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Optical analogue of the Mössbauer effect

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Abstract. **It is shown that in a transparent medium with a negative permittivity, absorption and stimulated emission of light by an atom can occur without changing its momentum due to transmission of a corresponding momentum in the medium.**

Keywords: *nonradiative processes, reactive components of the field, interference, recoil momentum.*

In near-field microscopy and nanoplasmonics, reactive components of an electromagnetic field source, whose contribution to a time-averaged energy flux is zero [1, 2], play an important role. This is caused by the fact that the phase difference of the reactive components of the electric and magnetic fields is equal to $\pi/2$. If a receiver is located near a radiation source, the interference of the source and receiver fields makes the phase difference between the total reactive components of the electric and magnetic fields other than $\pi/2$, resulting in a nonradiative energy transfer from the source to the receiver [3]. In this paper we consider the effects that are associated with the nonradiative transfer of a momentum, accompanying the transfer of energy from the source to the receiver.

The time-averaged force $\boldsymbol{F}$, with which a harmonic electromagnetic field with frequency ω acts on an atom, can be written in the form:

$$\boldsymbol{F} = \frac{1}{2}\mathrm{Re}\{(\boldsymbol{d}\nabla)\boldsymbol{E}^* + \frac{1}{c}[\dot{\boldsymbol{d}}\boldsymbol{H}^*]\}$$

$$= \mathrm{Re}\left\{-\mathrm{i}\frac{\omega\alpha'}{2c}[\boldsymbol{E}\boldsymbol{H}^*] + \frac{\omega\alpha''}{2c}[\boldsymbol{E}\boldsymbol{H}^*] + \frac{\alpha'}{2}(\boldsymbol{E}\nabla)\boldsymbol{E}^* + \mathrm{i}\frac{\alpha''}{2}(\boldsymbol{E}\nabla)\boldsymbol{E}^*\right\}$$

$$= \frac{\alpha'}{4}\nabla(\boldsymbol{E}\boldsymbol{E}^*) + \frac{\alpha''}{2}\mathrm{Re}\left\{\frac{\omega}{c}[\boldsymbol{E}\boldsymbol{H}^*] + \mathrm{i}(\boldsymbol{E}\nabla)\boldsymbol{E}^*\right\}$$

$$= \boldsymbol{F}_1(\alpha') + \boldsymbol{F}_2(\alpha''). \quad (1)$$

Here, $\alpha(\omega) = \alpha'(\omega) + \mathrm{i}\alpha''(\omega)$ is the complex polarisability of the atom; $\boldsymbol{d} = \alpha\boldsymbol{E}$ is the dipole moment of the atom induced by the electric field $\boldsymbol{E}$; $\boldsymbol{H}$ is the magnetic field; c is the speed of light; and the dot above denotes the time differentiation and the asterisk denotes a complex conjugate quantity. Transformations in (1) were produced using the formula for the gradient of a scalar product of two vector functions,

$$\nabla(\boldsymbol{E}\boldsymbol{E}^*) = (\boldsymbol{E}\nabla)\boldsymbol{E}^* + (\boldsymbol{E}^*\nabla)\boldsymbol{E} + [\boldsymbol{E}\nabla\boldsymbol{E}^*] + [\boldsymbol{E}^*\nabla\boldsymbol{E}], \quad (2)$$

and Maxwell's equations for harmonic fields:

$$[\nabla\boldsymbol{E}] = \mathrm{i}\frac{\omega}{c}\boldsymbol{H}. \quad (3)$$

The energy exchange between the atom and radiation is described by the coefficient α''; therefore, according to Newton's second law, the rate of change in the atom momentum $\dot{\boldsymbol{p}}$ during absorption and stimulated emission of light is equal to $\boldsymbol{F}_2(\alpha'')$. Using the expression for $\boldsymbol{F}_2(\alpha'')$ and the rate of change in energy W of the atom

$$\dot{W} = \frac{1}{2}\mathrm{Re}(\dot{\boldsymbol{d}}\boldsymbol{E}^*) = \frac{1}{2}\omega\alpha''|\boldsymbol{E}|^2, \quad (4)$$

we find that for a plane monochromatic wave and a damping monochromatic wave that emerge due to total internal reflection of the TE or TM waves from a transparent medium at their oblique incidence,

$$\frac{\dot{\boldsymbol{p}}}{\dot{W}} = \frac{\boldsymbol{F}_2(\alpha'')}{\dot{W}} = \frac{\boldsymbol{k}}{\omega}. \quad (5)$$

Here, $\boldsymbol{k}$ is the wave vector, which in the case of a damping wave is parallel to the plane of the interface between two media. In the case of total internal reflection of light from the surface of the insulator – vacuum interface, formula (5) has been experimentally confirmed in [4].

