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Asymptotic method for constructing a dynamic frequency response of a laser gyro

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Abstract. **We consider an equation describing the dynamics of the phase of a beat signal at the output of a ring laser gyro in the presence of an alternating-sign frequency biasing in the form of a superposition of two meanders with different amplitudes and periods. It is assumed that the amplitude of one of the meanders is much larger than that of the other as well as than the half-widths of the static lock-in and the measured difference of eigenfrequencies of a ring resonator. A method is proposed for approximately calculating the beat frequency as a function of the measured difference between the eigenrequencies of the resonator. The result of applying this method represents analytically a recursive algorithm. Its implementation on a computer has made it possible to construct a dynamic frequency (output) response of a laser gyro with a biasing of given form.**

Keywords: *laser gyroscope, ring resonator, phase equation, eigenfrequencies of a ring resonator, beat frequency, frequency response.*

1. Introduction

Despite significant advances in laser gyroscopy, possible ways have been studied up till now to overcome one of the most characteristic (and fundamental) drawbacks of ring laser gyros (LGs) – the emergence of a dead zone in the measurement of small angular velocities, i.e. at small differences of eigenfrequencies of a ring resonator (RR). In fact, all proposed methods for decreasing this zone (without decreasing backscatter in some way) are reduced to the use of a frequency biasing, i.e. to a known (controlled) additional splitting of RR eigenfrequencies that can move the LG from the dead zone of the LG. Usually, use is made of a variable-sign periodic frequency biasing [1, 2]. The form of the biasing on its period can vary from a simple harmonic to a frequency enriched curve (for example, a meander). As a rule, the amplitude of such a biasing considerably exceeds the width of the dead zone of the LG in its absence (the width of a static dead band), and in theoretical investigations this fact is often used for approximate asymptotic calculations [3]. Known asymptotic approaches are always adapted to specific types of biases. In this work, we will also propose an asymptotic method, intended for the construction of a frequency response of the LG with a combined biasing, which represents a superposition of two meanders with widely differing amplitudes Ω_r, Ω_s and periods $T_r = 2\pi/\nu_r$, $T_s = 2\pi/\nu_s$. In this case, the amplitude Ω_r of one of the meanders is much higher than all other parameters of the mathematical model with the frequency dimension. A combined biasing can be created, for example, in a Zeeman LG with a magnetic field applied [1]. This combined biasing not only makes it reasonable to use the asymptotic approach, but also determines its basic features.

In this paper, the asymptotic method is developed theoretically and used to construct a dynamic frequency response of the LG with a combined biasing on a personal computer (PC).

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2. Phase equation of an LG and an equivalent system of coupled equations with constant values of the bias

The dynamics of a single-mode (two-wave) LG with a gas mixture of special composition as an active medium is described by a system of three first-order differential equations (a system of truncated equations for slowly varying intensities and phase difference of counterpropagating waves [4]). These equations are coupled; however, under certain assumptions, in particular when a small amplitude modulation of these intensities is neglected, it is permissible to consider the phase equation to be independent. This assumption is introduced even when the alternating-sign biasing of the LG in question has a large amplitude. The possibility of using this approach is proved, for example, in [5] by integrating the coupled system of truncated equations on an analogue model. In this paper, the phase equation is also regarded as an independent one.

In using any periodic frequency biasing, the phase equation is a first-order nonlinear equation, the right-hand side of which is periodic both in time and with respect to the unknown function Ψ:

$$\frac{d\Psi}{dt} = \Omega_d(t) + \Omega + \Omega_L \cos\Psi. \quad (1)$$

Here $\Psi(t)$ is the difference between the counterpropagating waves (beat signal phase); in a particular case of a combined biasing, its change is given by the expression $\Omega_d(t) = \Omega_r \mathrm{sign}[\sin(\nu_r t)] + \Omega_s \mathrm{sign}[\sin(\nu_s t)]$; Ω_L is the half-width of the static locking band; and Ω is the difference between the RR eigenfrequencies (proportional to the measured angular velocity). The values of Ω_r, Ω_s, ν_r, ν_s and Ω_L are the parameters defining a specific variant of a dynamic frequency response, and Ω is an independent variable for any variant of the response. By the frequency response (curve of measurement conversion) we understand by definition the function of the variable Ω

