

NONLINEAR OPTICAL PHENOMENA

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Characteristics of SBS dynamics in single-mode optical fibres

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Abstract. **The characteristics of the gain of Stokes pulses in single-mode optical fibres by stimulated Brillouin scattering (SBS) of monochromatic and nonmonochromatic pump signals have been investigated by numerical simulation using a spectral approach. Conditions under which 'slow light' (caused by a group delay) can be implemented are found (it is reasonable to apply this term to a process in which a pulse is delayed with conservation of its shape). The plane-wave interaction model is shown to describe adequately the dynamics of this process in single-mode fibres. A number of gain modes are investigated for Stokes pulses with different time structures upon monochromatic and nonmonochromatic excitation. A new data transfer technique is proposed, which is based on the conversion of stepwise phase modulation of the input Stokes signal into amplitude modulation of the output signal.**

Keywords: *single-mode optical fibre, SBS, 'slow light', Kramers – Kronig relations.*

1. Introduction

The well-known Kramers – Kronig relations, which establish a one-to-one correspondence between the imaginary and real parts of active-medium permittivity, were widely used in numerous studies devoted to 'slow light' at stimulated Brillouin scattering (SBS) in single-mode optical fibres (see, e.g., [1 – 3]). This approach is obvious for monochromatic interacting waves in the constant pump approximation. At the same time, when calculating the SBS gain, the application of Kramers – Kronig relations for nonmonochromatic pumping is generally justified by the possibility of using the convolution of the pump spectrum with the scattering line profile [3, 4]. In fact, an analogy (which is far from complete) between the gain in a nonmonochromatic pump field at SBS and the gain in a medium with population inversion and inhomogeneously broadened line profile was used in the aforementioned studies. Indeed, even at the end of the 1960s and in the 1970s, it was experimentally and theoretically shown (see, e.g., [5, 6]) that the conditions under which the Stokes wave gain at a nonmonochromatic broadband pump field ($\Delta\omega_p \gg 1/T_2$, where $\Delta\omega_p$ is the pump linewidth and T_2 is the phonon lifetime) can be implemented are determined by the average pump intensity rather than the convolution. Note that these studies were devoted to stimulated Raman scattering (SRS); however, the results obtained are completely applicable for SBS, because the dynamic equations describing both processes are identical. In addition, there was an incorrect opinion, according to which the time delay is due to only the SBS nonstationarity [7]. The Stokes pulse delay caused by the SBS nonstationarity was, in particular, observed by us experimentally, and a quantitative theoretical description was developed for this effect [8]. Below we find the conditions under which the Stokes signal gain is determined by the aforementioned convolution at arbitrary $\Delta\omega_p$ values.

In this study, we consider also the influence of diffraction loss on the hypersound-wave damping. The regimes providing 'slow light' and generation of Brillouin solitons are investigated analytically and numerically, and adequate estimates of the transition time to the stationary scattering regime at stepwise switching of pumping are obtained.

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2. Analysis of the basic equations

Our analysis is based on the well known equations for slow amplitudes of the pump field, Stokes signal and acoustic phonons in the plane-wave approximation:

$$\left(\frac{\partial}{\partial z}+\frac{1}{V}\frac{\partial}{\partial t}\right)a_s=\frac{1}{2}gA_p(z,t)Q(z,t)^*,$$

$$\left(\frac{\partial}{\partial z}-\frac{1}{V}\frac{\partial}{\partial t}\right)A_p=-\frac{1}{2}ga_s(z,t)Q(z,t),\qquad (1)$$

$$T_2\frac{\partial Q(z,t)}{\partial t}+Q(z,t)=A_p(z,t)a_s^*(z,t).$$

Here, z is the longitudinal propagation coordinate; V is the group velocity; t is the time; g is the SBS gain; and a_s, A_p and Q are the amplitudes of the Stokes signal, pump signal and acoustic wave, respectively.