It follows from (1) that during absorption or stimulated emission of light the atomic momentum is preserved, if $\boldsymbol{F}_2(\alpha'') = 0$ for the harmonic field.

Let a plane monochromatic wave with frequency ω, polarised along the z axis, fall along the normal from the vacuum ($y < 0$) on a flat surface ($y = 0$) of a nonabsorbing medium with a permittivity $\varepsilon < 0$ (magnetic permeability $\mu = 1$) (Fig. 1). For the electromagnetic field of a refracted wave in a medium ($y > 0$)

$$E_{1z} = A\exp(-hy - \mathrm{i}\omega t), \quad H_{1x} = \mathrm{i}\frac{h}{k_0}A\exp(-hy - \mathrm{i}\omega t), \quad (6)$$

where $A = \mathrm{const}$; $h = k_0\sqrt{|\varepsilon|}$; and $k_0 = \omega/c$. If we place an atom with a complex polarisability $\alpha(\omega)$ in the region $y > 0$, then according to (1) the force $F_2(\alpha'')$, acting on the atom, is zero.

As shown in [5, 6], during absorption and stimulated emission of light by an atom, the exchange of energy, momentum and angular momentum between the atom and radiation is carried out by means of the corresponding interference fluxes, where the Poynting vector and the Maxwell stress ten-

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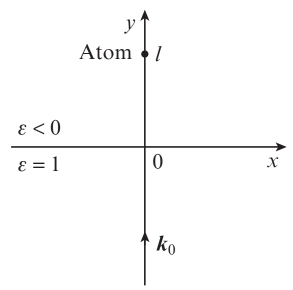

Figure 1.

sor are determined by the products of the fields of the incident wave and radiation of the induced atomic dipole. In this case, the averaged Poynting vectors for the fields of wave (4) and atomic dipole induced by the electric field of this wave are separately equal to zero. However, as in the case of the nonradiative energy transfer between atoms [3], the interference of the reactive components of the electromagnetic fields gives a nonzero energy flux along the y axis between an atom and the medium surface.

Let the coordinates of the atoms in the medium be $x = z = 0$, $y = l$. Using the well-known formulas for the fields $\boldsymbol{E}_2$ and $\boldsymbol{H}_2$ of the dipole in the medium [7], where we assume $\varepsilon < 0$, we can write the interference energy flux $I_{\text{int}\,1}$ through the plane $y = 0$ as follows:

$$I_{\text{int}\,1} = -\frac{c}{8\pi}\,\mathrm{Re}\iint_{-\infty}^{\infty}(E_{1z}H^*_{2x} + E_{2z}H^*_{1x})\,\mathrm{d}x\mathrm{d}z = -\frac{c}{8\pi}\alpha''|A|^2\mathrm{e}^{-hl}$$

$$\times\iint_{-\infty}^{\infty}\Big[k_0 l\Big(\frac{1}{r^3}+\frac{h}{r^2}\Big) - \frac{2h}{k_0\varepsilon}\frac{z^2}{r^2}\Big(\frac{1}{r^3}+\frac{h}{r^2}\Big) - \frac{x^2+l^2}{r^2}$$

$$\times\Big(\frac{1}{r^3}+\frac{h}{r^2}+\frac{h^2}{r}\Big)\Big]\mathrm{e}^{-hr}\,\mathrm{d}x\mathrm{d}z = -\frac{1}{2}\omega\alpha''|A|^2\mathrm{e}^{-2hl}, \qquad (7)$$

where $r = \sqrt{l^2 + x^2 + z^2}$; and the vector of the normal is directed against the y axis. The integrals can be easily calculated by going to the cylindrical coordinates $x = \rho\cos\alpha$, $z = \rho\sin\alpha$, and in the calculation we should take into account all the components of the dipole field. For the unexcited atom $\alpha'' > 0$, and so the energy flux is directed from the plane of the interface between the media to the atom and reduces the energy flux of the reflected wave. In the case of an excited atom $\alpha'' < 0$, and so the energy flux is directed from the atom to the surface of the interface between the media and enhances the reflected wave [8].