$$\Omega_{\text{beat}}(\Omega) = \frac{\Psi(T_s) - \Psi_0}{T_s}, \tag{2}$$

where $\Psi_0 = \Psi(0)$ is the phase of the beat signal at the beginning of the biasing period $0 \leqslant t \leqslant T_s$, and $\Psi(T_s)$ is the phase at the end of the mentioned interval (its dependence on Ω is obvious).

We assume that $K_r = T_s/T_r$ is a large integer. Then, T_s is the overall period of the combined biasing. In addition, the inequalities Ω_s/Ω_r, Ω/Ω_r, $\Omega_L/\Omega_r \ll 1$ are fulfilled. These relations determine the possibility of using the asymptotic approach to the construction of the frequency response. It follows from (2) that for the frequency response to be calculated it is sufficient to solve the Cauchy problem for equation (1) in the interval $0 \leqslant t \leqslant T_s$. A direct application of a PC for this purpose is ineffective, at least when using the MathCad 15 environment. Therefore, the most fundamental part of the work should be first made analytically.

Note that within a biasing period $0 \leqslant t \leqslant T_s$ there are intervals of duration $T_r/2$, on which the biasing takes one of the four fixed values: $\Omega_{dpq} = (-1)^{p-1}\Omega_r + (-1)^{q-1}\Omega_s$, where p, $q = 1, 2$. In the first half-period of the biasing (at $k = 1, \ldots, K_r/2$) we allocate odd intervals $(2k-2)T_r/2 \leqslant t \leqslant (2k-1)T_r/2$, where $\Omega_{d11} = \Omega_r + \Omega_s$, and even intervals $(2k-1)T_r/2 \leqslant t \leqslant 2k(T_r/2)$, where $\Omega_{d21} = -\Omega_r + \Omega_s$. When $k = (K_r/2+1), \ldots, K_r$ (i.e. on the second half-period of the biasing), we have $\Omega_{d12} = \Omega_r - \Omega_s$ and $\Omega_{d22} = -\Omega_r - \Omega_s$ on odd and even intervals, respectively. Therefore, on each of the following half-periods the phase equation (1) has a time constant in the right-hand side and can be written in the form

$$\frac{dt}{d\Psi} = \frac{1}{\Omega_{dpq} + \Omega + \Omega_L \cos\Psi}. \tag{3}$$

Equation (3) is integrated at an initial condition $t(\Psi_0) = 0$. It is obvious that each of the mentioned half-periods on the time axis should have a corresponding pair of boundary values on the Ψ axis. Let us explain the meaning of this statement by the example of the first three half-periods of time. On the first (odd) interval $0 \leqslant t \leqslant T_r/2$, the phase equation (3) has the form

$$\frac{dt}{d\Psi} = \frac{1}{\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi},$$

and its solution can be expressed as:

$$\frac{T_r}{2} = \int_{\Psi_0}^{\Psi_1} \frac{d\Psi}{\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi},$$

where Ψ_0 is an arbitrarily specified initial phase and Ψ_1 is an unknown phase. On the second (even) interval $T_r/2 \leqslant t \leqslant 2(T_r/2)$, the phase equation has the form

$$\frac{dt}{d\Psi} = \frac{1}{-\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi},$$

and its solution can be expressed as:

$$\frac{T_r}{2} = \int_{\Psi_1}^{\Psi_2} \frac{d\Psi}{-\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi},$$

where the boundary phases Ψ_1 and Ψ_2 are unknown. On the third (odd) interval $2(T_r/2) \leqslant t \leqslant 3(T_r/2)$, the phase equation again has the form

$$\frac{dt}{d\Psi} = \frac{1}{\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi},$$

and its solution can be expressed as:

$$\frac{T_r}{2} = \int_{\Psi_2}^{\Psi_3} \frac{d\Psi}{\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi},$$

where the boundary phases Ψ_2 and Ψ_3 are unknown.

