

NONLINEAR OPTICAL PHENOMENA

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Generation of solitary waves from continuous radiation in a nonlinear oppositely directed coupler

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Abstract. **We consider a nonlinear coupler formed by two tunnel-coupled waveguides, one waveguide being made of a conventional dielectric and the other – of a negative-index material. The possibility of the formation of solitary waves from continuous radiation having a constant intensity is shown provided that the radiation is coupled into the input of a negative-index coupler channel (on the back side of the waveguide system). With increasing intensity of the input light, the speed and amplitude of the generated solitary waves increase and the period of their formation is reduced.**

Keywords: *optical solitons, tunnel-coupled waveguides, forward and backward waves, metamaterials.*

1. Introduction

Nonlinear optics of negative-index media is being intensively developed at present [1–6]. Thus, it turns out that some well-known phenomena of nonlinear optics that take place in conventional positive-index media can be observed in negative-index media. As an example, we can mention parametric processes (harmonic generation [7–14] and parametric amplification [15–18]), optical bistability [19–23], solitons in arrays of coupled waveguides [24–32] and nonlinear surface waves [33–36]. Peculiarities of nonlinear optical phenomena in negative-index media are caused by the interaction of forward and backward waves. In contrast to forward waves, the phase velocity and the Poynting vector of backward waves are oppositely directed. A review of nonlinear phenomena in negative-index media is given in [37, 38] and in a recent book [39].

A simple optical device that provides the interaction of forward and backward waves is a system of two tunnel-coupled waveguides, the refractive index of one of them being positive, and the other – negative. In weak optical fields, when nonlinear properties of a waveguide can be neglected, the device acts as a distributed mirror: light entering into one of the waveguides is outcoupled from the second waveguide. If both waveguides are made of the same material, the propagation direction of light does not change. This device is known in integrated optics as a directional (or waveguide) coupler [40]. The coupler, changing the propagation direction of light, will be called an oppositely directed coupler. The authors of [24, 25] considered an extended nonlinear oppositely directed (anti-directional) coupler (ODC) and found solutions corresponding to a stationary electromagnetic wave propagating through tunnel-coupled waveguides in the form of a coupled solitary wave. Solitary waves are the waves that are localised at each instant of time in a finite region of space or localised at each point of space in a finite time interval. Based on the analogy between the properties of such pulses for a nonlinear Bragg waveguide and an ODC, a steady-state solitary wave in an ODC was also called a gap soliton.

As in many other cases, when there appears a steady-state solitary wave, including a soliton, the formation of a solitary wave requires that the energy of the initial pulse exceeds a certain threshold. The process of the formation of a gap soliton from an electromagnetic pulse coupled to the input of one of the ODC waveguides was considered in [30].

All steady-state solutions to equations describing the propagation of light in an ODC were obtained and listed in [29]. In addition to solutions in the form of solitary waves (solitons), there are periodic solutions describing cnoidal waves. Cnoidal waves can be generated from an initially periodically amplitude-modulated wave. The modulation instability of a wave with constant amplitude could lead to the formation of cnoidal waves [41]. Kudryashov et al. [29] found a solution which describes a wave localised in a finite time interval. Such waves are called compactons [42, 43]. An infinite array of compactons can also be attributed to periodic stationary waves. The question of how and from which initial field distribution steady-state (e.g., periodic) waves are formed is one of the main issues of the theory of nonlinear waves.

In this paper we have discovered a phenomenon of generation of steady-state solitary waves in an ODC from continuous radiation with constant amplitude, specified at the input to a negative-index channel, i.e. on the back side of the coupler. The number of solitary waves generated during the considered time interval depends on the amplitude of radiation and increases with its growth. In this case, the distance on the timeline between neighbouring solitary waves, called the period of their formation, is reduced. Thus, in an ODC a continuous wave can be transformed into a series of solitons.

2. Basic equations of the ODC model

We consider a pair of tunnel-coupled waveguides, one of which is made of a conventional nonlinear optical dielectric, and the

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other – of a material having linear optical properties and a negative refractive index. The linear properties of the first waveguide are determined by the dielectric constant $\varepsilon_1(\omega_0)$ at the carrier wave frequency ω_0, and its permeability is equal to unity. It is assumed that both waveguides are made of a material that is transparent at frequency ω_0. Existing negative-index materials exhibit losses, but active research is being conducted in the world to reduce and compensate for such losses. The propagation of waves in each channel is characterised by the group velocities v_{g1} and v_{g2} and tunnel coupling constants K_{12} and K_{21}. It is assumed that the waveguides are sufficiently small and therefore the second-order group velocity dispersion can be neglected.

