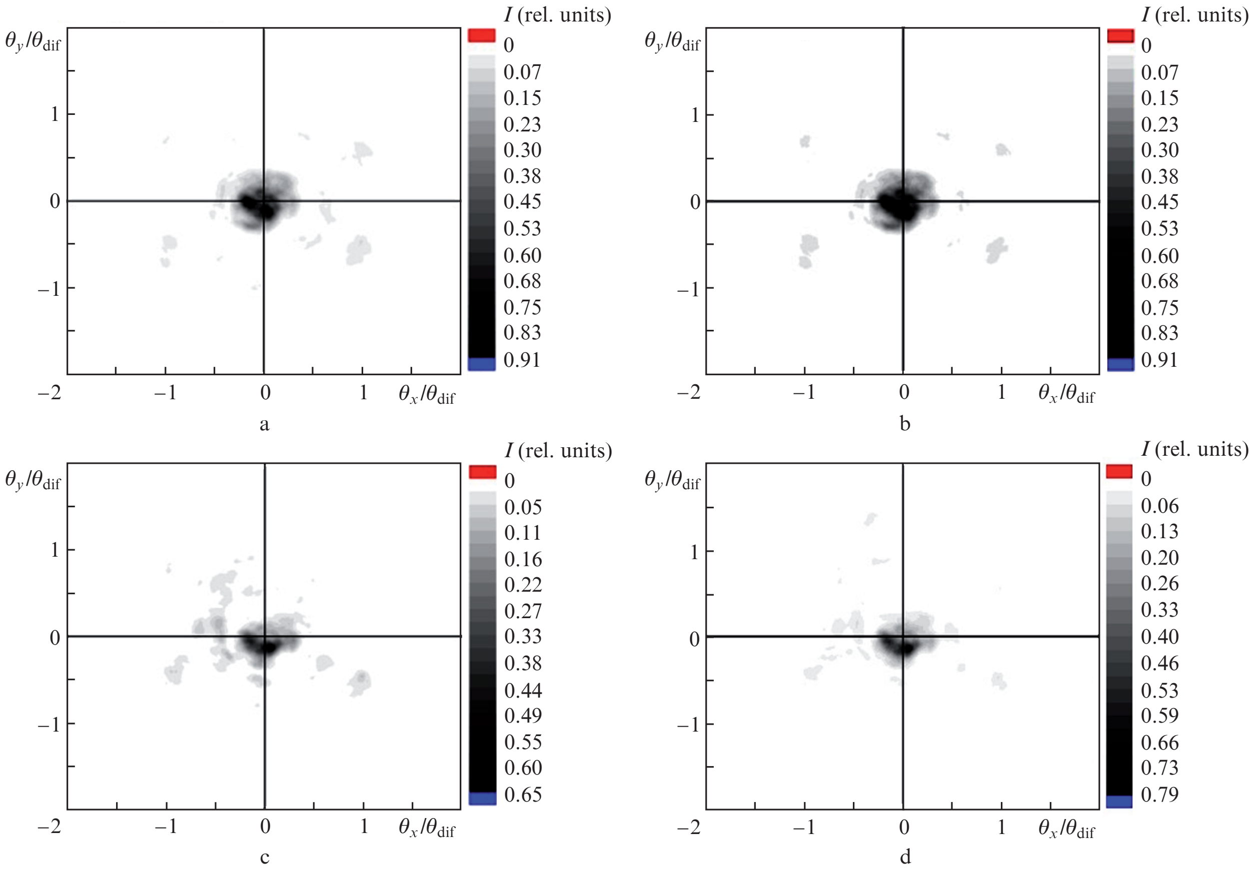


Figure 4. Intensity distributions I in the target plane with irradiation of the target in the feedback loop (a, c) without and (b, d) with local wavefront tilts; (a, b) with the lens and (c, d) without it at the system output; $L = 0.5L_{\rm dif}$, $r_0/d = 0.8$.