At $k = 1, \ldots, K_r/2$ we extend these formulas to other odd and even intervals of general form within the first half-period of the biasing having a duration $T_s/2$ (the total number of intervals is equal to K_r). On odd intervals $(2k-2)T_r/2 \leqslant t \leqslant (2k-1)T_r/2$, the phase equation has the form

$$\frac{dt}{d\Psi} = \frac{1}{\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi},$$

and its solution can be expressed as:

$$\frac{T_r}{2} = \int_{\Psi_{2k-2}}^{\Psi_{2k-1}} \frac{d\Psi}{\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi}, \tag{4}$$

where the boundary phases Ψ_{2k-2} and Ψ_{2k-1} are unknown. Only the initial phase Ψ_0 is specified. On even intervals $(2k-1)T_r/2 \leqslant t \leqslant 2k(T_r/2)$, the phase equation has the form

$$\frac{dt}{d\Psi} = \frac{1}{-\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi},$$

and its solution can be expressed as:

$$\frac{T_r}{2} = \int_{\Psi_{2k-1}}^{\Psi_{2k}} \frac{d\Psi}{-\Omega_r + \Omega_s + \Omega + \Omega_L \cos\Psi}, \tag{5}$$

where the boundary phases Ψ_{2k-1} and Ψ_{2k} are unknown.

Similarly, we consider odd and even half-intervals on the second half-period of the biasing $T_s/2 \leqslant t \leqslant 2(T_s/2)$ at $k = (K_r/2+1), \ldots, K_r$. On odd half-intervals $(2k-2)T_r/2 \leqslant t \leqslant (2k-1)T_r/2$, the phase equation has a solution

$$\frac{T_r}{2} = \int_{\Psi_{2k-2}}^{\Psi_{2k-1}} \frac{d\Psi}{\Omega_r - \Omega_s + \Omega + \Omega_L \cos\Psi}, \tag{6}$$

where the boundary phases Ψ_{2k-2} and Ψ_{2k-1} are unknown, but the phase Ψ_{K_r} has already been found (known). On even half-intervals $(2k-1)T_r/2 \leqslant t \leqslant 2k(T_r/2)$, the phase equation has a solution

$$\frac{T_r}{2} = \int_{\Psi_{2k-1}}^{\Psi_{2k}} \frac{d\Psi}{-\Omega_r - \Omega_s + \Omega + \Omega_L \cos\Psi}, \tag{7}$$

where the boundary phases Ψ_{2k-1} and Ψ_{2k} are unknown.

3. Recurrence relations between the boundaries of the intervals on the phase axis

The phase equation (1) yields the expression

$$T_s=\sum_{k=1}^{K_r/2}\left(\int_{\Psi_{2k-2}}^{\Psi_{2k-1}}\frac{d\Psi}{\Omega_r+\Omega_s+\Omega+\Omega_L\cos\Psi}+\int_{\Psi_{2k-1}}^{\Psi_{2k}}\frac{d\Psi}{-\Omega_r+\Omega_s+\Omega+\Omega_L\cos\Psi}\right)+\sum_{k=K_r/2+1}^{K_r}\left(\int_{\Psi_{2k-2}}^{\Psi_{2k-1}}\frac{d\Psi}{\Omega_r-\Omega_s+\Omega+\Omega_L\cos\Psi}+\int_{\Psi_{2k-1}}^{\Psi_{2k}}\frac{d\Psi}{-\Omega_r-\Omega_s+\Omega+\Omega_L\cos\Psi}\right).\quad(8)$$