The system of equations describing the interaction of waves in the ODC, obtained in [37] and used in [25], has the form:

$$\mathrm{i}\left(\frac{\partial}{\partial z}+\frac{1}{v_{g1}}\frac{\partial}{\partial t}\right)E_1+K_{12}E_2\mathrm{e}^{\mathrm{i}\Delta\beta z}+\frac{2\pi\omega_0}{c\sqrt{\varepsilon_1(\omega_0)}}\chi^{(3)}_{\mathrm{eff}}|E_1|^2E_1=0,$$
$$\mathrm{i}\left(\frac{\partial}{\partial z}-\frac{1}{v_{g2}}\frac{\partial}{\partial t}\right)E_2-K_{21}E_1\mathrm{e}^{-\mathrm{i}\Delta\beta z}=0, \quad (1)$$

where the parameter $\Delta\beta$ – the difference between the propagation constants of the waves localised in the adjacent waveguides – is a measure of a mismatch of phase velocities of these waves. Nonlinear properties of the first waveguide are characterised by the effective third-order nonlinear susceptibility $\chi^{(3)}_{\mathrm{eff}}$. For equations (1) to be solved numerically, it is convenient to use, instead of electric fields $E_{1,2}(z,t)$, normalised dimensionless fields $e_{1,2}(z,t)$:

$$E_1=\mathrm{A}_0e_1\mathrm{e}^{-\mathrm{i}\Delta\beta z},\ E_2=A_0\sqrt{K_{21}/K_{12}}\,e_2\mathrm{e}^{\mathrm{i}\Delta\beta z},$$

as well as normalised spatial ζ and time τ variables:

$$\zeta=z/L_{\mathrm{c}},\ \tau=t_0^{-1}(t-z/V_0),$$

where

$$L_{\mathrm{c}}=(K_{12}K_{21})^{-1/2},\ t_0=L_{\mathrm{c}}(v_{g1}+v_{g2})/(2v_{g1}v_{g2}),$$

$$V_0^{-1}=(v_{g2}-v_{g1})/(2\,v_{g1}v_{g2}).$$

It is further assumed that the condition of the wave synchronism $\Delta\beta=0$ is met.

The system of equations for the normalised slowly varying envelopes of the electric fields e_1 and e_2 in a positive-index waveguide and a negative-index waveguide, respectively, has the form:

$$\mathrm{i}\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\tau}\right)e_1+e_2+r|e_1|^2e_1=0,$$
$$\mathrm{i}\left(\frac{\partial}{\partial\zeta}-\frac{\partial}{\partial\tau}\right)e_2-e_1=0, \quad (2)$$

where the coefficient

$$r=\frac{2\pi\omega_0A_0^2\chi^{(3)}_{\mathrm{eff}}}{c\sqrt{\varepsilon_1(\omega_0)}K_{12}K_{21}} \quad (3)$$

is a dimensionless parameter characterising the nonlinearity of the positive-index waveguide.

In [24, 25] we have found solutions corresponding to a stationary pulse of the electromagnetic field, which propagates in tunnel-coupled waveguides as a whole. This solitary wave was called a gap soliton, because the spectrum of linear waves in the vicinity of the carrier wave frequency has a band gap, similar to that in a Bragg waveguide.

3. Steady-state solutions in the form of a solitary wave

Steady-state solutions in the form of solitary waves for the model in question were found in [25]. The normalised electric field strengths $e_{1,2}$ can be represented as fields with real amplitudes $a_{1,2}$ and phases $\phi_{1,2}$: $e_1=a_1\mathrm{e}^{\mathrm{i}\phi_1}$, $e_2=a_2\mathrm{e}^{\mathrm{i}\phi_2}$. To find the steady-state solutions in the form of a travelling wave, we assume that these solutions depend on one variable

$$\eta=\frac{\zeta+\beta\tau}{\sqrt{1-\beta^2}},$$

where β is the propagation velocity of a solitary wave. The resulting system of ordinary real differential equations allows one to find amplitudes and phases. The amplitudes $a_{1,2}$ of the solitary wave components in positive and negative-index waveguides are defined by the relationships:

$$a_1^2(\eta)=\frac{4}{\Theta(1+\beta)\cosh 2(\eta-\eta_0)},$$
$$a_2^2(\eta)=\frac{4}{\Theta(1-\beta)\cosh 2(\eta-\eta_0)}. \quad (4)$$