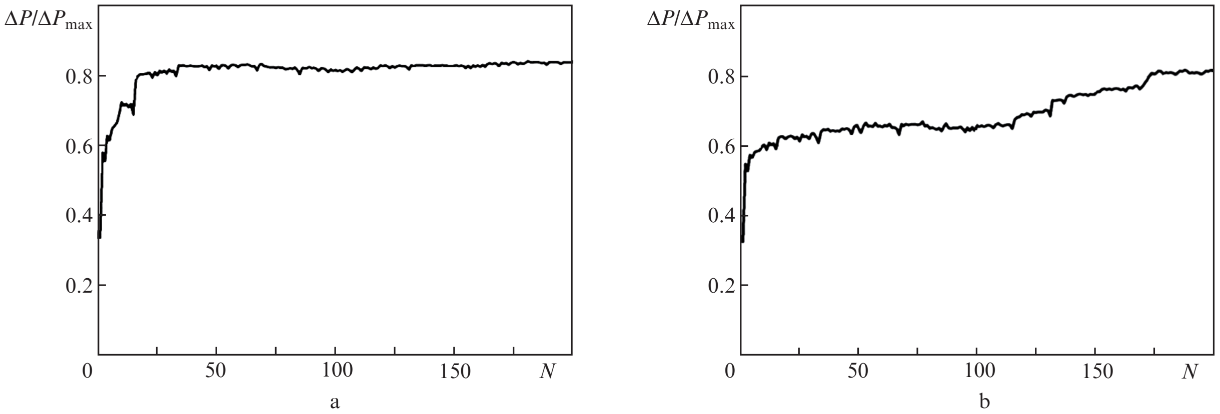


Figure 5. Typical dependence of part of the power in the diffraction angle on the iteration number N of the SPG algorithm in the process of phasing (a) with the collecting lens at the system output and (b) without the lens.

$$\sigma^2 \approx C_n^2 k^{7/6} L^{11/6} \approx \left[\frac{d^2}{r_0^2}\frac{L}{L_{\rm dif}}\right].$$

One can see that at a greater $L/L_{\rm dif}$ this parameter increases. In the case of scintillations, more actual becomes the employment of the amplitude–phase correction (i.e., phase conjugation) for phase matching [17].

From the results presented in Figs 6a–c and 7a–c one may conclude that if there is a focusing lens at the system output, a high efficiency of focusing of multichannel radiation can only be discussed in the conditions where the coherence radius of a spherical wave r_0 is approximately equal to or above the sub-aperture size d. At a lower ratio r_0/d, i.e., higher turbulence, both phase conjugation and target irradiation in the feedback loop lose the efficiency. In this case, the main effect is provided by a proper phase shift in channels, whereas the additional allowance made for local wavefront tilts on the sub-aperture weakly affects the result. One can also assert that focusing of the seven-channel laser radiation on the target in the feedback loop through an optically heterogeneous medium is, anyway, as good in efficiency as phase conjugation.

