Quantum Electronics **46** (2) 159–162 (2016)

NANOOPTICS DOI: 10.1070/QEL15861

Extremely tight focusing of light at the nanoapex of a metal microtip

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Abstract. **We study the focusing of an optical electromagnetic wave into a nanoscale spatial region in the vicinity of the nanoapex of a metal microtip, arising due to a convergent surface plasmon wave. The metal boundary near the nanoapex is approximated by a paraboloid of revolution. It is proved that an increase in the metal absorption in approaching the nanoapex, associated with an increase in the frequency of collisions of electrons with the surface, is an essential but not limiting factor for this method of light nanofocusing. It is shown that the minimum possible size of the focusing region can be about of 1 nm.**

Keywords: *nanofocusing, surface plasmons, plasmon waveguide.*

1. Introduction

Nanofocusing of light is the key problem of modern nanophotonics. Unfortunately, in a uniform space it is impossible to obtain under normal focusing conditions a focusing spot size smaller than the Rayleigh diffraction limit for conventional optical instruments [1]. Nevertheless, it was shown that by using surface electromagnetic waves at the metal surface, one can localise light fields of unusually high intensity on geometric singularities of the surface and to focus the light energy into a spot with dimensions much smaller than the wavelength of a plane light wave in vacuum [2–5].

Experiments demonstrate that the best nanofocusing is observed when a TM-mode symmetrical surface plasmon wave converges towards the tip apex [6]; therefore, in this paper we will discuss a focused field with this kind of symmetry. It is believed [7] that the TM wave 'survives' during nanofocusing at the microtip apex even when surface plasmons are excited using diffraction gratings, located on one side of a metal microtip. Problems arising in nanofocusing of a TE-mode surface plasmon wave will be addressed in subsequent studies.

In paper [8] we determined in the quasi-static approximation the focused fields in the vicinity of the nanoapex of a metal microtip, whose boundary is approximated by an axisymmetric paraboloid of revolution. It was assumed that the dielectric constant of the metal is described by the lossless Drude formula. It turned out that in this approximation the size of the focal distribution of the electric field in the vicinity of the microtip nanoapex in spatial coordinates normalised to the radius of nanoapex curvature is determined only by the ratio of the frequency of focused plasmons to the plasma frequency of the metal. In the present work it is shown that in approaching the nanoapex an increase in absorption in the metal, associated with an increase in the frequency of collisions of electrons with the surface, is a limiting factor at frequencies close to the critical frequency of the existence of surface plasmons rather than at low (compared to the plasma frequency of the nanoapex metal) frequencies for the nanofocusing method in question.

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Received 15 May 2015; revision received 26 October 2015
Kvantovaya Elektronika **46** (2) 159–162 (2016)
Translated by I.A. Ulitkin

2. Electric field distribution near the nanoapex of a metal microtip in the quasi-static approximation

Consider a metal microtip, whose surface near the apex is described by an axisymmetric paraboloid of revolution: $z = R/2 - (x^2 + y^2)/(2R)$ (Fig. 1). The complex dielectric constants of the metal and the external homogeneous medium are denoted by ε_m and ε_d, respectively.

The expression for the electric field near the tip apex in the quasi-static approximation was obtained in papers [8, 9]. The representation of harmonic electromagnetic fields with a temporal dependence of form $\exp(-i\omega t)$, where ω is the cyclic frequency of the field, was assumed complex. In the quasi-static approximation the electric field potential should obey the Laplace equation, and the normal and tangential components of the field on the surface of the tip should meet the known boundary conditions. We found an axially symmetric solu-

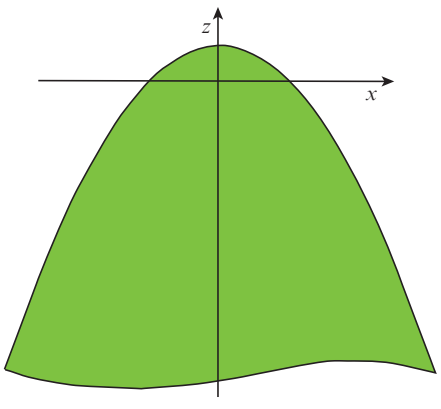

Figure 1. Geometry of the problem.

tion having a maximum at the tip apex and corresponding to the focusing of a TM-mode surface plasmon wave at the microtip [8].