The problem is to find the last phase Ψ_{2K_r} in the coupled phase sequence

$$\Psi_0,\Psi_1,\ldots,\Psi_{K_r},\Psi_{K_r+1},\ldots,\Psi_{2K_r-1},\Psi_{2K_r}.\quad(9)$$

To solve this problem, first we use the asymptotic transformation of integrals in (4), for example as follows:

$$\frac{T_r}{2}=\int_{\Psi_{2k-2}}^{\Psi_{2k-1}}\frac{d\Psi}{\Omega_r+\Omega_s+\Omega+\Omega_L\cos\Psi}=\frac{1}{\Omega_r}\int_{\Psi_{2k-2}}^{\Psi_{2k-1}}\frac{d\Psi}{1+(\Omega_s+\Omega+\Omega_L\cos\Psi)/\Omega_r}\quad(10)$$
$$\approx\frac{1}{\Omega_r}\left(1-\frac{\Omega_s+\Omega}{\Omega_r}\right)(\Psi_{2k-1}-\Psi_{2k-2})-\frac{\Omega_L}{\Omega_r^2}\int_{\Psi_{2k-2}}^{\Psi_{2k-1}}\cos\Psi\,d\Psi.$$

In the zero approximation in Ω_L/Ω_r we obtain from (10)

$$\Psi_{2k-1}=\Psi_{2k-2}+\frac{\Omega_r}{1-(\Omega_s+\Omega)/\Omega_r}\frac{T_r}{2}.$$

In the first approximation in the Ω_L/Ω_r it follows from (10) that the boundary phases are related by the recurrence expression

$$\Psi_{2k-1}=\Psi_{2k-2}+\frac{1}{1-(\Omega_s+\Omega)/\Omega_r}\left\{\Omega_r\frac{T_r}{2}+\frac{\Omega_L}{\Omega_r}\left\{\sin\left[\Psi_{2k-2}+\frac{\Omega_r}{1-(\Omega_s+\Omega)/\Omega_r}\frac{T_r}{2}\right]-\sin\Psi_{2k-2}\right\}\right\}.\quad(11)$$

We use the asymptotic transformation of integrals in (5):

$$\frac{T_r}{2}=\int_{\Psi_{2k-1}}^{\Psi_{2k}}\frac{d\Psi}{-\Omega_r+\Omega_s+\Omega+\Omega_L\cos\Psi}\quad(12)$$
$$\approx-\frac{1}{\Omega_r}\left(1+\frac{\Omega_s+\Omega}{\Omega_r}\right)(\Psi_{2k}-\Psi_{2k-1})-\frac{\Omega_L}{\Omega_r^2}\int_{\Psi_{2k-1}}^{\Psi_{2k}}\cos\Psi\,d\Psi.$$

In the zero approximation in Ω_L/Ω_r we obtain from (12)

$$\Psi_{2k}=\Psi_{2k-1}-\frac{\Omega_r}{1+(\Omega_s+\Omega)/\Omega_r}\frac{T_r}{2}.$$

In the first approximation in Ω_L/Ω_r it follows from (12) that the boundary phases are related by the recurrence expression

$$\Psi_{2k}=\Psi_{2k-1}+\frac{1}{1+(\Omega_s+\Omega)/\Omega_r}\left\{-\Omega_r\frac{T_r}{2}-\frac{\Omega_L}{\Omega_r}\left\{\sin\left[\Psi_{2k-1}-\frac{\Omega_r}{1-(\Omega_s+\Omega)/\Omega_r}\frac{T_r}{2}\right]-\sin\Psi_{2k-1}\right\}\right\}.\quad(13)$$