According to (4), expression (7) corresponds to the total power, which characterises the rate of energy exchange between the atom and the electromagnetic field. Thus, flux (7) is the total energy flux that passes only through the plane $y = 0$.

The momentum flux ($P_y > 0$) through the plane $y = 0$ of the total field of the refracted waves and induced dipole is determined by the component $\sigma_{\text{int}\,yy}$ of the interference Maxwell stress tensor

$$\sigma_{\text{int}\,ik} = \frac{1}{8\pi}\mathrm{Re}\{\varepsilon(E_{1i}E^*_{2k} + E^*_{2i}E_{1k}) + H_{1i}H^*_{2k} + H^*_{2i}H_{1k}$$

$$-\,\delta_{ik}[\varepsilon(\boldsymbol{E}_1\boldsymbol{E}^*_2) + (\boldsymbol{H}_1\boldsymbol{H}^*_2)]\}$$

($i, k = x, y, z$; δ_{ik} is the Kronecker symbol) and is described by the expression

$$I_{\text{int}\,2} = \iint_{-\infty}^{\infty}\sigma_{\text{int}\,yy}\,\mathrm{d}x\mathrm{d}z = -\frac{1}{8\pi}$$

$$\times\mathrm{Re}\iint_{-\infty}^{\infty}(\varepsilon E_{1z}E^*_{2z} + H_{1x}H^*_{2x})\,\mathrm{d}x\mathrm{d}z = -\frac{1}{8\pi}\alpha'|A|^2\mathrm{e}^{-hl}$$

$$\times\iint_{-\infty}^{\infty}\Big[\frac{r^2-l^2}{r^2}\Big(\frac{1}{r^3}+\frac{h}{r^2}\Big) - \frac{r^2+l^2}{2r^2}\Big(\frac{1}{r^3}+\frac{h}{r^2}+\frac{h^2}{r}\Big)$$

$$-\,hl\Big(\frac{1}{r^3}+\frac{h}{r^2}\Big)\Big]\mathrm{e}^{-hr}\,\mathrm{d}x\mathrm{d}z = \frac{1}{2}\alpha' h|A|^2\mathrm{e}^{-2hl}. \qquad (8)$$

Equation (8) yields an averaged gradient force $F_1(\alpha') = -I_{\text{int}\,2}$ acting on the atom in the direction of the interface between the two media. This expression takes into account all components of the dipole, which are proportional to $1/r$, $1/r^2$ and $1/r^3$, where r is the distance to the dipole. It is obvious that this force has nothing to do with the absorption or stimulated emission of the atom and causes the motion of the atom to the interface between the two media, and its work increases the kinetic energy of the atom.

In accordance with (1) and equality $F_2(\alpha'') = 0$, expression (8) determines the total force with which the electromagnetic field acts on the atom. Consequently, flux (8) is the total momentum flux passing only through the plane $y = 0$.

It does not take into account the contribution to the induced processes of the electromagnetic field of a dipole reflected from the interface between the two media, since its value is proportional to e^{-4hl} and is assumed small compared to (8). Allowance for the reflected field is fundamentally important when considering the spontaneous emission of excited atoms residing in a medium with $\varepsilon < 0$ near the interface with the medium, where $\varepsilon > 0$ [9]. In this case, the energy flux to the interface between the two media is formed only due to the interference of the radiation fields of the atom and the radiation reflected from the surface of the two media. Here, the spontaneous emission of the excited atom is not considered.

The atom with the field of the induced dipole, the medium and incident, refracted and reflected waves form a closed system. Note that Carniglia and Mandel [10], who considered quantisation of the electromagnetic waves near the interface between two media, used the model of a 'composite' photon, which is based on a simultaneous account for the fields of incident, refracted and reflected waves. According to the law of conservation of momentum, at $\alpha'' > 0$ a reflecting medium acquires a momentum of an absorbed part of the incident wave, while at $\alpha'' < 0$ – a recoil momentum associated with the amplification of the reflected wave. In both cases, according to (8) the momentum of the atom does not change, and the medium acquires a momentum $P_y > 0$. In contrast to the Mössbauer effect, where the nucleus is elastically connected with the crystal lattice, here the main role is played by the specifics of the energy and momentum transfer by the reactive components of the electromagnetic field which are the functions of the coordinates, velocity and acceleration of charges.