For brevity, we omit the details of the potential calculations and present the result, i.e. the potential distribution Φ in the xz plane (Fig. 1):

$$\Phi(x,z) = J_0\left(q_*\sqrt{\sqrt{\tilde{x}^2+\tilde{z}^2}-\tilde{z}}\right) \times K_0\left(q_*\frac{|\tilde{x}|}{\sqrt{\sqrt{\tilde{x}^2+\tilde{z}^2}-\tilde{z}}}\right), \quad \frac{|\tilde{x}|}{\sqrt{\sqrt{\tilde{x}^2+\tilde{z}^2}-\tilde{z}}} \geqslant 1,$$

$$\Phi(x,z) = \left[\frac{K_0(q_*)}{I_0(q_*)}\right] J_0\left(q_*\sqrt{\sqrt{\tilde{x}^2+\tilde{z}^2}-\tilde{z}}\right) \times I_0\left(q_*\frac{|\tilde{x}|}{\sqrt{\sqrt{\tilde{x}^2+\tilde{z}^2}-\tilde{z}}}\right), \quad \frac{|\tilde{x}|}{\sqrt{\sqrt{\tilde{x}^2+\tilde{z}^2}-\tilde{z}}} \leqslant 1, \tag{1}$$

where $\tilde{x} = x/R$, $\tilde{z} = z/R$ are the coordinates normalised to the radius R of the microtip curvature; J_0 is the zero-order Bessel function of the first kind; I_0 and K_0 are the modified zero-order Bessel functions of the first and the second kind; and q_* is the solution of the equation [8]

$$\varepsilon_d I_0(q)K_1(q) + \varepsilon_m K_0(q)I_1(q) = 0, \tag{2}$$

where K_1 is the modified first-order Bessel function of the second kind. Distribution (1) enables us to find the potential (and hence the electric field) in the vicinity of the metal tip nanoapex with an accuracy up to a constant.

3. Focal distribution of the electric field near the microtip nanoapex. Effect of losses in the metal

The dielectric constant of the metal is approximately described by the Drude formula $\varepsilon_m = 1 - \omega_p^2/(\omega^2 + i\omega\Gamma)$, where ω_p is the plasma frequency of the metal, and Γ is the coefficient taking into account losses. For example, for silver [10] $\omega_p \approx 1.36 \times 10^{16}$ s^{-1}, $\Gamma \approx 2 \times 10^{14}$ s^{-1}. Then, equation (2) can be rewritten for the metal tip bordering the vacuum as follows:

$$I_0(q)K_1(q) + \left(1 - \frac{1}{\tilde{\omega}^2 + i\tilde{\omega}\gamma}\right)K_0(q)I_1(q) = 0, \tag{3}$$

where $\tilde{\omega} = \omega/\omega_p$ is the cyclic frequency of the field, normalised to the plasma ferquency; and $\gamma = \Gamma/\omega_p$ is the absorption coefficient, normalised to the plasma frequency. The absence of losses corresponds to $\gamma = 0$.

Calculations have shown that at frequencies $\omega < \omega_p/\sqrt{2}$ (where $\omega_p/\sqrt{2}$ is the upper limiting frequency of existence of surface plasmons) in the absence of losses ($\gamma = 0$), equation (3) has a unique and purely real solution q. In this case, special functions appearing in (3) can be regarded as function of the real variable. In the presence of losses ($\gamma \neq 0$), the solution is also unique, but complex. To find numerically a complex solution of equation (3), it is necessary to express analytically the special functions of Eqn (3) on the complex plane. This was done by using the following well-known integral representations for these functions:

$$I_n(q) = \frac{1}{\pi}\int_0^{\pi}\cos(n\theta)\exp(q\cos\theta)\,d\theta,$$

$$K_n(q) = \frac{1}{\pi}\int_0^{\infty}\cosh(n\theta)\exp(-q\cosh\theta)\,d\theta.$$

