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OPTICAL TRAPS

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Magneto-optical trap formed by elliptically polarised light waves for Mg atoms[*]

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Abstract. **We consider a magneto-optical trap (MOT) formed by elliptically polarised waves for ^{24}Mg atoms on a closed optical $^3P_2 \to {}^3D_3$ (λ = 383.8 nm) transition in the $\varepsilon-\theta-\bar{\varepsilon}$ configuration of the field. Compared with a known MOT formed by circularly polarised waves ($\sigma_+-\sigma_-$ configuration), the suggested configuration of the trap formed by fields of $\varepsilon-\theta-\bar{\varepsilon}$ configuration allows deeper sub-Doppler cooling of trapped ^{24}Mg atoms, which cannot be implemented in a conventional trap formed by fields of $\sigma_+-\sigma_-$ configuration.**

Keywords: *magneto-optical trap, sub-Doppler cooling, laser cooling, elliptical polarisation.*

1. Introduction

The use of ^{24}Mg atoms offers the possibility to develop a new generation of optical frequency standards due to the presence of narrow spectral lines (so-called clock transitions) associated with forbidden transitions from the ground state $^1S_0$ to the lowest triplet level $^3P_{0,1,2}$ (Fig. 1) as well as due to the presence of optical transitions with sufficiently broad spectral lines that are available for laser cooling. Thus, a closed optical $^1S_0 \to {}^1P_1$ transition can be used for Doppler cooling of ^{24}Mg atoms to a temperature T_D = 1.2 mK. However, the ground state of the optical transition in question is nondegenerate (with a total angular momentum $J = 0$), which makes it impossible to use the mechanisms of sub-Doppler laser cooling.

The authors of [1, 2] suggested using a closed optical $^3P_2 \to {}^3D_3$ (λ = 383.8 nm) transition to achieve lower temperatures. Since the lower state $^3P_2$ is degenerate with respect to the angular momentum projection ($J_g = 2$), this should lead to extremely low temperatures in sub-Doppler laser cooling [3]. However, the experimental implementation of laser cooling of ^{24}Mg atoms in a magneto-optical trap (MOT) formed by the fields with $\sigma_+-\sigma_-$ configuration, resonant to the optical $^3P_2 \to {}^3D_3$ transition, did not lead to the expected results [1]. ^{24}Mg atoms could be cooled to a temperature of only 1 mK, which even exceeds the Doppler limit of the laser cooling temperature (for this transition T_D = 425 mK). A recent analysis [2] revealed the reasons for limitations of laser cooling of ^{24}Mg atoms in the $\sigma_+-\sigma_-$ fields. They are mainly caused by imperfection of semiclassical methods that predict deep sub-Doppler cooling. However, the full quantum analysis [2] shows a limitation in temperature, which is in qualitative agreement with the results of [1]. As stated in [2], this is due to an insufficient smallness of the semiclassical parameter for the atomic transition in question (the ratio of the recoil energy to the radiation width of the upper level), which is a prerequisite for the employment of the semiclassical approximation [4–8]. In this paper, we report a MOT formed by light waves with elliptical polarisations to achieve sub-Doppler temperatures of laser-cooled ^{24}Mg atoms.

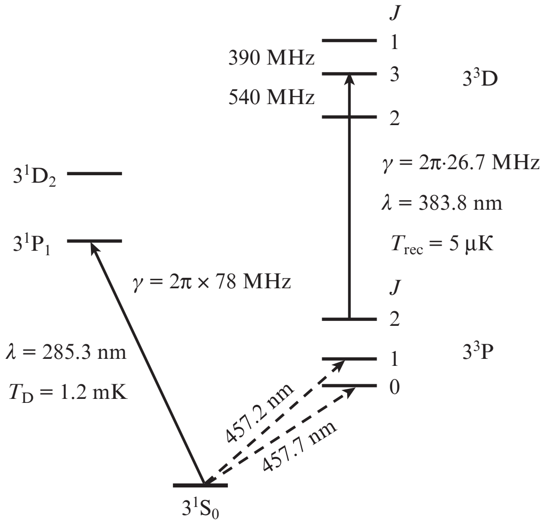

Figure 1. Diagram of some energy levels of the ^{24}Mg atom. The solid arrows indicate the optical transitions, which can be used for laser cooling, and the dashed arrows show possible 'clock' transitions.