Parameter Θ depends on the nonlinearity coefficient r and is defined as

$$\Theta=\frac{r}{1+\beta}\sqrt{\frac{1-\beta}{1+\beta}}.$$

This indicates that the parameter β is limited in quantity: $|\beta|<1$. The solitary wave phases $\phi_{1,2}$

$$\phi_1(\eta)=\phi_1(-\infty)+3\arctan[\exp 2(\eta-\eta_0)],$$
$$\phi_2(\eta)=\phi_2(-\infty)+\arctan[\exp 2(\eta-\eta_0)] \quad (5)$$

must satisfy the condition

$$\cos[\phi_1(-\infty)-\phi_2(-\infty)]=0, \quad (6)$$

which is fulfilled at $\phi_1(-\infty)=0$, and $\phi_2(-\infty)=-\pi/2$. The parameter η_0 determines the position of the centre of a solitary wave.

Note that it follows from the definition of the variable η that in light-cone coordinates (ζ,τ) the propagation velocity of a soliton is $-\beta$. And the same parameter β determines

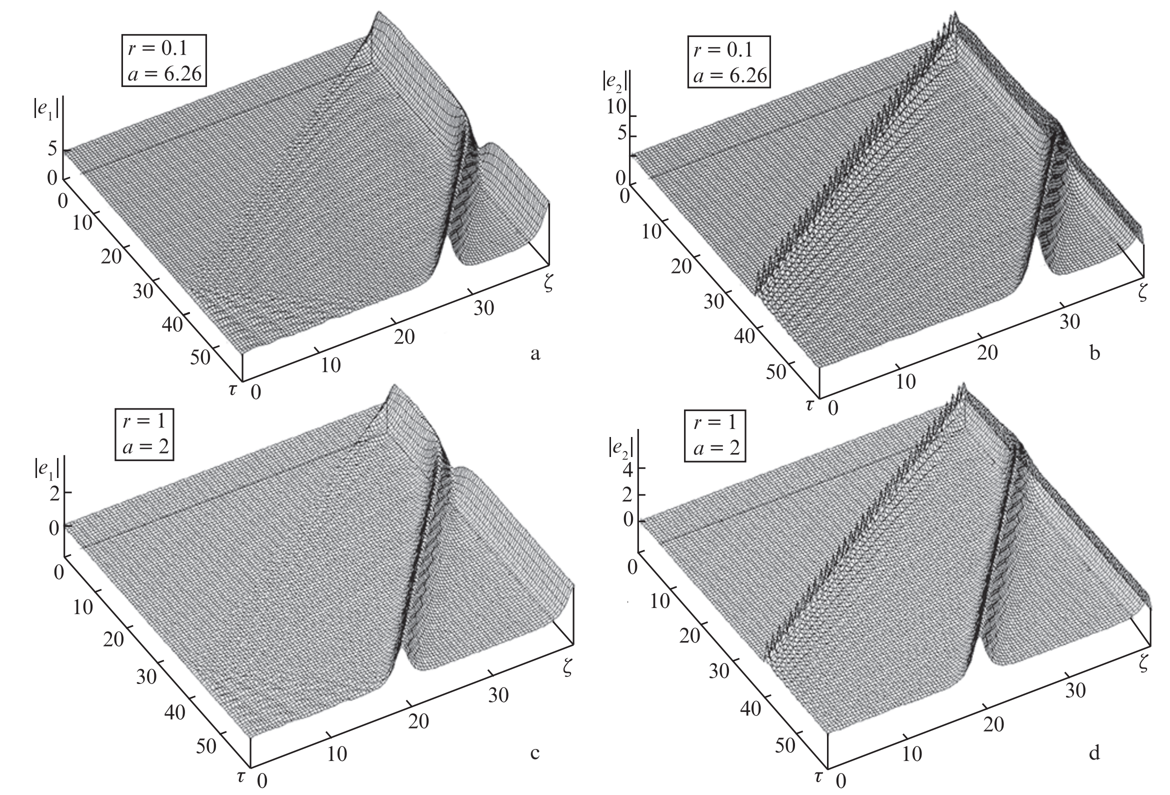

Figure 1. Generation of a single solitary wave in the ODC at (a, b) the nonlinearity parameter $r = 0.1$ and amplitude $a = 6.26$ and (c, d) $r = 1$ and $a = 2$ in (a, c) a positive-index channel and (b, d) in a negative-index channel.

