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Parametric generation of radiation in a dynamic cavity with frequency dispersion

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Abstract. **A numerical simulation of the parametric generation of electromagnetic radiation in a cavity with periodically oscillating mirrors and Lorentz-type frequency dispersion has been performed. It is shown that initially weak seed radiation can be transformed into intense short pulses, the shape of which under steady-state conditions changes periodically when reflecting from mirrors and, depending on the dispersion characteristics, corresponds to uni- or bipolar pulses.**

Keywords: *dynamic cavity, frequency dispersion, parametric oscillator.*

The dynamic Casimir effect (generation of photons in a cavity with oscillating mirrors) was predicted in 1970 [1], and then its quantum theory has been developed in many studies (see review [2]). The existence of this effect was experimentally confirmed in only one study [3] for superconducting quantum interference devices. However, the possibility of parametric generation of electromagnetic radiation in a cavity with oscillating mirrors was shown even earlier in [4] using classical Maxwell equations. The field energy is increased due to the transfer of the kinetic energy of the mirrors to the field, and the physics of this effect is close to that of the parametric Mandel'shtam–Papaleksi oscillator [5] – an electric circuit with periodically (mechanically) varying inductance or capacitance, in which initial current fluctuations may be significantly accelerated. Therefore, the dynamic Casimir effect has an important classical component. Although its classical theory has not been developed so intensively as the quantum theory and is qualitative to a greater extent (see also [6]), one can reveal a number of important aspects within the classical approach, which are difficult to take into account in the quantum theory. The purpose of our study was to perform a numerical analysis (in terms of classical electrodynamics) of the influence of the form of cavity frequency dispersion on the parametric generation of radiation, which is necessary for implementing new, more efficient schemes of this type.

The model is a cavity with two plane-parallel mirrors (a dynamic cavity with spherical mirrors was considered in [7]) located on the z axis (Fig. 1a). Radiation (plane waves) propagates along the z axis and is linearly polarised. The right mirror, positioned at $z = L(t)$ (t is the time) is assumed to be ideal. At small accelerations, the following boundary condition is imposed on this mirror: the tangential component of the wave electric field must turn to zero (a conventional condition) in the coordinate system moving jointly with the mirror. In the laboratory frame of reference, this equality has the form

$$\left(E + \frac{v}{c}H\right)\Big|_{z = L(t)} = 0, \qquad (1)$$

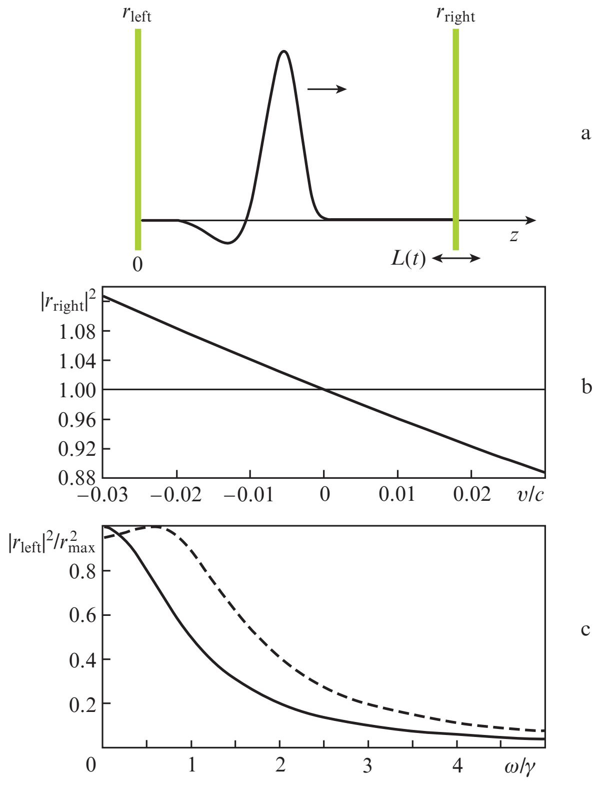

Figure 1. (a) Schematic of the dynamic cavity, (b) the dependence of the reflection coefficient of the right mirror on its instantaneous speed and (c) the frequency dependences of the reflection coefficient of the dispersive mirror at $\omega 0/\gamma = 0$ (solid line) and 0.628 (dashed line).