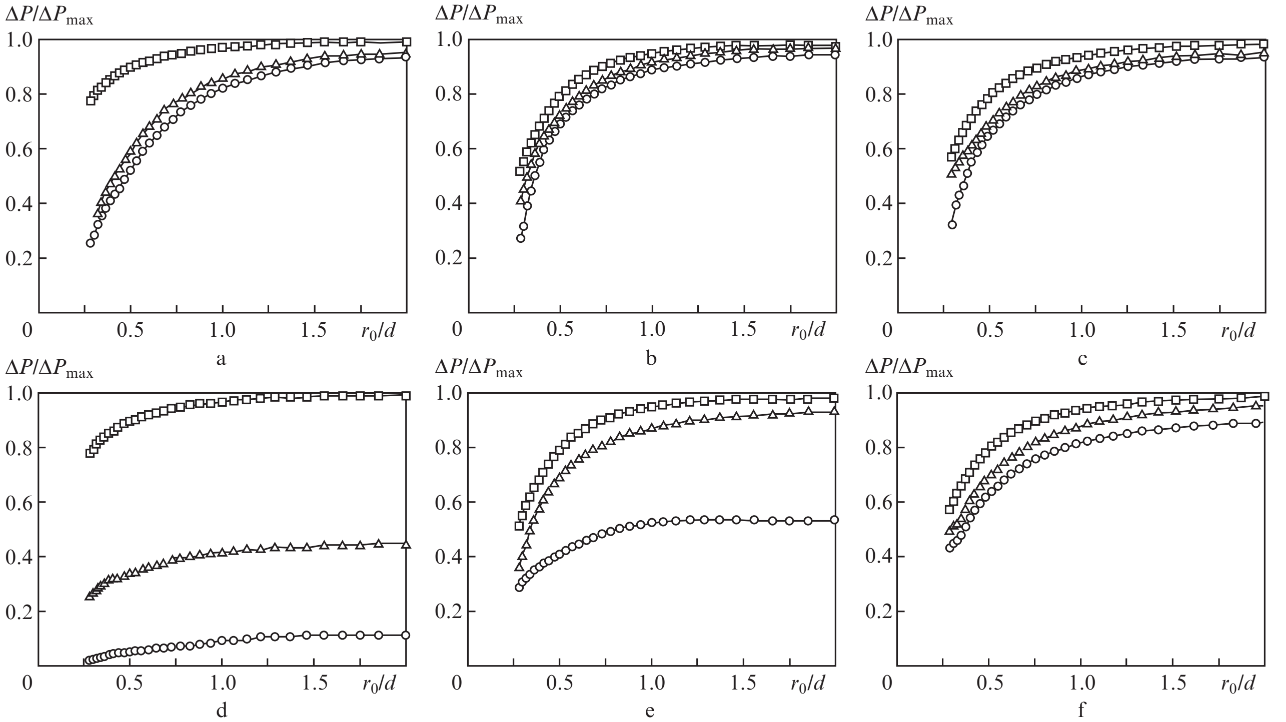


Figure 6. Dependences of $\Delta P/\Delta P_{\max}$ at the target on the ratio of the spherical wave coherence radius r_0 to the sub-aperture size d at $L/L_{\rm dif}$ = (a, d) 0.13, (b, e) 0.5 and (c, f) 1, (a–c) with the lens at system output and (d–f) without the lens, obtained at the exact phase conjugation (squares) and at approximate phase conjugation with the local wavefront tilts taken into account (triangles) and neglected (circles).

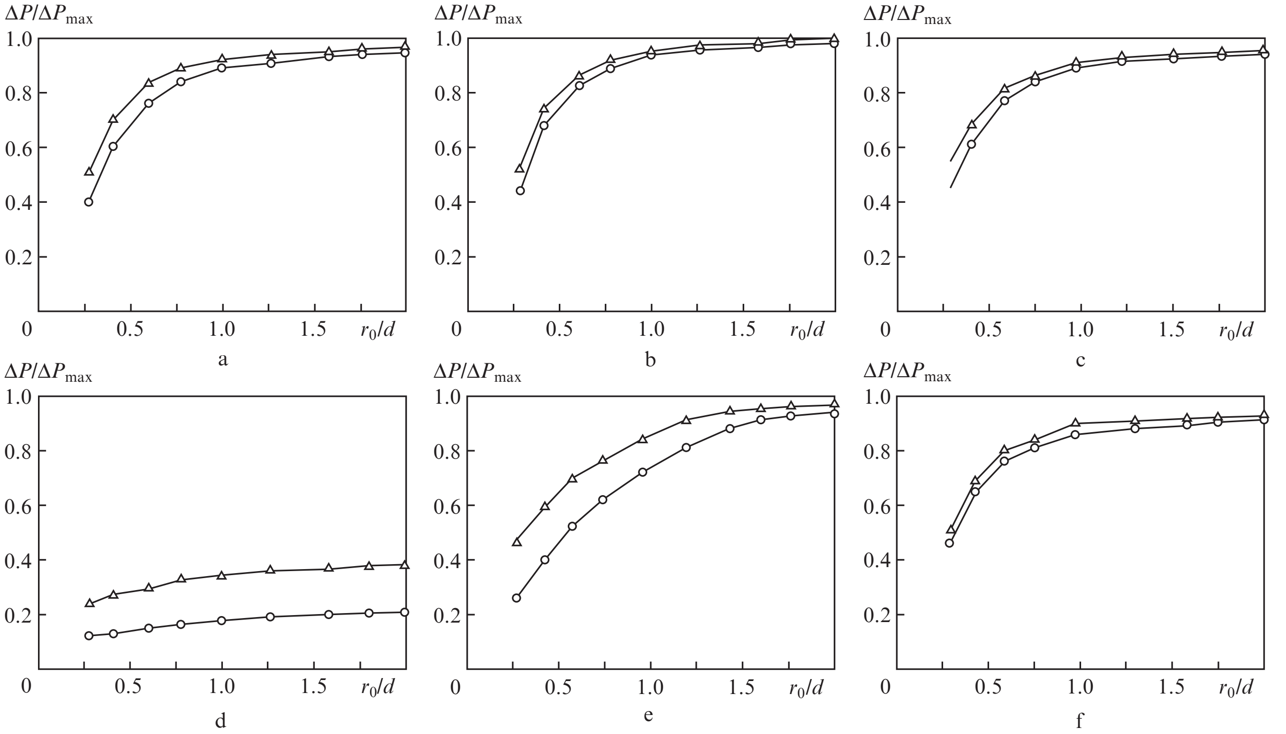


Figure 7. Similar to Fig. 6 but with target irradiation in the feedback loop in the case of the SPG phase control.

Without the collecting lens at the system output (Figs 6d–f and 7d–f) the situation is more complicated. The efficiency of focusing to relatively short distances (less than half the diffraction length $L_{\rm dif}$) is low for all ratios r_0/d; the allowance made for wavefront tilts increases it, however, to moderate values. In focusing multichannel laser radiation to long distances (of about half the $L_{\rm dif}$) taking into account wavefront tilts becomes the key factor and makes it possible to reach a high efficiency at $r_0/d \geqslant 1$. Finally, in focusing to long distances (approximately equal to $L_{\rm dif}$ or longer) one may neglect the wavefront tilts; a high efficiency of focusing may be provided at $r_0/d \geqslant 1$ by choosing only phase shifts in channels.