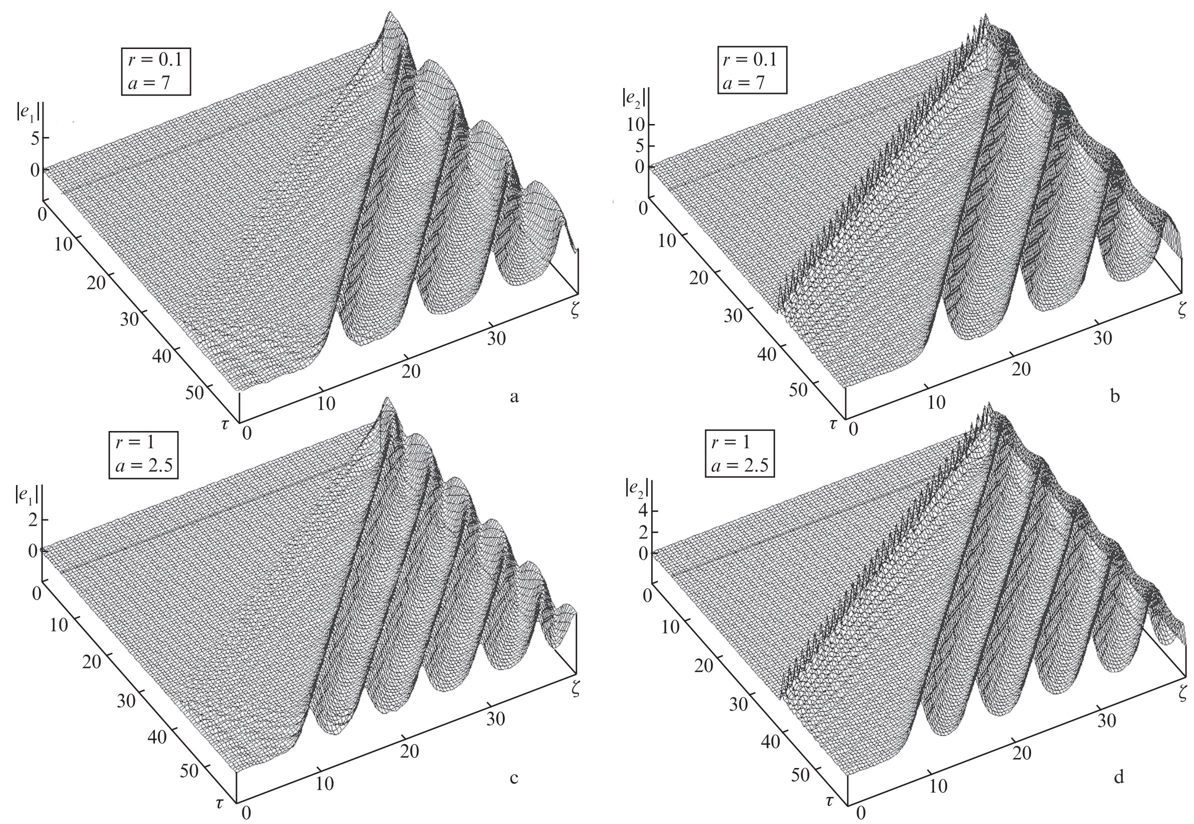

Figure 2. Generation of several solitary waves from continuous radiation with constant amplitude a, specified (at $\zeta_L = 40$) at the input to the negative-index ODC channel at (a, b) $r = 0.1$, $a = 7$ and (c, d) $r = 1$, $a = 2.5$ in (a, c) a positive-index channel and (b, d) in a negative-index channel.

the amplitude of partial waves coupled into a soliton and each localised in its waveguide. This means that solitons with the same amplitude move with the same velocity. For the soliton velocity (more precisely, for the projection of the velocity vector on the z axis) V_s we can obtain an expression

$$\frac{1}{V_s} = \frac{1}{2v_{g1}v_{g2}}\left[v_{g2}\left(1-\frac{1}{\beta}\right) - v_{g1}\left(1+\frac{1}{\beta}\right)\right],$$

defining it through the group velocities of the partial waves v_{g1}, v_{g2} and the parameter $|\beta| < 1$.