Next, we use the asymptotic transformation of integrals in (6):

$$\frac{T_r}{2}=\int_{\Psi_{2k-2}}^{\Psi_{2k-1}}\frac{d\Psi}{\Omega_r-\Omega_s+\Omega+\Omega_L\cos\Psi}\quad(14)$$
$$\approx\frac{1}{\Omega_r}\left(1-\frac{-\Omega_s+\Omega}{\Omega_r}\right)(\Psi_{2k-1}-\Psi_{2k-2})-\frac{\Omega_L}{\Omega_r^2}\int_{\Psi_{2k-2}}^{\Psi_{2k-1}}\cos\Psi\,d\Psi.$$

In the zero approximation in Ω_L/Ω_r we obtain from (14)

$$\Psi_{2k-1}=\Psi_{2k-2}+\frac{\Omega_r}{1-(-\Omega_s+\Omega)/\Omega_r}\frac{T_r}{2}.$$

In the first approximation in Ω_L/Ω_r it follows from (14) that the boundary phases are related by the recurrence expression

$$\Psi_{2k-1}=\Psi_{2k-2}+\frac{1}{1-(-\Omega_s+\Omega)/\Omega_r}\left\{\Omega_r\frac{T_r}{2}+\frac{\Omega_L}{\Omega_r}\left\{\sin\left[\Psi_{2k-2}+\frac{\Omega_r}{1-(-\Omega_s+\Omega)/\Omega_r}\frac{T_r}{2}\right]-\sin\Psi_{2k-2}\right\}\right\}.\quad(15)$$

We use the asymptotic transformation of integrals in (7)

$$\frac{T_r}{2}=\int_{\Psi_{2k-1}}^{\Psi_{2k}}\frac{d\Psi}{-\Omega_r-\Omega_s+\Omega+\Omega_L\cos\Psi}\quad(16)$$
$$\approx\frac{1}{\Omega_r}\left(1+\frac{-\Omega_s+\Omega}{\Omega_r}\right)(\Psi_{2k}-\Psi_{2k-1})-\frac{\Omega_L}{\Omega_r^2}\int_{\Psi_{2k-1}}^{\Psi_{2k}}\cos\Psi\,d\Psi.$$

In the zero approximation in Ω_L/Ω_r we obtain from (16)

$$\Psi_{2k}=\Psi_{2k-1}-\frac{\Omega_r}{1+(-\Omega_s+\Omega)/\Omega_r}\frac{T_r}{2}$$

In the first approximation in Ω_L/Ω_r it follows from (16) that the boundary phases are related by the recurrence expression

$$\Psi_{2k}=\Psi_{2k-1}+\frac{1}{1+(-\Omega_s+\Omega)/\Omega_r}\left\{-\Omega_r\frac{T_r}{2}-\frac{\Omega_L}{\Omega_r}\left\{\sin\left[\Psi_{2k-1}-\frac{\Omega_r}{1-(-\Omega_s+\Omega)/\Omega_r}\frac{T_r}{2}\right]-\sin\Psi_{2k-1}\right\}\right\}.\quad(17)$$

4. Construction of a dynamic frequency response

As follows from expressions (11), (13), (15), (17), one can find all boundary phases in sequence (9), starting from a given Ψ_0 and ending with a desired $\Psi_{2K_r}=\Psi_\Omega$. The dependence of the last phase Ψ_Ω of the iterative process on Ω is evident from the above recurrence relations. Next, we determine the beat frequency $\Omega_{beat}(\Omega)=(\Psi_\Omega-\Psi_0)/T_s$ as a function of the variable Ω (RR frequency difference), i.e., the frequency response by definition.

In the method under consideration, we used only the first-order approximation in Ω_L/Ω_r, i.e. the accuracy of the method is limited. However, when the conditions Ω_r/Ω, Ω_r/Ω_s, $\Omega_r/\Omega_L \gg 1$, sufficient for the application of the asymptotic method, are fulfilled, the accuracy of determining the dynamic frequency response is quite satisfactory. The speed of the algorithm implementation on the PC is extremely high.

