# Investigation of band-gap properties in one-dimensional ternary photonic crystals with a single defect layer

Ru-Yi Tang, Jian-Wei Wu, Bikash Nakarmi

*Abstract.* We report one-dimensional ternary photonic crystals containing a single defect layer, whose photonic band-gap properties including the bandwidth and defect mode are analysed by changing the thickness and position of the defect layer, the number of unit cell repetitions in photonic crystals and the initial incidence angle. The results obtained show that the defect layer thickness can strongly affect the transmittance and the number of defect modes. The defect mode can disappear by increasing the number of periodic layers. The position of the defect layer in the photonic crystals can change the wavelength and transmittance of the defect mode. In addition, both the bandwidth and the defect mode properties of TE and TM waves are strongly dependent on the incidence angle.

*Keywords:* one-dimensional photonic crystal, transfer matrix method, band-gap, defect mode.

## 1. Introduction

Over the past three decades, photonic crystals (PCs) that are artificial dielectric or metallic structures, whose refractive index is changed periodically in space, have been intensively investigated by many research groups at home and abroad [1-6]. PCs can be generally classified into three groups such as one-, two-, and three-dimensional periodic structures. Among these groups, one-dimensional photonic crystals (1D PCs) have played an important role and attracted much attention owing to their simple configuration and theoretical modelling. To date, being widely presented in optics, 1D PCs have demonstrated their potential in various applications such as optical switching [7, 8], low-threshold lasers [9, 10], omnidirectional mirrors [11], tunable filters [12-14], etc. Analysis of literature shows that in devices based on 1D PCs use is made of two very significant properties of such PCs: photonic bandgaps and defect modes.

**Ru-Yi Tang** College of Physics and Electronic Engineering, Chongqing Normal University, Chongqing 401331, P.R. China; **Jian-Wei Wu** College of Physics and Electronic Engineering, Chongqing Normal University, Chongqing 401331, P.R. China; State Key Laboratory of Millimeter Waves, Nanjing 210096, P.R. China; School of Electrical Engineering, Korea Advanced Institute of Science and Technology, Daejeon 305714, Republic of Korea; e-mail: jwwu@kaist.ac.kr;

**Bikash Nakarmi** School of Electrical Engineering, Korea Advanced Institute of Science and Technology, Daejeon 305714, Republic of Korea

Received 23 September 2015; revision received 30 March 2016 *Kvantovaya Elektronika* **46** (7) 640–643 (2016) Submitted in English

The band-gap is induced by the interference of the waves experiencing Bragg scattering resulting from perfect periodic lattices, which forbid some frequencies to pass through the PCs [15]. In reality, the fabricated PCs are not perfect, because they contain some defect layers along the periodic directions with the result that obvious defect modes can be observed in the band-gap [16]. Defect modes caused by defect layers in PCs have rich potential applications. Consequently, it is very interesting to study the reflectance characteristics of band-gaps of 1D PCs containing some layers. Of course, both the bandgap and defect mode of one-dimensional binary photonic crystals with defect layers have thoroughly discussed by many research groups [17-21]. However, 1D PCs consisting of more than three dielectrics materials require more attention due to the presence of some significant properties compared to the case of 1D binary structures.

In this study, we present one-dimensional ternary photonic crystals with a single defect layer, whose band-gap and defect mode characteristics are theoretically investigated by changing various parameters including the number of periodic structure layers, position and thickness of the defect layer, and the initial incidence angle. Some results of these studies are presented in Section 3.

#### 2. Theory

The schematic diagram of a one-dimensional ternary photonic crystal with a single defect layer  $[(ABC)^N ABCD(ABC)^N]$  is shown in Fig. 1, in which the defect layer, *D*, breaks the perfect periodic structure. Films of *A*, *B* and *C* materials form the basic unit that is orderly organised along the special direction. As can be seen from Fig. 1, the number of repetitions is equal to 2N + 1, and the defect layer is placed into the middle section of the 1D PCs, whose position can be shifted to P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub> and P<sub>4</sub> in this study. Obviously, if optical waves at various frequencies are introduced into the left port of the presented device, they may be reflected or transmitted depending on the band-gap, defect mode and initial incidence angle. In general, the transmission matrix method is an effective numerical process to deal with the optical wave transmission through 1D PCs, which is expressed by [21]