2. Basic equations

Evolution of atoms in a resonance electromagnetic field in the presence of a magnetic field is described by the equation

$$\frac{\partial}{\partial t}\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] - \hat{\Gamma}\{\hat{\rho}\} \tag{1}$$

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for the atomic density matrix $\hat{\rho}$. Here, $\hat{H}$ is the Hamiltonian of the atom in the field, and $\hat{\Gamma}\{\hat{\rho}\}$ describes the relaxation of the populations and coherences of the energy levels in the processes of a spontaneous decay (see, e.g., [8, 9]). To date, there are many methods that allow one to solve the light–atom interaction problem described by equation (1), taking into account recoil effects in the processes of absorption and emission of photons of the field. All developed methods are methodologically divided into semiclassical [4–8] and quantum [10–17] ones. Semiclassical approaches are based on the expansion of equation (1) in the recoil parameter (the ratio of the photon momentum $\hbar k$ to the atomic momentum dispersion Δp) and are applicable under certain conditions [4–8]. In this case, the equation for the evolution of the atomic density matrix $\hat{\rho}$ (1) is reduced to the semiclassical Fokker–Planck equation for the distribution of the atom density in the phase space $W(r,p) = \mathrm{Tr}\{\hat{\rho}(r,p)\}$:

$$\left(\frac{\partial}{\partial t} + \sum_i \frac{p_i}{M}\nabla_i\right)W = -\sum_i \frac{\partial}{\partial p_i}F_i(\boldsymbol{r},\boldsymbol{p})W +$$

$$+\sum_{ij}\frac{\partial^2}{\partial p_i \partial p_j}D_{ij}(\boldsymbol{r},\boldsymbol{p})W, \qquad (2)$$

where $F_i(\boldsymbol{r},\boldsymbol{p})$ and $D_{ij}(\boldsymbol{r},\boldsymbol{p})$ are the Cartesian components of the force acting on a moving atom and of the atomic diffusion tensor in the phase space, respectively; and M is the mass of the atom.

The existing quantum methods also make use of various simplifications and approximations. In particular, the method described in [16, 17] (so-called band theory) relies on a secular approach, which involves the use of the fields with a sufficiently large detuning of their frequency $\delta = \omega - \omega_0$ from the atomic resonance frequency ω_0, when $\sqrt{U_0/E_\mathrm{R}} \ll 6|\delta|/\gamma$, where U_0 is the depth of the optical potential, and $E_\mathrm{R} = \hbar^2 k^2/(2M)$ is the recoil energy (energy acquired by an atom as a result of emission of a photon) [16]. Advanced approaches additionally involve an approximation based on the simplification of the original equation for the atomic density matrix, i.e. on the reduction of the basic equation to the equation for the atomic density matrix only in the lower state $\hat{\rho}^{gg}$, which also leads to a number of limitations on the applicability of the results [18].

We have recently been proposed an alternative method that, for the one-dimensional problem in the single-particle approximation $n\lambda^3 \ll 1$ (n is the concentration of cold atoms, and λ is the wavelength of the cooling radiation), allows one to find numerically the stationary solution for the density matrix of atoms (1) with degenerate (over the angular momentum projection) levels in a light field formed by counterpropagating waves of arbitrary intensity and polarisation. The proposed method can accurately take into account the translational motion of atoms and recoil effects arising from photon absorption and emission processes [18, 19]. In contrast to the previously discussed approaches, the proposed quantum approach is limited only by the frame of applicability of the quantum kinetic equation (1).