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where E and H are, respectively, the electric and magnetic field strengths; $v = \mathrm{d}L/\mathrm{d}t$ is the instantaneous mirror velocity; and c is the speed of light in vacuum. Thus, we neglect the effects of charge carrier inertia (which depend on the mirror microstructure) of the Tolman–Stewart effect type [8], which is justified at small accelerations. The amplitude reflection coefficient of radiation from this mirror is $r_{\rm right} = -(1 - v/c) \times (1 + v/c)^{-1}$ [9, 10] (Fig. 1b). A wave can be amplified when reflecting from a mirror moving towards the incident wave ($v < 0$). The left mirror is assumed to be immobile (located at $z = 0$) and characterised by frequency dispersion of the reflection coefficient. Correspondingly, the amplitudes of the electric fields incident on this mirror ($E_{\rm i}$) and reflected from it ($E_{\rm r}$) are related by the expression

$$E_{\rm r}(t) = \int_0^\infty K(\tau) E_{\rm i}(t - \tau)\mathrm{d}\tau.$$

We assume that $K(\tau) = K_0\exp(-\gamma\tau)\cos(\omega_0\tau)$. Then, for monochromatic incident radiation with a frequency ω, we obtain the mirror reflection coefficient in the form

$$|r_{\rm left}|^2 = \left(\frac{K_0}{2\gamma}\right)^2 \left\{\left[\frac{1}{1 + (\omega - \omega_0)^2/\gamma^2} + \frac{1}{1 + (\omega + \omega_0)^2/\gamma^2}\right]^2 + \left[\frac{(\omega - \omega_0)/\gamma}{1 + (\omega - \omega_0)^2/\gamma^2} + \frac{(\omega + \omega_0)/\gamma}{1 + (\omega + \omega_0)^2/\gamma^2}\right]^2\right\}. \quad (2)$$

Expression (2) describes a Lorentzian spectral profile; for an ideally reflecting mirror (without dispersion), $\gamma = \infty$. At $\omega_0/\gamma < \alpha = (\sqrt{5} - 2)^{1/2} = 0.486$, the reflection coefficient (2) reaches a maximum at $\omega = 0$ and monotonically decreases with an increase in ω. At $\omega_0/\gamma > \alpha$, frequency $\omega = 0$ corresponds to a local minimum of the reflection coefficient, while the position of the maximum shifts, gradually approaching the value $\omega = \omega_0$ (Fig. 1c).

The propagation of radiation in the vacuum gap between the mirrors is described by the one-dimensional Maxwell equations:

$$\frac{\partial H}{\partial z} = -\frac{1}{c}\frac{\partial E}{\partial t}, \quad \frac{\partial E}{\partial z} = -\frac{1}{c}\frac{\partial H}{\partial t}. \quad (3)$$

The d'Alembert solution is a superposition of two counterpropagating waves:

$$E(z,t) = f_1\left(t - \frac{z}{c}\right) + f_2\left(t + \frac{z}{c}\right), \; H(z,t) = f_1\left(t - \frac{z}{c}\right) - f_2\left(t + \frac{z}{c}\right). \quad (4)$$

Relations (4), jointly with the aforementioned boundary conditions imposed on the mirrors, allow one to trace the evolution of the initial field distribution with time. Below we present the results of numerical calculations for the case where the motion of the right mirror is set in the form $L(t) = L_0[1 + \mu\cos(\Omega t + \varphi)]$, where $\mu \ll 1$ and Ω are, respectively, the modulation depth and frequency. In order to make the description of the effects more illustrative, we use below fixed values of parameters: $\mu = 0.03$ and $r_{\max} = 0.99$. To make the parameters dimensionless, we normalise the coordinate z to the average cavity length L_0 and the time t to the average light transit time through the cavity: L_0/c. Correspondingly, the dimensionless frequency of the low-order mode of static cavity (at $\mu = 0$) is $\omega_{\rm s} = \pi$.

Figures 2a and 2b show the result of reflection of a Gaussian video pulse from the right mirror vibrating with a frequency $\Omega = \omega_{\rm s}$. The initial conditions at $t = 0$ have the form

$$f_1 = E_0\exp[-(z - 0.5)^2/w^2], \quad f_2 = 0, \quad w = 0.2. \quad (5)$$

It can be seen that, at a fixed initial position of pulse, the result depends strongly on the initial phase of mirror vibrations. The largest amplification is obtained for a short incident pulse at the instant of its passage through the middle position with a negative and maximum (in modulus) velocity $v = -\mu L_0\omega$ [at $\Omega t = \pi(2n + 1/2)$].