4. Generation of solitary waves from continuous radiation with constant amplitude

The system of equations (2) has been solved numerically with the following boundary conditions:

$$e_1(\tau, \zeta = 0) = 0, \quad e_2(\tau, \zeta_L) = a.$$

The coefficient a determines the amplitude of the constant input radiation, which is set at $\zeta_L = L/L_c$ in a negative-index ODC channel of length L. In the numerical simulation the nonlinearity parameter r in the system of equations (2) has been set in the range of 0.1 to 1, and the dimensionless length of the ODC was $\zeta_L = 40$.

Solitary waves in the ODC with nonlinearity in a positive-index channel are produced from a continuous wave with constant amplitude, specified at the input to a negative-index channel of the coupler, i.e. the back side of the waveguide system. At $r = 0.1$ and $a < 6$, and at $r = 1$ and $a < 2$ solitary waves in the ODC were not formed, and the main part of radiation, partially penetrating into the ODC, remained localised near ζ_L. It is known that the ODC acts like a mirror for the pulse with an amplitude that is less than a certain threshold [30, 44]. In this case we also observed the existence of the threshold required for the generation of a solitary wave from continuous radiation with constant intensity. If the amplitude of this radiation exceeds a certain threshold value a_{th}, then due to the nonlinear phase modulation there occurs generation of solitary waves propagating in the ODC.

To assess a_{th}, we will use the following assumptions. At some point in time the value of the radiation intensity in both waveguides will be the same because of the interaction of the waves in the waveguides. In this case, the instantaneous frequency of radiation, as follows from equations (2), is shifted by $0.5\,ra^2$. If the system of equations (2) is linearised, it is possible to determine in a standard way (see [25]) the spectrum of the linear waves in the coupler under study: $\nu^2 = 1 + q^2$ (here ν is the frequency in terms of the carrier frequency ω_0 and q is the wavenumber in terms of the wavenumber of the carrier wave). This formula implies the existence of a gap (band gap) in the spectrum of the linear waves, the width of the gap being equal to 2 in units of ω_0. The threshold intensity can be estimated by setting the value of $0.5\,ra_{th}^2$ equal to the width of the gap in the spectrum of the linear waves, then $a_{th}^2 \approx 4/r$. At $r = 0.1$, $a_{th} \approx 6.32$, and at $r = 1$, $a_{th} \approx 2$. These values of a_{th} are close to those obtained in the calculation of the threshold amplitude of radiation at which solitary waves are formed in the ODC (see Figs 1 and 2).

Figures 1–3 shows the plots of the distributions of $|e_1|$ and $|e_2|$ in the positive-index and negative-index ODC channels, respectively.