3. Limits of laser cooling of ^{24}Mg atoms

Our method allows one to compare different configurations of light fields and identify the parameters that are optimal for laser cooling of atoms to the lowest temperature. Strictly speaking, the distribution of atoms in the electromagnetic field is far from Maxwellian. However, we define the temperature as a measure of the average kinetic energy, which in the one-dimensional problem gives the ratio $k_\mathrm{B}T = \langle p^2 \rangle / M$. Below we present the results for the temperature in $\hbar\gamma$ units, where $k_\mathrm{B}T/(\hbar\gamma) = 1$ corresponds to the temperature $T \approx 1.28$ mK ($\gamma = 2\pi\cdot 26.7$ MHz for a closed optical $^3\mathrm{P}_2 \rightarrow {}^3\mathrm{D}_3$ transition). In addition, we consider the quantity $N_{|p|<3\hbar k}$ – a fraction of cold atoms with a momentum $|p| < 3\hbar k$ of the total number of atoms in a trap.

First, we present the results for the temperature of the laser cooling of ^{24}Mg atoms on a closed optical $^3\mathrm{P}_2 \rightarrow {}^3\mathrm{D}_3$ transition in a standard field ($\sigma_+ - \sigma_-$ configuration), obtained on the basis of the method proposed in [18, 19]. It turns out that the laser cooling temperature in this case does not reach extremely low sub-Doppler values (Fig. 2a), which is in qualitative agreement with the results of [1]. The use of the lin$\perp$lin configuration of the light field formed by counterpropagating light waves with orthogonal linear polarisations leads to temperatures of laser-cooled atoms well below the limit (Fig. 3a) and to a significantly greater fraction of cold atoms $N_{|p|<3\hbar k}$. However, it should be noted that this configuration of the light field cannot be used for the implementation of the MOT, as it leads to a zero magneto-optical effect.

For deep sub-Doppler laser cooling of ^{24}Mg atoms in the MOT we propose the $\varepsilon-\theta-\bar{\varepsilon}$ configuration of the light field

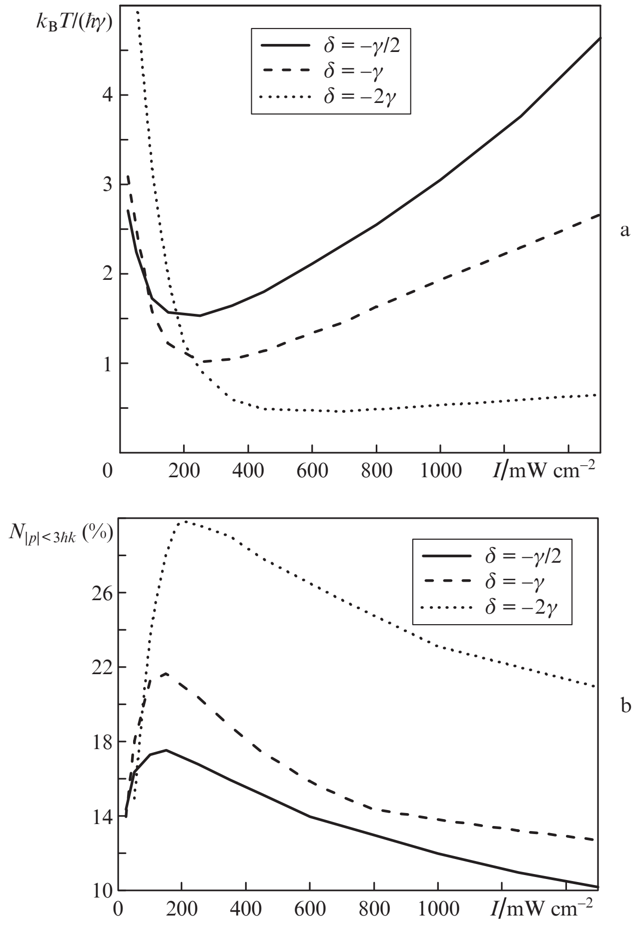

Figure 2. (a) Temperature of ^{24}Mg atoms in the $\sigma_+ - \sigma_-$ field and (b) fraction of cold atoms $N_{|p|<3\hbar k}$ as functions of the intensity of the light waves.