An analytical consideration will be performed in the constant pump approximation. Then, the right-hand side of the pumping equation in system (1) is zero. Let us make a standard change of variables ($\theta = t + z/V$) and search for a solution to system (1) by representing the pump and Stokes signal fields as expansions in series [9]:

$$A_{\rm p} = \sum A_{n0}\exp({\rm i}\Omega n\theta), \quad a_{\rm s} = \sum a_m(z)\exp({\rm i}\Omega m\theta).$$

Here, n, $m = 0, \pm1, \pm2, \ldots$ are the numbers of the frequency components of interacting fields with boundary conditions at $z = 0$:

$$A_{\rm p}(t) = \sum A_{n0}\exp({\rm i}\Omega nt), \quad a_{\rm s}(t) = \sum a_m(0)\exp({\rm i}\Omega mt),$$

which correspond to the application of a discrete Fourier transform to the input amplitudes in the periodicity interval $T = 2\pi/\Omega$. Obviously, one can choose an interval T sufficiently large to overlap completely all characteristic times inherent in the system under consideration. In our opinion, this circumstance makes the model in use adequate to physical reality.

The solution for Q will be presented as Duhamel's integral:

$$Q = \frac{1}{T_2}\int_0^\infty \exp(-\theta_1/T_2)A_{\rm p}(\theta-\theta_1,z)a_{\rm s}^*(\theta-\theta_1,z)\,{\rm d}\theta_1.$$

Substituting the corresponding expansions into the expression for Q, we obtain

$$Q^* = \sum_{n,m}\exp[-{\rm i}\Omega(n-m)\theta]A_n^*a_m\frac{1}{1-{\rm i}\Omega(n-m)\,T_2}.$$

Let us introduce $k = m - n$. Then the first equation of system (1) yields

$$\frac{\rm d}{{\rm d}z}\sum_l a_l(z)\exp({\rm i}\Omega l\theta) + \frac{2}{V}\sum_l a_l(z)\exp({\rm i}\Omega l\theta){\rm i}\Omega l$$

$$= \frac{1}{2}g\sum_j A_j\exp({\rm i}\Omega j\theta)\sum_{n,k}\exp({\rm i}\Omega k\theta)A_n^*a_{n+k}(z)\frac{1}{1+{\rm i}\Omega kT_2}. \quad (2)$$

Having equated the terms with identical exponentials in (2) and replaced variables $a_l' = a_l\exp({\rm i}\Omega z2l/V)$ [this replacement is equivalent to the transition to 'travelling' time $\theta = t - z/V$ in coordinate representation (1)], we arrive at

$$\frac{{\rm d}a_l'}{{\rm d}z} = \frac{1}{2}g\sum_k A_{l-k}\frac{1}{1+{\rm i}\Omega T_2k}$$

$$\times\sum_m A_m^*a_{m+k}'\exp[{\rm i}\Omega(m-l+k)z(2/V)]. \quad (3)$$

It follows from (3) that the gain of each spectral component (a_l') depends, generally speaking, on the presence or absence of other spectral components of the Stokes field and on the phase relations between the Stokes and pump field components. At $\Omega^{-1} \gg T_2$, system (3) can be reduced to the system of equations giving a model description of stimulated scattering of broadband pumping, which was analysed in detail in [9]. Here, we consider the case of an arbitrary relation between Ω^{-1} and T_2. Let us take into account only the Bragg terms with indices $(m - l + k) = 0$ in system (3):

$$\frac{{\rm d}a_p'}{{\rm d}z} \cong \frac{1}{2}ga_p'\sum_k\frac{|A_{p-k}|^2}{1+{\rm i}\Omega kT_2},$$

$$p = 0, \pm1, \pm2, \ldots; \quad k = 0, \pm1, \pm2, \ldots \quad (4)$$

At transition to a continuous spectrum in integral sum (4), we obtain the expression (note that the limiting transition from a discrete spectrum to a continuous one is the same as in [10]):

$$\frac{{\rm d}a'(\omega)}{{\rm d}z} \cong \frac{1}{2}ga'(\omega)\int_{-\infty}^{\infty}\frac{|A(\omega-\nu)|^2}{1+{\rm i}\nu T_2}{\rm d}\nu. \quad (5)$$

The integral in (5) is a convolution of a Lorentzian gain profile with the pump line profile. Here, $a'(\omega)$ and $A(\omega)$ are the corresponding amplitude spectral densities.