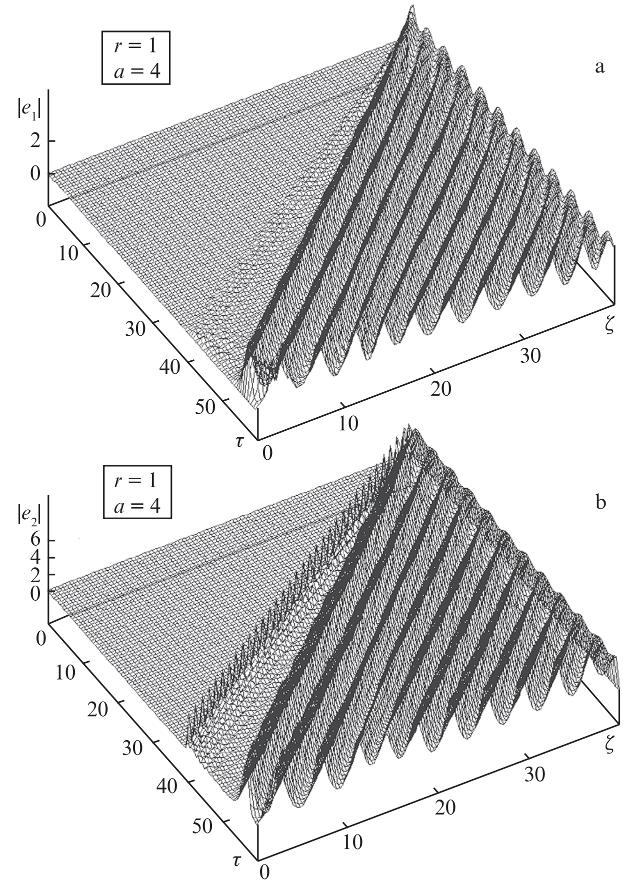

Figure 3. Solitary waves arising in (a) positive- and (b) negative-index ODC channels at $r = 1$, $a = 4$.

The value of a_{th}, at which the first solitary wave is generated, decreases with increasing nonlinearity parameter r (see Figs 1 and 4). An increase in the amplitude a reduces the time required for the formation of the first solitary wave (see Figs 1 and 2), as well as the period of the formation of solitary waves T, i.e. the time interval (along the τ axis) necessary for the emergence of each of the following solitary waves (Fig. 4a). Figure 4b demonstrates the dependence of the solitary wave velocity parameter β on the amplitude a. For the same values of a, the parameter β of the produced solitary waves is greater at larger values of the nonlinearity parameter r. An increase in a (for a given value of r) increases the propagation velocity and amplitude of solitary waves formed from the input radiation with amplitude a.

5. Conclusions

We have considered the interaction of forward and backward waves in a nonlinear oppositely directed coupler which represents two closely spaced waveguides, one of which is made of a nonlinear positive-index dielectric, and the other – of a linear negative-index material. Because this coupler (being infinitely long) does not transmit weak waves, acting as a distributed mirror [25], it is necessary for the input intensity to exceed the threshold value. Then, a coupled pair of solitary waves is formed, each localised in its coupler channel.

Increasing the intensity of continuous radiation at the input to a negative-index ODC channel increases the number

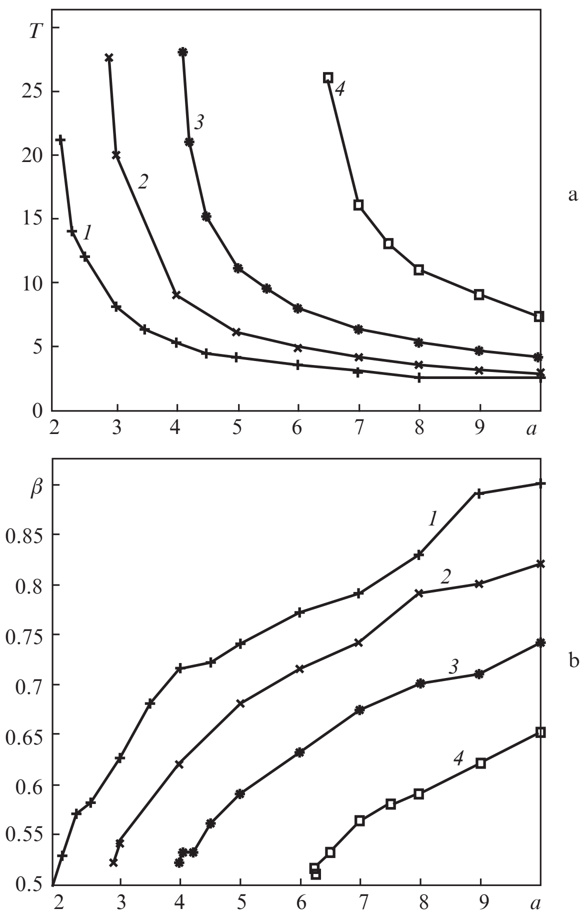

Figure 4. (a) Period T, during which solitary waves are generated, and (b) velocity parameter β of solitary waves as functions of amplitude a of a continuous wave at $r = 1$ (*1*), 0.5 (*2*), 0.25 (*3*) 0.25 and 0.1 (*4*).

of solitary waves appearing on the considered time interval. The velocity and the amplitude of the solitary waves increase with increasing amplitude of the input continuous radiation; however, the velocity of the solitary wave does not exceed the velocity of the linear wave. The threshold value of the amplitude of continuous radiation, at which the first solitary wave is formed, decreases with increasing nonlinearity parameter r of a positive-index channel of a nonlinear oppositely directed coupler.

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