One can see from equations (1) and (3) that for the metal in the absence of losses ($\gamma = 0$), the size of the region of the field maximum near the nanotip apex (focal spot), expressed in units of the radius of the nanoapex curvature, depends only on the normalised frequency $\tilde{\omega}$. The closer the $\tilde{\omega}$ to $\tilde{\omega}_c = 1/\sqrt{2}$, i.e. the normalised critical frequency of existence of surface plasmons on a flat metal surface, the smaller the focal spot size. In the presence of losses ($\gamma \neq 0$) the normalised size of the region of the field maximum near the nanotip apex will be a function of two parameters, namely, $\tilde{\omega}$ and γ.

It is known from the theory of light scattering on small spherical metal particles [11] that for a given metal the effective coefficient Γ depends on the radius of the particle. This is due to the fact that the decay constant is proportional to the frequency of collisions of conduction electrons, resulting in a loss of the energy of the directional motion of the electron gas and conversion of the energy of the directional motion to the energy of the thermal motion of the electrons. Near the surface, apart from collisions with bulky scatterers we should additionally take into account collisions with the surface. Under the assumption that the boundary scatters diffusely, the coefficient Γ can be represented in the form $\Gamma = \Gamma_{bulk} + v_F/L$, where Γ_{bulk} is the loss factor in the bulk of the metal; v_F is the Fermi velocity; and L is some effective length.

To assess the impact of this effect on the focusing of light in the vicinity of the microtip nanoapex, we can use the radius R of the tip curvature as an effective length L. Figure 2 shows the dependence of $\Gamma/\Gamma_{bulk} = 1 + v_F/(R\Gamma_{bulk})$ on R for silver ($v_F = 1.392 \times 10^6$ m s^{-1}). It can be seen that by decreasing the radius of the nanoapex curvature down to 1 nm the effective loss factor increases by eight times compared with the case $R \approx 1$ μm.

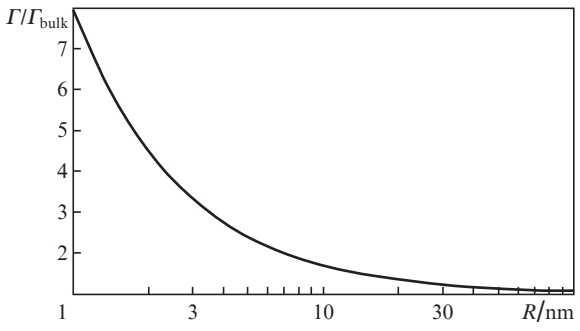

Figure 2. Dependence of $\Gamma/\Gamma_{bulk} = 1 + v_F/(R\Gamma_{bulk})$ on the radius of curvature R for silver ($v_F = 1.392 \times 10^6$ m s^{-1}).

We investigate the impact of the parameter γ on the distribution of the maximum of the electric field vector modulus E_a in the vicinity of the focus at certain fixed frequencies $\tilde{\omega}$. Considering the distribution with increasing γ, we will be able to assess how the distribution changes with decreasing radius of the nanoapex curvature.

Let us describe in detail the method for calculating E_a at each spatial point of the $\tilde{x}\tilde{z}$ plane. First, at the point under study we found complex components $E_{\tilde{x}} = -\partial\Phi/\partial\tilde{x}$ and $E_{\tilde{z}} = -\partial\Phi/\partial\tilde{z}$ of the complex vector of the electric field $\boldsymbol{E} = -\nabla\Phi$. Note that, because in (1) the potential (and hence, the

electric field) is determined to within a constant, we can perform differentiation in the normalised coordinates $\tilde{x}$ and $\tilde{z}$. Then, we found the real terms of the components $\mathrm{Re}[E_{\tilde{x}}\exp(-\mathrm{i}\omega t)]$ and $\mathrm{Re}[E_{\tilde{z}}\exp(-\mathrm{i}\omega t)]$ at some point in time t. Finally, we calculated the length of the instantaneous electric field vector and determined its maximum value for the period

$$E_{\mathrm{a}} = \max_{0 \leqslant \omega t \leqslant 2\pi} \sqrt{\{\mathrm{Re}[E_{\tilde{x}}\exp(-\mathrm{i}\omega t)]\}^2 + \{\mathrm{Re}[E_{\tilde{z}}\exp(-\mathrm{i}\omega t)]\}^2}\,. \quad (4)$$