Here is an example of calculating the dynamic frequency response $\Omega_{beat}(\Omega)$ in the range $0 \leqslant \Omega \leqslant 3\nu_s$ with the following parameters: $\Omega_s = 2\pi{\cdot}154$, $\Omega_r = 2\pi{\cdot}60 \times 10^3$, $\nu_s = 2\pi{\cdot}4$, $K_r = 126$, $\nu_r = K_r\nu_s$, $\Omega_L = 2\pi{\cdot}600$. The ideal frequency response corresponds to $\Omega_{beat}(\Omega) = \Omega$. As can be seen from Fig. 1, the calculated frequency response (because of the presence of a component with a large amplitude in the alternating-sign biasing) is virtually identical to the ideal one because the distortions of the calculated frequency response are extremely small. To illustrate the nature and magnitude of distortions, Fig. 2 shows the dependence of deviation of the calculated frequency response from the ideal one.

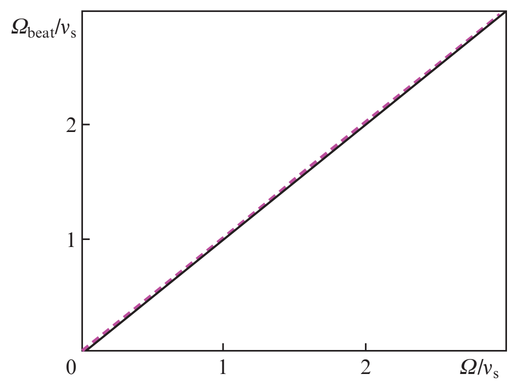

Figure 1. Calculated dynamic frequency response of the LG with a combined biasing (dashed line) and ideal frequency response (solid line). The parameters used in the calculation are given in the text.

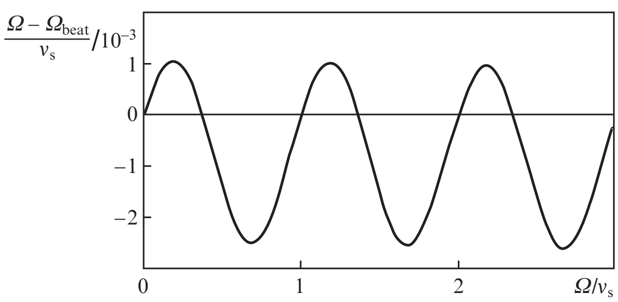

Figure 2. Deviation of the calculated dynamic frequency response of the LG with a combined biasing from the ideal one at the same parameters as those in Fig. 1.

5. Calculation of the dynamic frequency response of the LG with a biasing in the form of a meander by the asymptotic method and according to the Floquet theory

As we know nothing about the published results of calculations of the dynamic frequency response of the LG with a combined biasing, it is impossible to check the results, obtained here by the asymptotic method, by comparing them with the results obtained by other methods. However, the accuracy of the asymptotic method can be estimated by comparing the results of the calculation of the dynamic frequency response of the LG with a biasing in the form of a meander by this method and by the Floquet theory [6].

In some works, the influence of a biasing in the form of a meander with an amplitude Ω_r and a frequency ν on the formation of a dynamic frequency response was investigated in detail. For example, Birman et al. [6] used the Floquet theory to construct a frequency response in the entire range of Ω variation (constant component of the frequency biasing), i.e., both within the locking bands (lower orders) as well as between these bands. Birman et al. [6] noted that the results obtained are confirmed by comparing them with similar results from [7–9], in which the frequency response was investigated only for local regions of Ω variation near the locking bands and between the bands but far from their boundaries. Therefore, we will use the results of Ref. [6], as the most common and reliable source of information about the dynamic frequency response in the case of a biasing in the form of a meander. In this paper, we have obtained a theoretical frequency response for the case when $\Omega_r \gg \Omega_L \gg \nu$ (Fig. 3). According to [6], the locking bands within the boundaries $\Omega_n \pm S_n/2$ $(n = 1, 2, \ldots)$ have centres at