Note that the neglect of oscillating terms in (3) is obviously equivalent to averaging over the longitudinal coordinate z on the assumption of a large interaction length [$L \gg L_{\rm coh}$, where $L_{\rm coh} = V(2\Delta\omega_{\rm p})^{-1}$ is the coherent interaction length] and small-signal gain on the coherent interaction length ($gI_{\rm p}L_{\rm coh} \ll 1$, where $I_{\rm p}$ is the pump intensity); this situation corresponds to the incoherent interaction regime [9]. It follows directly from (4) and (5) that the gain of each spectral component in this regime is exponential and independent of the other Stokes spectral components. Specifically this circumstance makes it possible to interpret the Stokes wave gain in terms of variation in the refractive index and apply the Kramers–Kronig relations. In practice, the aforementioned conditions are generally satisfied in sufficiently long optical fibres, and only for this reason the theoretical estimates [3, 4] are adequate to the experimental data. For example, for two-mode pumping with an intermodal distance $\sim 1/T_2$ at $T_2 \approx 10$ ns, the coherent interaction length is on the order of 1 m. At $L > 200$ m, the condition of small-signal gain on the coherent length is obviously satisfied even for a total gain increment of ~ 20, corresponding to the Stokes radiation generation threshold from spontaneous noise. Nevertheless, the exact equality in systems (4) and (5) is possible only in the limiting case of an infinitely large interaction length or under monochromatic pumping and arbitrary interaction length.

3. Structure of wave vectors in a single-mode cylindrical optical fibre

It is well known that the transverse field distribution for the central low-order mode of a cylindrical optical fibre is described by the zero-order Bessel function $A(r) \backsim J_0(\xi r)$,

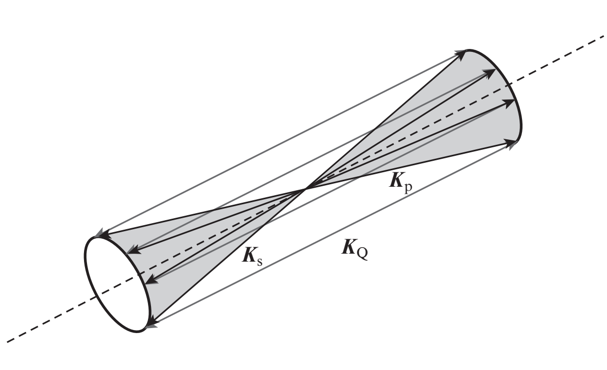

Figure 1. Conical geometry of the pump and Stokes signal wave vectors in a single-mode fibre: $\boldsymbol{K}_{\rm s}$ are the Stokes signal wave vectors, $\boldsymbol{K}_{\rm p}$ are the pump wave vectors, and $\boldsymbol{K}_Q$ is the acoustic wave vector; the dotted line is the fibre axis.

where ξ is determined from the zero boundary condition [$J_0(\xi r_0) = 0$ and r_0 is the fibre core radius]. Note that a similar distribution is characteristic of the plane-wave focusing by a conical lens; therefore, the pattern presented in Fig. 1 is observed at each point of a single-mode fibre in the wave-vector space. On the assumption of the symmetry of the problem, one can conclude that any pair of wave vectors of pump and Stokes signals in an arbitrary axial cross section interacts with the same acoustic wave (with an amplitude Q), propagating along the fibre axis.

Let us introduce the densities of interacting-wave amplitudes: $A(\varphi) \backsim \sqrt{I_p/2\pi}$ and $a(\varphi) \backsim \sqrt{I_s/2\pi}$, where I_p and I_s are, respectively, the pump and Stokes signal intensities and φ is the rotation angle in a fibre cross section with respect to some zero direction. For monochromatic pump and Stokes signal fields, we have

$$Q(1 - \mathrm{i}\omega T) = \int_0^{2\pi} A(\varphi)a^*(\varphi)\mathrm{d}\varphi,$$

$$\frac{\mathrm{d}a(\varphi)}{\mathrm{d}z} = \frac{1}{2}gA(\varphi)Q^*,$$

$$\frac{\mathrm{d}a(\varphi)}{\mathrm{d}z} = \frac{1}{2}gA(\varphi)\int_0^{2\pi} A^*(\varphi)a(\varphi)\mathrm{d}\varphi\frac{1}{1+\mathrm{i}\omega T_2}$$

$$= \frac{1}{2}g\frac{1}{1+\mathrm{i}\omega T_2}I_p a(\varphi).$$

Then the gain increment is $\bar{\Gamma} = {}^1\!/_2 g I_p L(1 + \mathrm{i}\omega T_2)^{-1}$.