Note that the calculation by the formula $E_{\mathrm{a}} = \sqrt{|E_{\tilde{x}}|^2 + |E_{\tilde{z}}|^2}$ would give a correct result only if the complex values $E_{\tilde{x}}$ and $E_{\tilde{z}}$ had equal phases. In general, this is not the case, an so the calculations in this paper were carried out by using a strict formula (4).

Figure 3 shows the calculated dependences $E_{\mathrm{a}}(\tilde{x})$ at the line $\tilde{z} = 1/2$, which corresponds to the intersection of the $\tilde{x}\tilde{z}$ plane and the focal plane. The dependences were obtained for $\tilde{\omega} = 0.62252$ and γ values corresponding to the absence of losses ($\gamma = 0$), losses in silver ($\gamma = \gamma_{\mathrm{Ag}}$ =0.01471), as well as to four times larger losses ($\gamma = 4\gamma_{\mathrm{Ag}}$) and eight times larger losses ($\gamma = 8\gamma_{\mathrm{Ag}}$). The frequency $\tilde{\omega} = 0.62252$ is chosen such that the radius of the focal distribution was approximately equal to the radius of the nanoapex curvature [8]. One can see from Fig. 3 that, although the focal spot expands with increasing γ, this expansion is small. Consequently, the electric field distributions in the focal plane at the line $\tilde{z} = 1/2$ are weakly dependent on the absorption in the metal at the selected frequency $\tilde{\omega} = 0.6225$.

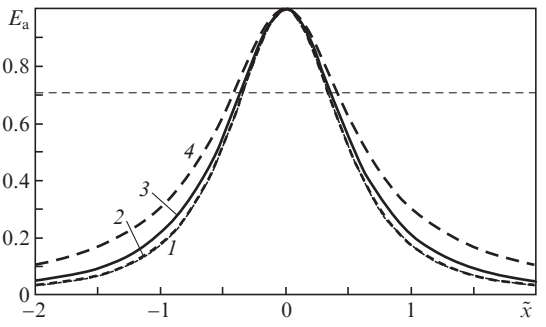

Figure 3. Normalised maximum of the electric field vector modulus E_{a} at the line $\tilde{z} = 1/2$ as a function of $\tilde{x}$ for the normalised frequency $\tilde{\omega} = 0.62252$ and normalised absorption coefficient $\gamma = 0$ (*1*), $\gamma = \gamma_{\mathrm{Ag}}$ (*2*), $\gamma = 4\gamma_{\mathrm{Ag}}$ (*3*) and $\gamma = 8\gamma_{\mathrm{Ag}}$ (*4*).

In addition to Fig. 3, Fig. 4 shows the distributions of E_{a}, i.e. the maximum of the electric field vector modulus in the $\tilde{x}\tilde{z}$ plane near the tip apex at the same values of the damping constant γ. From these distributions we can draw the following conclusion: although the distributions in the focal plane weakly change with increasing γ, an increase in absorption leads to a greater damping of the wave converging to the tip apex, and, obviously, to a decrease in the absolute value E_{a} at the maximum of the focal distribution (at the same intensity of convergent waves).

A similar but stronger dependence is observed for a higher, normalised working frequency ($\tilde{\omega} = 0.6742$), which is closer to the critical frequency of existence of surface plasmons $\tilde{\omega}_{\mathrm{c}} = 1/\sqrt{2}$ (Fig. 5). At this higher frequency, although the spatial oscillation wavelength and the focal spot size are smaller, the influence of the attenuation of the waves converging to the tip apex is enhanced. Thus, even at $\gamma = 4\gamma_{\mathrm{Ag}}$ the electric field amplitude is significantly reduced within the rounded apex of the microtip.