$$\mathbf{M}_{j} = \begin{bmatrix} \cos \delta_{j} & -(i/\eta_{j}) \sin \delta_{j} \\ -i\eta_{j} \sin \delta_{j} & \cos \delta_{j} \end{bmatrix},$$
(1)

where the subscript *j* is the number of the dielectric layer;  $\delta_j = (2\pi/\lambda) n_j h_j \cos\theta$  is the phase;  $n_j = n_j \cos\theta$  (TE wave), or  $n_j = (\cos\theta)/n_j$  (TM wave);  $\lambda$  is the wavelength of incident light; *n* is the refractive index; and *h* and  $\theta$  are, respectively, the thickness and the incidence angle for each layer. If all the PCs are surrounded by air,  $\theta_0$  is the initial angle of incidence on the PCs, and for each media layer, the referred trigonometric function is given by

$$\cos\theta = \cos\theta_i = \sqrt{1 - (\sin^2\theta_0)/n_i^2},$$
(2)

where i = A, B, C, D.

Each the transfer matrix  $M_0$  for ABC layers can be written as

$$\mathbf{M}_0 = \mathbf{M}_A \mathbf{M}_B \mathbf{M}_C. \tag{3}$$

If the defect layer is located in position  $P_4$ , the full transfer matrix M of the PCs with a defect layer can be expressed by

$$\mathbf{M} = (\mathbf{M}_0)^N \mathbf{M}_A \mathbf{M}_B \mathbf{M}_C \mathbf{M}_D (\mathbf{M}_0)^N = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix},$$
(4)

where  $M_D$  is the transfer matrix of the defect layer. For other cases shown in Fig. 1, the corresponding transfer matrix M is similar to Eqn (4), in which the position of  $M_D$  is shifted to the position corresponding to the case in question.



Figure 1. Schematic diagram of a one-dimensional ternary photonic crystal containing a defect layer.

Using the matrix elements in Eqn (4), the reflectance of the PC is given by

$$r = \frac{(\mathbf{M}_{11} + \mathbf{M}_{12}\eta_{\text{out}})\eta_{\text{in}} - (\mathbf{M}_{21} + \mathbf{M}_{22}\eta_{\text{out}})}{(\mathbf{M}_{11} + \mathbf{M}_{12}\eta_{\text{out}})\eta_{\text{in}} + (\mathbf{M}_{21} + \mathbf{M}_{22}\eta_{\text{out}})},$$
(5)

where  $\eta_{in} = \eta_{out} = \cos \theta_0$  because the PCs are surrounded by air. The corresponding power reflectance is given by

$$R = |r|^2. ag{6}$$

Based on above theoretical equations, we can demonstrate and study the photonic band-gap and defect mode of the 1D PCs containing a single defect layer as a function of such parameters as the repetition number N, thickness and position of the defect layer and initial incidence angle.

### 3. Results and discussion

By changing the defect layer thickness  $h_D$ , various photonic band-gaps can be obtained (Fig. 2), where N = 10, the refractive indices of the materials A, B, C and D are, respectively, fixed at  $n_A = 1.5$ ,  $n_B = 2$ ,  $n_C = 2.5$ ,  $n_D = 2.3$ , and the layer thicknesses are, respectively,  $h_A = 100$  nm,  $h_B = 75$  nm and  $h_C = 60$  nm. Because the defect layer thickness is small (10 nm), the influence of the defect layer on the band-gap can be ignored. As a result, a full band-gap of ~250 nm in bandwidth is shown in Fig. 2a. One can see from Fig. 2a that the reflectance band-



Figure 2. Photonic band-gap at a thickness  $h_D =$  (a) 10, (b) 50, (c) 150 and (d) 220 nm of the defect layer located in P<sub>4</sub> position; N = 10.

width is remarkably broadened and the central wavelength is red shifted as compared to that of conventional one-dimensional binary photonic crystals. In the case of  $h_D = 50$  nm, in the band-gap there appears a defect mode (Fig. 2b), which, with increasing defect layer thickness to 150 nm, is shifted to longer wavelengths (Fig. 2c). However, an interesting phenomenon is observed in Fig. 2d, in which the wavelength of the defect mode is again shifted to short wavelengths at  $h_D =$ 



Figure 3. Photonic band-gap at a thickness  $h_D = (a) 800$ , (b) 1200, (c) 1600 and (d) 2000 nm of the defect layer located in P<sub>4</sub> position; N = 10.