$$\Omega_n = n\nu\sqrt{1 - \frac{\Omega_L^2}{\Omega_r^2 - n^2\nu^2}}. \tag{18}$$

The widths of these bands are

$$S_n = 2(2n-1)\frac{\nu^2}{\Omega_L}. \tag{19}$$

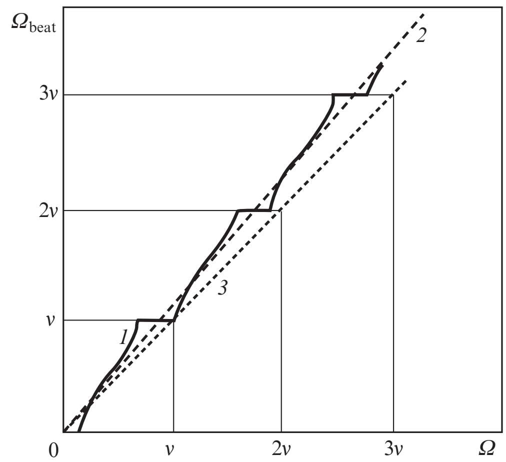

Figure 3. (*1*) Dynamic, (*2*) quasi-static and (*3*) ideal frequency response of the LG at low rotation velocities [6].

From (18) it follows that the centres of the locking bands are located nonequidistantly and are always shifted to the left with respect to frequencies $n\nu$ (the shift increases with increasing Ω_L). One can see from Fig. 3 that it is manifested in the fact that the centres of the locking bands lie on a quasi-static frequency response (conventionally straight line) rather than on an ideal frequency response. According to [6], the widths of the bands have an oscillatory dependence on their number n, and the oscillation amplitudes increase, with approaching the region of strong linear distortions ($\Omega \approx \Omega_r$), up to a value comparable to ν. Expression (19) is only an approximation for the widths of the lower order bands and does not reflect this peculiarity.

Let us use the asymptotic method to calculate the dynamic frequency response of the LG with a biasing in the form of a meander. The calculation results are presented in Fig. 4. The comparison of the dependences in Figs 3 and 4 shows that the asymptotic calculation method yields the same result as the method (Floquet theory) used in [6]. In view of the above remark, the accuracy of the results obtained by the asymptotic method is also confirmed by the results of calculations [7–9].

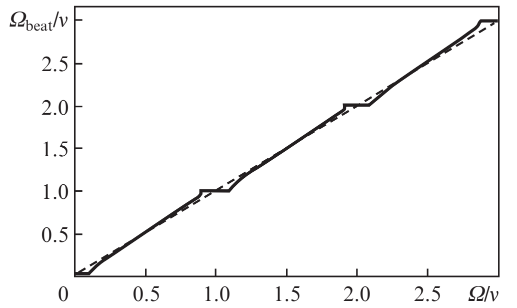

Figure 4. Dynamic frequency response of the LG with a biasing in the form of a meander, calculated by the asymptotic method at $\Omega_r = 2\pi\cdot 2 \times 10^3$, $\nu = 2\pi\cdot 50$, $\Omega_L = 2\pi\cdot 600$ (solid line), and an ideal frequency response (dashed line).

Dependences in Figs 1, 2 and 4 lead to the following conclusion: Compared with a biasing in the form of a meander, a combined biasing largely 'straightens' the dynamic frequency response by reducing the widths of the locking bands (brings it to an ideal frequency response).

6. Conclusions

The proposed method of calculation of a dynamic frequency response of the LG has allowed us to construct it on a PC using a MathCad 15 environment. A specific feature of the method is the possibility (under the condition Ω_L/Ω_r, $\Omega_s/\Omega_r \ll 1$) of presenting the solution of the phase equation in the form of recurrence sequences processed on a PC. The algorithm for the PC using this method makes it possible to construct a frequency response within a few seconds, i.e. can be used in the development or testing of the device in real time. This proves the possibility of an on-line comparative analysis of various frequency response variants.

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