For the experimental observation of the effect described above, we need a medium that has $\varepsilon'(\omega) < 0$ and $\varepsilon''(\omega) \ll 1$ in a certain frequency range. The existence of such media does not contradict the integral Kramers–Kronig relations for $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ in the entire interval of frequencies [11]. Some examples of such media are discussed in [9]. These media include gases with the transition frequency near the frequency of the electromagnetic field. In addition, the success of modern technologies in the development of artificial media with unusual optical properties, such as negative-index media [12], makes us believe in the creation of media suitable for the observation of the optical analogue of the Mössbauer effect. The contribution of absorption is determined by the quantity $\omega l\varepsilon''/(c\sqrt{|\varepsilon'|})$, and therefore the absorption and reflected radiation of the dipole can be neglected if the condition $1/2h' = c/(2\omega\sqrt{|\varepsilon'|}) \ll l \ll c\sqrt{|\varepsilon'|}/(\omega\varepsilon'')$ is met.

At oblique incidence of a wave in the region of total internal reflection, the absorbing atom acquires only the component of the momentum of the incident wave that is parallel to the interface between the two media [6, 7]. Let a plane monochromatic wave fall from a transparent medium with $\varepsilon > 0$ at the interface of vacuum with a residing atom. The force $\boldsymbol{F}_2(\alpha'')$ acting on the atom is parallel to the interface between the two media, and so the atom acquires a momentum that is parallel to the surface for which relation (5) is valid [6]. In this case, the force $\boldsymbol{F}_2(\alpha')$ is still perpendicular to the interface between the two media and is not related with the absorption of the atom; therefore, in accordance with the law of conservation of momentum, the medium from which a plane monochromatic wave falls, acquires a component of the momentum of the incident wave, which is perpendicular to the interface between the two media.

Consider the transfer of a momentum between the excited and unexcited atoms residing in vacuum at a distance l from each other. Let the x axis be directed from the excited atom 1 with the electric dipole moment $d_{1z} = d_{10}e^{-i\omega t}$, $d_{1x} = d_{1y} = 0$ to the unexcited atom 2, where the electric field E_{1z} of the dipole induces the dipole moment $d_{2z} = \alpha E_{1z}$. Here, ω is the frequency of the transition from the excited state of atom 1 to its ground state.

In general, the transfers of energy and momentum between atoms are determined by the interference fluxes with allowance for both radiative ($\propto 1/r$) and reactive ($\propto 1/r^2$ and $\propto 1/r^3$) field components of the dipoles. The total rates of change in energy $\dot{W}_2$ and the resulting absorption of the momentum $\dot{P}_{2x}$ of the second atom are described by the formulas:

$$\dot{W}_2 = \frac{1}{2}\mathrm{Re}(\dot{d}_{2z}E^*_{1z}) = \frac{1}{2}\left(\frac{1}{l^6} - \frac{k_0^2}{l^4} + \frac{k_0^4}{l^2}\right)\alpha''\omega|d_{10}|^2, \quad (9)$$

$$\dot{P}_{2x} = F_2(\alpha'') = \frac{1}{2}\left(-2\frac{k_0^3}{l^4} + \frac{k_0^5}{l^2}\right)\alpha''|d_{10}|^2. \quad (10)$$

One can see that for distances $0 < l < \sqrt{2}/k_0$, at which the contribution of the reactive components of the dipole field is significant, the direction of the momentum acquired by atom 2 in the absorption of a photon is opposite to the direction of the energy transfer. A similar situation occurs in a negative-index medium [12]. At $l = \sqrt{2}/k_0$, the energy is transferred without a change in the momentum of atom 2. If atom 2 is in the far field where $l \gg 1/k_0$ and the energy and momentum are transferred only by interference fluxes of radiative field components of both dipoles, the ratio $\dot{P}_{2x}/\dot{W}_2 = k_0/\omega$ becomes the same as for a plane monochromatic wave.

Thus, the results obtained show that the determination of the momentum associated with the photon energy of the electromagnetic field generally depends on both radiative and reactive components of the field. The same conclusion with respect to the angular momentum of the radiating atom on the basis of Maxwell's equations was made in [13]. In conclusion, we note that the nonradiative energy transfer has been studied in detail both theoretically and experimentally [14]. However, there are almost no works devoted to the nonradiative transfer of the momentum. We hope that the effects discussed here will stimulate further research in this area and new spatial structures of harmonic electromagnetic fields can be found, for which the energy exchange with the atom is not accompanied by an exchange of momenta.

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