Equations of types (4) and (5) can be obtained in a similar way. Hence, they are analogous to the plane-wave equations. Thus, the generally accepted plane-wave model, which is used to describe theoretically the processes taking place in optical fibres [11, 12], is adequate to the experimental situation. It can easily be shown that the Brillouin shift is smaller than the shift in the case of exact backscattering, Ω_{MB}, by a value on the order of $\Delta = {}^1\!/_2\Omega_{MB}(\mathrm{NA}/n_c)^2$, where NA is the fibre numerical aperture and n_c is the fibre core refractive index. Assuming that $\Omega_{MB} \approx 10$ GHz, we obtain the following estimate for typical NA values (0.1–0.3): $\Delta = 2-200$ MHz.

Obviously, the interaction of any other pair of pump and Stokes signal wave vectors with a sound wave results in off-axis components of the acoustic wave. However, the absence of spatial resonance makes their contribution into the amplification process much smaller than the contribution of the axial component, and these components can be neglected in the first approximation.

Let us estimate the influence of diffraction loss on the hypersound decay constant. To this end, we can use the following circumstance: as was shown above, an axial component of a hypersonic wave is generated in a single-mode fibre. Since the hypersound wavelength is $\lambda_{hyp} = \lambda_p/2$ (λ_p is the pump wavelength), the relative amplitude loss on the Fresnel length can be estimated as $\lambda_p(nd_0^2)^{-1}$, where d_0 is the fibre core diameter. Assuming the diffraction loss and the loss related to hypersound absorption to be additive, we determine the damping coefficient: $\alpha = \alpha_0 + \lambda_p(nd_0^2)^{-1}$, where α_0 is the hypersound absorption coefficient. Then the total inverse damping time is $1/\tau = 1/T_2 + V_{hyp}/l_{Fr}$. Here, l_{Fr} is the Fresnel length for a sound wave and V_{hyp} is the speed of sound in the active medium.

4. Analytical properties of the output Stokes signal

Based on the equations for the Stokes spectral components, (4) and (5), one can draw a number of qualitative conclusions about the characteristic features of the temporal behaviour of output Stokes signal for signals with different spectra at the input of the active medium. Equations (4) and (5) can be written as

$$\frac{\mathrm{d}a(\omega)}{\mathrm{d}z} = \frac{1}{2}g(\omega)a(\omega)\langle I_p\rangle, \qquad (6)$$

where

$$\langle I_p\rangle = \int_{-\infty}^{\infty}|A_p(\omega)|^2\mathrm{d}\omega$$

is the mean pump intensity and $g(\omega)$ is the effective gain. The expression for $g(\omega)$ is the simplest in the case of a pump spectrum described by a Lorentzian profile, because a convolution of two Lorentzian profiles in expression (5) yields again a Lorentzian profile with the following parameters:

$$g(\omega) = g\frac{\Delta\omega_L}{\Delta\omega_L + \Delta\omega_p}\frac{1}{1+\mathrm{i}\omega\tau}. \qquad (7)$$

Here, $\Delta\omega_L = 2/T_2$ is the Brillouin resonance linewidth and $\tau = 2/(\Delta\omega_L + \Delta\omega_p)$. The equation for the spectral components (6) can easily be integrated:

$$a(\omega, L) = a(\omega, 0)\exp\left[\frac{\Gamma/2}{1+\omega^2\tau^2}\right]\exp\left[-\mathrm{i}\omega\tau\frac{\Gamma/2}{1+\omega^2\tau^2}\right], \qquad (8)$$

where

$$\Gamma = g\frac{\Delta\omega_L}{\Delta\omega_L + \Delta\omega_p}\langle I_p\rangle L \qquad (9)$$

is the stationary gain increment.

Expression (8) allows one to draw certain conclusions about the temporal behaviour of the output Stokes signal, depending on the input signal spectrum. For example, at a spectral width of the input Stokes signal smaller than the bandwidth of the SBS medium, $\Delta\omega \backsim (\sqrt{\Gamma}\tau)^{-1}$, and $\Gamma \gg 1$, the phase of the last factor in (8) depends almost linearly on frequency. This means that, in the case of the inverse Fourier transform, we have an output signal reproducing the shape of the input signal, with a temporal shift with respect to L/V by the 'slow light' value: $\sim\tau\Gamma/2$. Hence, one can see that the 'slow light' effect is related to the linear phase modulation in the gain bandwidth rather than to the scattering nonstationarity, as was erroneously suggested in [7]. Specifically this circumstance makes it possible to explain the temporal shift in terms of variation in the Stokes signal group velocity.