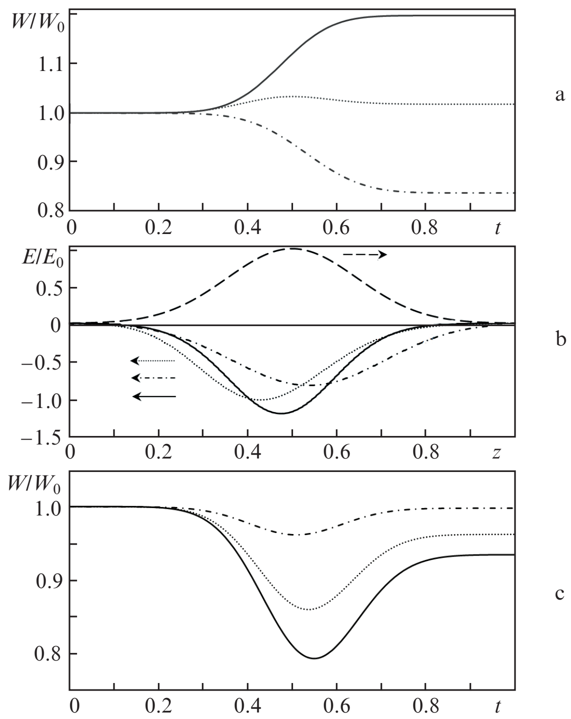

Figure 2. Reflection of a Gaussian video pulse from (a, b) vibrating and (c) immobile dispersive mirrors: (a, c) the time dependence of field energy W in the cavity and (b) the profile of the electric field strength in reflected pulse at $t = 1$ and $\Omega = \omega_{\rm s}$ for the initial vibration phase $\varphi = 0$ (solid line), $\pi/2$ (dotted line), and π (dash–dotted line); the initial profile is shown by a dashed line, the arrows indicate (hereinafter) the pulse motion direction.

The reflection of a Gaussian video pulse from the left mirror is illustrated in Fig. 2c. In this case, the initial conditions at $t = 0$ have the form

$$f_1 = 0, \quad f_2 = E_0\exp[-(z - 0.5)^2/w^2], \quad w = 0.2. \quad (6)$$

The radiation energy in vacuum is constant when the pulse is far from the mirror. The energy dip is caused by temporal energy transfer to the dispersive mirror. The dip is absent for an ideal mirror ($\gamma = \infty$); the smaller the γ value, the more pronounced the dip is.

When a pulse multiply passes through a cavity, its shape and energy change at each reflection from mirrors and are stabilised far from them. A parametric increase in the pulse energy may occur if the mirror vibration frequency is approximately multiple of the fundamental mode frequency: $\Omega \approx N\omega_{\rm s}$, where N is an integer.

Figure 3a shows the temporal change in radiation energy in the vacuum gap of the cavity in the case of fundamental resonance $\Omega = \omega_s$ and $\omega_0 = 0$, where the radiation loss is smaller at low frequencies. Here, the initial condition (5) is used. With the accepted parameters, the amplification on the vibrating mirror compensates for the loss on the left mirror; hence, the field energy in the cavity increases on the whole. The quasi-periodic increases in energy correspond to the pulse reflection from the moving mirror, the drops are related to the reflection from the immobile dispersive mirror, and the plateau regions corresponds to the motion far from the mirrors. In the absence of dispersion ($\gamma = \infty$), the increase in energy (on the whole) continues unlimitedly with a simultaneous unlimited decrease in the pulse duration. With allowance for the dispersion, the smaller the γ value, the earlier the systematic rise in energy stops, because losses increase for high frequencies (the contribution of which increases with a decrease in the pulse width). A unipolar pulse is set at long times; its shape is independent of the initial field profile [6]. However, with an increase in ω_0, the initial video pulse (5) is transformed into a bipolar pulse with time. This can be seen in Figs 3b and 3c, which present the change in the pulse shape after complete passage through the cavity in the steady mode.