220 nm. Therefore, one can conclude that, when the defect layer thickness varies from 10 nm to 220 nm, the wavelength of a defect mode with a narrow bandwidth is strongly dependent on  $h_D$ . With increasing  $h_D$  from 800 to 2000 nm (other parameters being the same as in Fig. 1), some surprising results with two defect modes are observed as shown in Fig. 3. In the case of  $h_D = 800$  nm (Fig. 3a), two sharp defect modes are observed, and with increasing  $h_D$  to 1200 nm, the wavelength spacing is shortened. Similarly, with increasing defect thickness from 1600 to 2000 nm, the number of defect modes increases (Figs 3c and 3d), and the wavelength interval between them is gradually compressed. In reality, the defect modes are the direct result of oscillations in the defect layer. Hence, we can conclude that a single defect layer can lead to more defect modes by growing its thickness and the result is significant since a greater number of defect modes are important for such potential applications as multi-channel filters.

Let us now examine the effect of the repetition number Nof the main element of the PCs' structure on the reflectance at a fixed defect layer thickness. With the defect layer of 300 nm in thickness, various band-gaps are plotted as a function of the repetition number (Fig. 4). As the repetition number N is increased, the defect mode in the band-gap disappears gradually. In the case of N = 5, the defect mode with a high transmittance and wide bandwidth is observed. This behaviour implies that the defect layer has important influence on the photonic band-gap. At N = 10 and N = 11, the thickness of the defect layer is very small compared to the total length of the photonic crystal. As a consequence, the transmittance of the defect mode is quickly decayed, and its bandwidth is remarkably narrowed. The influence of the defect layer on the reflectance band-gap, in which the defect mode exits, can be ignored in the case of N = 15. Figure 4d shows a ~250-nmwide band-gap. It is worth noting that the position of the defect mode does not change with varying the repetition



**Figure 4.** Photonic band-gap with a thickness  $h_D = 300$  nm of the defect layer in P<sub>4</sub> position at a repetition number N = (a) 5, (b) 10, (c) 11 and (d) 15.



**Figure 5.** Photonic band-gap at position (a)  $P_1$ , (b)  $P_2$ , (c)  $P_3$  and (d)  $P_4$  of a 300-nm-thick defect layer; N = 10.

number. Hence, the produced defect mode is tunable by judiciously selecting the corresponding repetition number and defect thickness.

Figure 5 shows the dependence of the reflectance on the wavelength for a single defect layer placed into various positions  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_4$  at N = 10 and  $h_D = 300$  nm. One can see that both the position and transmittance of the defect mode is slightly changed at different positions of the defect layer (Fig. 1). Here, it should be noted that the defect mode may disappear if the defect layer is inserted into other positions except the four cases described in this paper.

The next parameter, the initial angle of incidence on the photonic crystal, plays an important role for TE and TM waves under the condition of oblique incidence. The behaviour of the band-gap and the defect mode for TE and TM waves are shown in Fig. 6 at the incidence angle varying from 0 to 85°. The effective optical length for each layer decreases with increasing incidence angle; as a result, the reflectance band-gap and the defect mode are always shifted to short wavelengths. Some differences between the TE wave and the TM wave consist in the fact that the band-gap width for the TE wave is slightly broadened, which the band-gap width for the TM wave is remarkably narrowed with increasing incidence angle from 0 to 85°. However, the trend of the defect mode growth with increasing incidence angle is similar for TE and TM waves. At  $\theta_0 = 85^\circ$ , the defect modes for TE and TM waves are strongly suppressed and their bandwidths are broadened as compared to the case of normal incidence. Consequently, a defect mode with high transmittance and narrow bandwidth can be achieved by tuning the initial incidence angle.