Furthermore, we assume that the input signal amplitude $a(\omega)$ is an even function of frequency (which generally corresponds to its bell-shaped time envelope) and its spectral width exceeds the bandwidth of the SBS medium. Then, the product of the first two factors in (7) is also an even function. At $\Gamma \gg 1$, the phase, as in the case of 'slow light', depends linearly on frequency; however, the cutoff of the signal spectrum by the bandwidth distorts the temporal behaviour of the output pulse in comparison with the behaviour of the input signal. Therefore, a signal with a distorted shape is recorded at the output. Nevertheless, its intensity peak is shifted in time by the same value: $\sim \tau\Gamma/2$. It is noteworthy that the case with an odd function $a(\omega)$ of the input signal corresponds to the formation of Brillouin solitons, i.e., implies a stepwise change in the phase of the input signal by π [13]. Repeating the above considerations, one can conclude that the zero of the output signal intensity should also be delayed with respect to the input signal zero by $\tau\Gamma/2 + L/V$.

5. Some results of computer simulation

Unfortunately, simple analytical expressions for $g(\omega)$ cannot be obtained in the general case of an arbitrary pump spectrum. Therefore, to confirm the above considerations, we carried out computation of the SBS gain process for pump spectra of different shapes. System (1) was numerically solved within the constant pump approximation, with the corresponding initial and boundary conditions. The interaction domain length was taken to be 100 m; the acoustic phonon lifetime was $T_2 = 8$ ns. To obtain amplitudes with Lorentzian and Gaussian pump spectra, we used a function in the form $f_n = \text{rnd}(1)\exp(2\pi i\cdot\text{rnd}(1))$, which served (after the Fourier transform, subsequent transmission through the corresponding virtual-frequency filter, and inverse Fourier transform) as a model of stationary random process with a Lorentzian or Gaussian spectrum. Here, n is the number of steps in the programme of numerical solution of system (1) (in our case, $n = 6750$ on a time interval of 125 ns) and rnd(1) is a function generating a quasi-random number with a uniform probability distribution from 0 to 1 at each addressing operation.

To verify our calculation, we compared the results of numerical simulation for monochromatic pumping with the similar results obtained using the exact formula [9]. The comparison showed complete adequacy of the numerical simulation in the range of variation in the stationary gain increment from 0 to 35. Note that the time response of the stationary scattering $T_{\text{eqv}} = \Gamma\tau/2$, which was reported in [14, 15], is in fact the response time of the system to a Δ-shaped pulse of input Stokes signal under constant pumping; it is shorter by a factor of at least 2 than the real response time (Fig. 2; see also [8]). Note that the input Stokes pulse in all figures is shifted by a value equal to the light travel time along the fibre: L/V. Figures 3 and 4 show the results of numerical calculation of the gain of a 20-ns Stokes signal with a Gaussian shape of the input pulse without a sudden phase change (Fig. 3) and with a sudden phase change by π at the centre of the input pulse (Fig. 4); these data confirm completely the qualitative conclusions drawn in Section 4. Specifically, in correspondence with relations (7)–(9), we have $\Gamma = {}^1/_4 g\langle I_{\text{p}}\rangle L$ and $\tau = {}^1/_4 T_2$ for a Lorentzian pump spectrum of width $\Delta\omega_{\text{p}} = 3\Delta\omega_{\text{L}}$ at $T_2 = 8$ ns. Therefore, the numerical results for the mean gain increment (equal to 32) should be equivalent to the results for monochromatic pumping with a gain increment $\Gamma = gI_{\text{p}}L = 8$ and $\tau = 2$ ns.