Thus, when the working frequency $\tilde{\omega}$ approaches the critical frequency $\tilde{\omega}_{\mathrm{c}} = 1/\sqrt{2}$, the influence of the damping on the absolute value of the maximum electric field near the nanotip apex increases. Although the focal spot size decreases with increasing frequency, the focused surface plasmon wave energy at such frequencies is strongly absorbed at the periphery of the nanotip.

De Angelis et al. [2] used the light with a wavelength λ = 532 nm in free space. Surface plasmons were focused at the nanoapex of a metal microtip. For the silver tip the wavelength λ = 532 nm corresponds to the frequency $\tilde{\omega} = 0.26053$. This frequency is much less than critical, and so one can

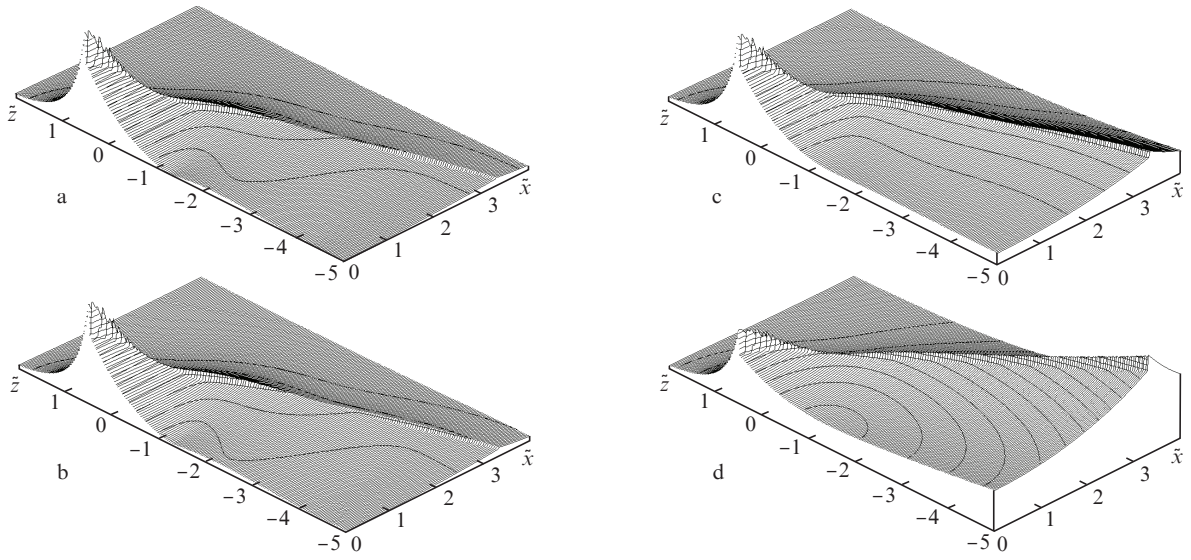

Figure 4. Distributions of the normalised maximum of the electric field vector modulus E_{a} in the vicinity of the nanoapex of the silver microtip for $\tilde{\omega} = 0.62252$, $\gamma = 0$ (a), $\gamma = \gamma_{\mathrm{Ag}}$ (b), $\gamma = 4\gamma_{\mathrm{Ag}}$ (c) and $\gamma = 8\gamma_{\mathrm{Ag}}$ (d).