#### 4. Conclusions

We have presented and numerically analysed a one-dimensional ternary photonic crystal structure,  $(ABC)^N ABCD(ABC)^N$ , containing a single defect layer. Both the reflectance band-



Figure 6. Photonic band-gap (the defect layer is in P<sub>4</sub> position) of the TE wave (on the left) and the TM wave (on the right) at various incidence angles  $\theta_0 = (a) 0$ , (b) 30°, (c) 60° and (d) 85°; N = 10,  $h_D = 300$  nm.

gap and the transmittance defect mode are discussed in detail by changing such parameters as the defect thickness, repetition number, defect position and initial incidence angle that can be judiciously selected to obtain various band-gaps and defect modes. The presented structure has some prominent applications in optical switching, filters and laser technologies.

Acknowledgements. This work was supported by the National Natural Science Foundation of China (Grant No. 61205111), Scientific and Technological Research Programme of Chongqing Municipal Education Commission (Grant No. KJ130633), the Open Foundation of State Key Laboratory of Millimeter Waves (Grant No. K201513) and the West Project of China Scholarship Council (Grant No. 201408505054).

### References

- Joannopoulos J.D., Johnson S.G., Winn J.N., Meade R.D. *Photonic Crystals: Molding the Flow of Light* (Princeton: University Press, 2008).
- Lončar M., Doll T., Vučković J., Scherer A. *IEEE J. Lightwave Technol.*, 18 (10), 1402 (2000).
- Kuznetsova T.I., Raspopov N.A. Kvantovaya Elektron., 45 (11), 1055 (2015) [Quantum Electron., 45 (11), 1055 (2015)].
- 4. Gao Y.H., Xu X.S. Chin. Phys. B, 23 (11), 114205 (2014).
- 5. Yablonovitch E. J. Phys.: Condens. Matter., 5, 2443 (1993).
- Mohebbi Z., Nozhat N., Emami F. Opt. Commun., 355, 130 (2015).
- 7. Chang K.D., Liu C.Y. Opt. Commun., 316, 10 (2014).
- Beggs D.M., White T.P., O'Faolain L., Krauss T.F. Opt. Lett., 33 (2), 147 (2008).
- Loncar M., Yoshie T., Scherer A., Gogna P., Qiu Y. Appl. Phys. Lett., 81 (5), 2680 (2002).
- Nomura M., Lwamoto S., Kumagai N., Arakawa Y. Phys. E: Low-dimensional Systems and Nanostructures, 40 (6), 1800 (2008).
- 11. Bruyant A., Lerondel G., Reece P.J., Gal M. *Appl. Phys. Lett.*, **82** (19), 3227 (2003).
- Qiu M., Mulot M., Swillo M., Anand S., Jaskorzynska B., Karlsson A., Kamp M., Forchel A. *Appl. Phys. Lett.*, 83 (25), 5121 (2003).

- Costa R., Melloni A., Martinelli M. *IEEE Photon. Technol. Lett.*, 15 (3), 401 (2003).
- Cos J., Ferre-Borrull J., Pallares J., Marsal L.F. Opt. Commun., 282 (6), 1220 (2009).
- 15. Nusinsky I., Hardy A.A. Phys. Rev. B, 73, 125104 (2006).
- 16. King T.C., Wu C.J. *Phys. E: Low-dimensional Systems and Nanostructures*, **69**, 39 (2015).
- 17. Li X., Xie K., Jiang H.M. Opt. Commun., 282 (21), 4292 (2009).
- 18. Wu C.J., Wang Z.H. Prog. Electromagn. Res., 103, 169 (2010).
- Fan H.M., Wang T.B., Liu N.H., Liu J.T., Liao Q.H., Yu T.B. J. Opt., 16, 125005 (2014).
- Lu Y.H., Huang M.D., Park S.Y., Kim P.J., Lee Y.P. J. Korean Phys. Soc., 51 (4), 1550 (2007).
- 21. Born M., Wolf E. *Principle of Optics* (Cambridge: University Press, 2013).