To compare the results obtained using nonmonochromatic pumping of different spectral compositions, we calculated numerically the gain for pump fields with Lorentzian and Gaussian spectra. It should be noted that, at identical FWHM values for Gaussian and Lorentzian pumping spectra, their correlation functions have different widths, i.e., different time statistics. It was found that the similarity criterion for this case is the equality of correlation function

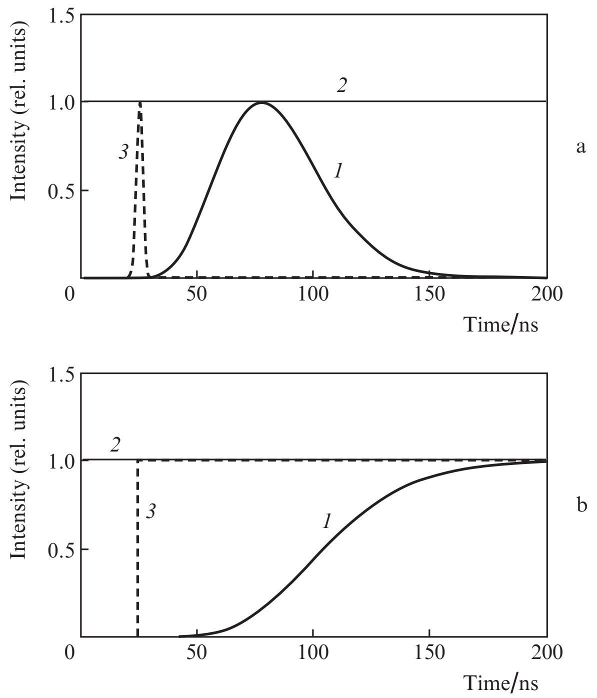

Figure 2. Response of a SBS amplifier to (a) Δ-shaped and (b) step input Stokes pulses: (*1*) output Stokes signal, (*2*) pump level, and (*3*) input Stokes signal. The stationary gain increment is $\Gamma = 16$; $T_2 = 8$ ns.

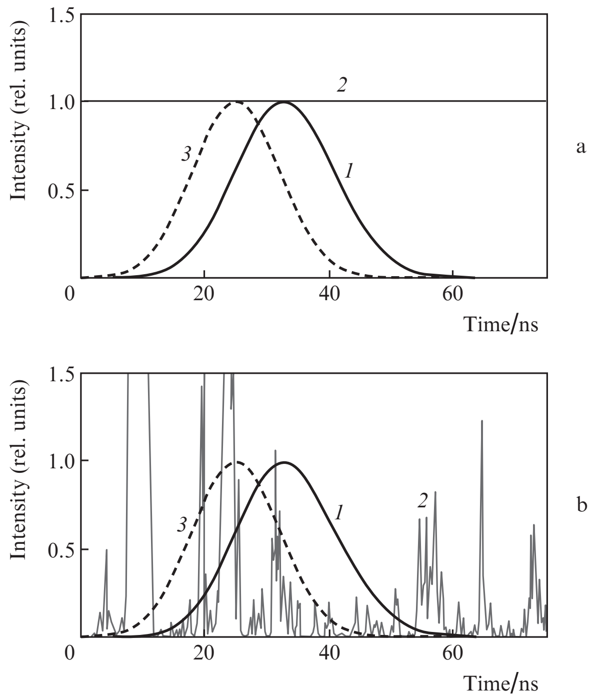

Figure 3. Amplification of a Stokes pulse with a Gaussian shape and duration $\tau = 20$ ns in a 100-m-long fibre (a) under monochromatic pumping at a gain increment $\Gamma = 8$ and $T_2 = 2$ ns and (b) under pumping with a Lorentzian spectral profile having a spectral width $3\Delta\omega_{\text{L}}$ at $T_2 = 8$ ns and mean gain increment $\langle\Gamma\rangle = 32$. The other designations are the same as in Fig. 2.

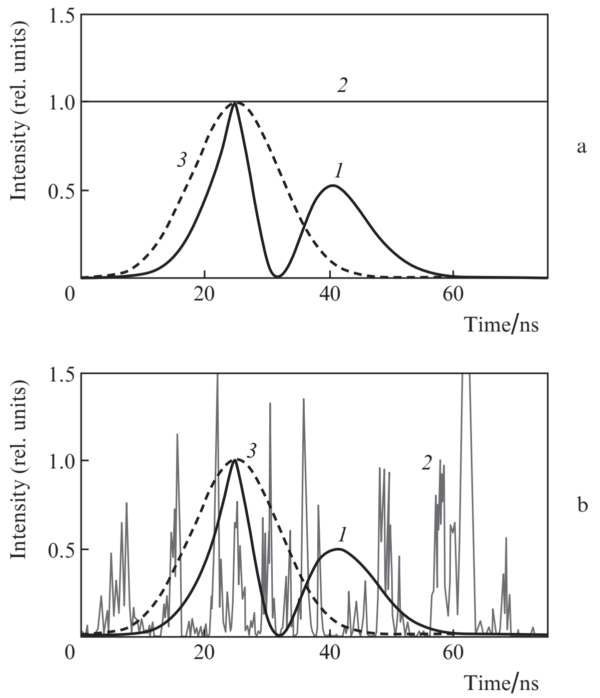

Figure 4. Same as in Fig. 3 but with a phase shift by π in the maximum of input Stokes pulse.