6. Conclusions

The present work is devoted to a numerical study of the problem of phase matching of multichannel laser radiation to a target through an optically heterogeneous medium (turbulent atmosphere). The efficiencies of target illumination under phase conjugation and in the feedback loop have been compared in various approximations. The employment of the phase conjugation method requires a preliminary formation of a reference beam, which, propagating from the target to the input aperture of the laser system, transfers information about optical heterogeneities of the path. Formation of the wavefront in an initial beam, which is conjugate to the wavefront of the reference beam, compensates for the influence of an optically heterogeneous medium. While illuminating the target in the feedback loop, the phase of initial radiation is controlled based on the SPG algorithm by maximising certain parameters of the initial beam passed to the target (or reflected from it).

Analysis of the results obtained shows that target irradiation in the feedback loop is highly competitive with the method of phase conjugation by the efficiency of focusing of multichannel laser radiation through an optically heterogeneous medium. This efficiency is high if the coherence radius of the spherical wave r_0 is approximately equal to or greater than the sub-aperture d and the collecting lens is set at the system output. In this case, the main contribution to the efficiency is made by phase shifts in the channels, whereas the allowance made for local wavefront tilts on sub-apertures relatively weakly affects the result.

Without a focusing lens, a high focusing efficiency cannot be attained at the focusing length substantially less than the diffraction sub-aperture size L_{dif} at all ratio values r_0/d. In focusing a multichannel laser beam to long distances greater than L_{dif}, a high efficiency can obtained at $r_0/d \geqslant 1$ by choosing only the phase shifts in channels, neglecting local wavefront tilts.

References

1. Likhanskii V.V., Napartovich A.P. *Usp. Fiz. Nauk*, **160** (3), 101 (1990) [*Sov. Phys. Usp*., **33** (3), 228 (1990)].
2. Fan T.Y. *IEEE J. Sel. Top. Quantum Electron*., **11**, 567 (2005).
3. Lhermite J., Suran E., Kermene V., et al. *Opt. Express*, **18**, 4783 (2010).
4. Anderegg J., Brosnan S., Cheung E., et al. *Proc. SPIE Int. Soc. Opt. Eng*., **6102**, 61020U (2006).
5. Volkov M.V., Garanin S.G., Dolgopolov Yu.V., Kopalkin A.V., Kulikov S.M., Starikov F.A., Sukharev S.A., Tyutin S.V., Khokhlov S.V., Chaparin D.A. *Kvantovaya Elektron*., **44**, 1039 (2014) [*Quantum Electron*., **44**, 1039 (2014)].
6. Lukin V.P. *Usp. Fiz. Nauk*, **184**, 599 (2014) [*Phys. Usp*., **57**, 556 (2014)].
7. Weyrauch T., Vorontsov M., Carhart G.W., Beresnev L.A., Rostov A.P., Polnau E.E., Liu J.J. *Opt. Lett*., **36**, 22 (2011).
8. Xinyang L., Geng C., Zhan X., Rao C. *Proc. SPIE Int. Soc. Opt. Eng*., **8178**, 81780M (2011).
9. Volkov V.A., Volkov M.V., Garanin S.G., Dolgopolov Yu.V., Kopalkin A.V., Kulikov S.M., Starikov F.A., Sukharev S.A., Tyutin S.V., Khokhlov S.V., Chaparin D.A. *Kvantovaya Elektron*., **43**, 852 (2013) [*Quantum Electron*., **43**, 852 (2013)].
10. Ladagin V.K. *Vopr. Atom. Nauki Tekh., Ser. Algoritmy Metody Chislennogo Resheniya Zadach Mat. Fiziki*, (1), 19 (1985).
11. Feit M.D., Fleck J.A. *J. Opt. Soc. Am. B*, **7**, 633 (1988).
12. Fried D.L. *J. Opt. Soc. Am*., **56** (10), 1380 (1966).
13. Kandidov V.P. *Usp. Fiz. Nauk*, **166**, 1309 (1996) [*Phys. Usp*., **39**, 1243 (1996)].
14. Tatarskii V.I. *Rasprostranenie voln v turbulentnoi atmosphere* (Wave Propagation in Turbulent Atmosphere) (Moscow: Nauka, 1967).
15. Lukin V.P., Fortes B.V. *Opt. Atmos. Okeana*, **9**, 1492 (1996) [*Atmos. Ocean. Opt*., **9** (11), 948 (1996)].
16. Isimaru A. *Wave Propagation and Scattering in Random Media* (New York: Acad. Press, 1978; Moscow: Nauka, 1966).
17. Bogachev V.A., Garanin S.G., Dolgopolov Yu.G., Kopalkin A.V., Kulikov S.M., Starikov F.A., Sukharev S.A., Feoktistov V.V. *Kvantovaya Elektron*., **42**, 531 (2012) [*Quantum Electron*., **42**, 531 (2012)].