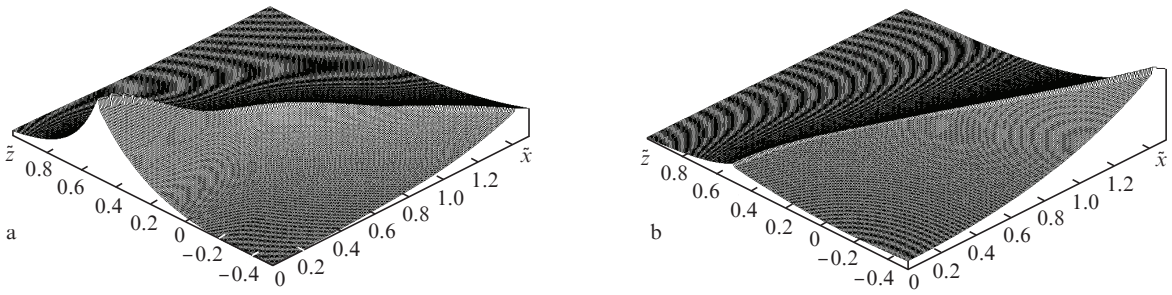

Figure 5. Same as in Fig. 4, but for $\tilde{\omega} = 0.6742$, $\gamma = \gamma_{Ag}$ (a) and $\gamma = 4\gamma_{Ag}$ (b).

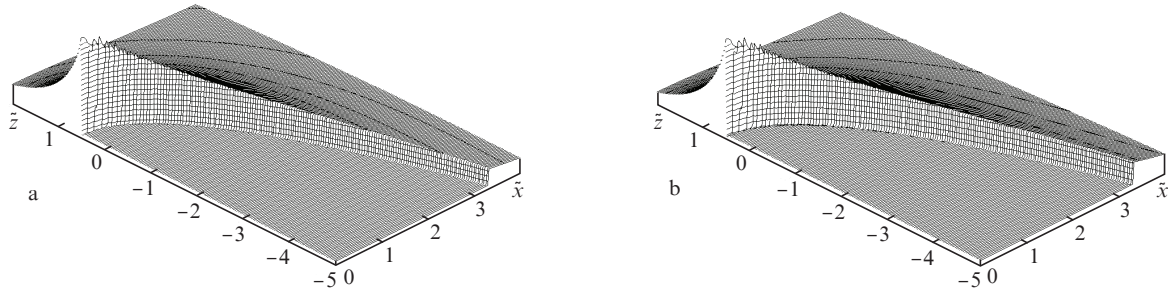

Figure 6. Same as in Fig. 4, but for $\tilde{\omega} = 0.26053$, $\gamma = \gamma_{Ag}$ (a) and $\gamma = 32\gamma_{Ag}$ (b).

expect a weaker influence of absorption on the focusing. Calculations show that this is true.

Figure 6 shows the distributions of E_a in the vicinity of the nanoapex at $\tilde{\omega} = 0.26053$ and two values of γ. Although the absorption in the second case (Fig. 6 b) is substantial, the electric field distribution is almost unchanged in the vicinity the nanoapex. Inside the metal the field is virtually uniform and low, while outside the metal the distribution is close to the static distribution of the electric field near the apex of the metal tip. From this it can be concluded that at these frequencies the absorption of the metal does not significantly affect the focal distribution in the vicinity of the nanoapex.

4. Conclusions

We have considered nanofocusing of a surface plasmon wave at the nanoapex of a metal microtip whose boundary near the tip is approximated by a paraboloid of revolution. It is shown that the absorption in a well-conductive metal of the microtip with a nanoapex radius up to 1 nm has virtually no effect on the size of the focal field distribution in the vicinity of the nanoapex up to $\omega \approx 0.6\omega_p$. When the frequency approaches the maximum frequency of existence of surface plasmons $\omega_c = \omega_p/\sqrt{2}$, focusing properties of the microtip decrease due to absorption in the metal. For an optical frequency range (much less than critical) and highly conductive metals with $\omega_p \sim 10^{16}$ s^{-1}, the effect of absorption on nanofocusing can be neglected. The focal spot size in this case is only determined by the radius of the curvature of the microtip nanoapex and can be about 1 nm.

It is obvious that the observed phenomenon is extremely important in the development of nanooptics devices, which use nanofocusing of surface plasmons at the nanoapex of the microtip. In particular, the studied method of nanofocusing has been used to design a variety of electro-optical gradient thin film structures in order to control thin-film interference systems.

Acknowledgements. This work was supported by a grant of the RF Ministry of Education and Science (Research Project 'Creation of electro-optical gradient thin-film structures for precision optics and analytical instrumentation'; grant agreement No. 14.579.21.0066 with the RF Ministry of Education and Science dated 10.23.2014, unique identifier RFMEFI 57914 X 0066).

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