widths rather than the identity of pump spectral widths (at identical spectral widths, a Gaussian pulse is more 'monochromatic' than a Lorentzian one, because it has not any wide high-frequency wings). Figure 5 shows the output

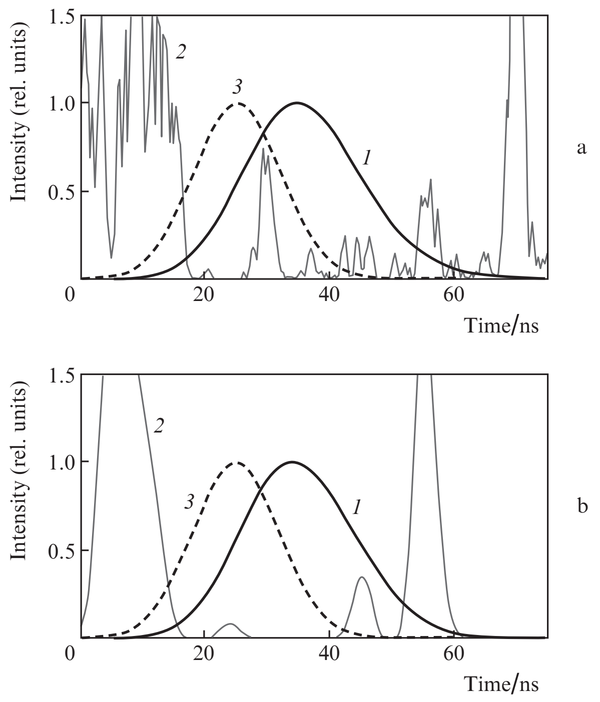

Figure 5. Comparison of the time characteristics of a SBS amplifier under pumping with (a) Lorentzian and (b) Gaussian spectral profiles at identical widths of correlation functions. The designations are the same as in Figs 2–4.

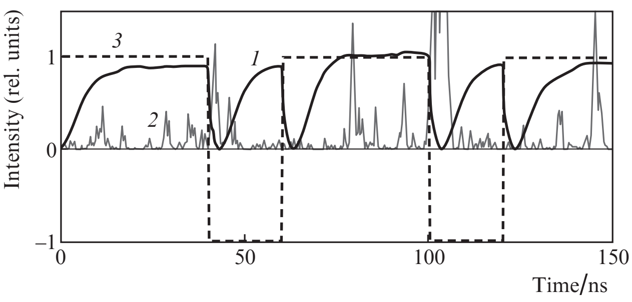

Figure 6. Conversion of a π-phase modulated Stokes pulse amplitude into an amplitude-modulated signal in an SBS amplifier. The designations are the same as in Figs 2–5.

Stokes signals for Lorentzian and Gaussian pump spectra at identical widths of correlation functions at the e^{-1} level. Similar results were obtained at other values of the gain increment and pump spectral widths. It is noteworthy that the gain of a π-phase modulated Stokes signal can be used to convert phase modulation into amplitude modulation (Fig. 6), i.e., to detect information.

6. Conclusions

Our study showed that the convolution of a pump spectrum with SBS profile can be used to estimate the gain efficiency and band in sufficiently long active SBS media. Each spectral component of the Stokes signal is amplified independently. Due to this circumstance, one can interpret an additional delay of the Stokes signal at the output of the active medium in terms of variation in the group refractive index. It was also shown that the process can be theoretically described using the plane-wave interaction approximation in the SBS dynamic equations. The diffraction of sound waves is an additional damping mechanism, which may cause broadening of the SBS gain line. In addition, it was shown that a π-step-phase modulated Stokes signal with its subsequent detection in a Brillouin amplifier can be used to transfer information along single-mode fibres.

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