Finally, Nth-order resonances lead to the occurrence of not one but N pulses in the cavity; each pulse arrives at the vibrating mirror at the instant corresponding to the largest instantaneous reflection coefficient. This is shown in Fig. 4 at

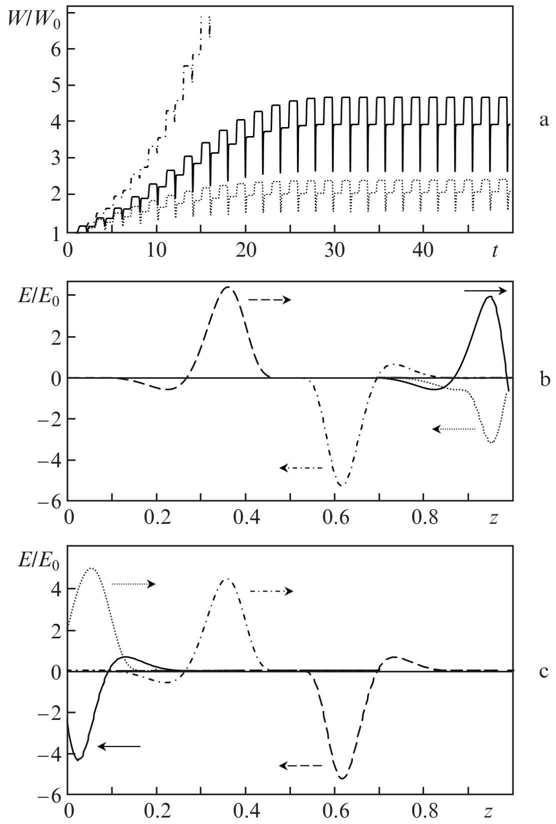

Figure 3. (a) Time dependencs of the field energy in cavity at (dash–dotted line) $\gamma = 100$, $\omega_0 = 0$; (solid line) $\gamma = 22$, $\omega_0 = 4\omega_s$; and (dotted line) $\gamma = 25$, $\omega_0 = 0$; (b) electric field strength profiles at t = (dashed line) 30, (solid line) 30.6, (dotted line) 30.65 and (dash–dotted line) 31; (c) the same at t = (dashed line) 31, (solid line) 31.6, (dotted line) 31.7 and (dash–dotted line) 32; $\gamma = 25$, $\omega_0 = 4\omega_s$ (b, c); $\Omega = \omega_s$, $\varphi = 0$.

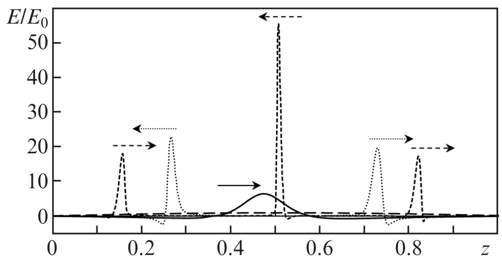

Figure 4. Electric field strength profiles at $\gamma = 100$, $\omega_0 = 16\omega_s$, $\varphi = 0$, and $t = 20$ for Ω = (solid line) ω_s, (dotted line) $2\omega_s$ and (dashed line) $3\omega_s$; the initial profile is shown by a long-dash line.

γ =100 (dispersion-free mirror) and the initial condition $f_1 = f_2 = E_0\sin(\pi z)$ at $t = 0$. Since this problem is linear, the ratio of the energies of these pulses may be arbitrary (determined by the initial conditions).

Thus, a consideration within classical electrodynamics allows parametric generation of electromagnetic radiation in a dynamic cavity; at long times, the field characteristics depend strongly on the form of the cavity frequency dispersion. The radiation generated in the case of amplification of small fluctuations or regular seed pulses is presented by not only unipolar (video) pulses but also bipolar pulses; note that the degree of bipolarity increases when the dispersion discrimination of higher frequencies weakens. In the steady mode, pulses periodically change shape when reflect off from mirrors and are stabilised at a large distance from the mirrors; under these conditions, the shapes of the pulses moving both along the cavity axis and in the opposite direction are different in view of the asymmetry of the scheme. This effect can be implemented experimentally using nanomechanics schemes [11] or 'plasma mirrors', the formation and motion of which are provided by intense laser beams [12].

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