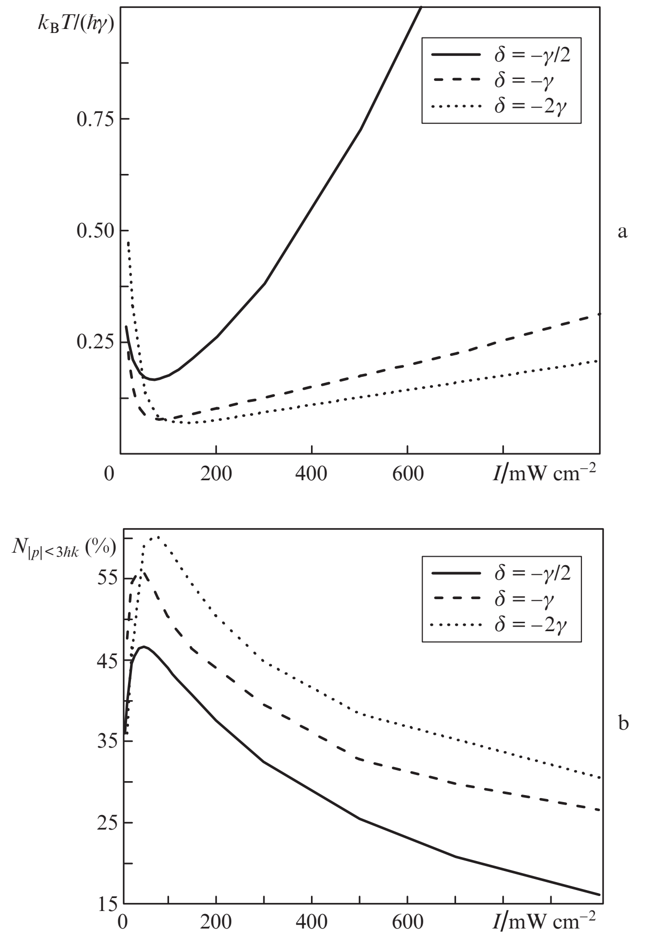

Figure 3. (a) Temperature of ^{24}Mg atoms in the lin$\perp$lin field and (b) fraction of cold atoms $N_{|p|<3\hbar k}$ as functions of the intensity of the light waves.

(Fig. 4). A preliminary study of the MOT formed by elliptically polarised waves was carried out by us in [9], where we pointed out the features of laser cooling of atoms in this field and the peculiarities of MOT operation related to the difference of the polarisations of the used light waves from linear and circular [20].

Figure 5 shows the dependences of the temperature of cooled ^{24}Mg atoms in the $\varepsilon-\theta-\bar{\varepsilon}$ field on the intensity of the counterpropagating waves in the particular case of their linear polarisation ($\varepsilon_0 = 0$) at a mutual orientation angle $\theta = -\pi/4$. The negative sign of the angle θ is selected such that the magneto-optical force was negative at the output of the atoms from the equilibrium point (regions with a zero magnetic field), i.e. formed a trapping magneto-optical potential of the MOT. Figure 6 shows the dependences of the temperature of laser-cooled ^{24}Mg atoms and the fraction of cold atoms $N_{|p|<3\hbar k}$ on the ellipticity parameter of the counterpropagating waves. The values $\varepsilon_0 = \pm\pi/4$ correspond to the left- and right-hand circular polarisations of the light waves, when the temperature takes the values corresponding to the temperature of the $\sigma_+-\sigma_-$ field. The observed asymmetry with respect to the ellipticity parameter ε_0 is determined by anomalous polarisation contributions to the frictional force [20].

In [20] we showed that anomalous contributions to the friction force dominate in the region of small detunings ($|\delta| \ll \gamma$), and so in the considered detunings $\delta = -\gamma$, these contribu-

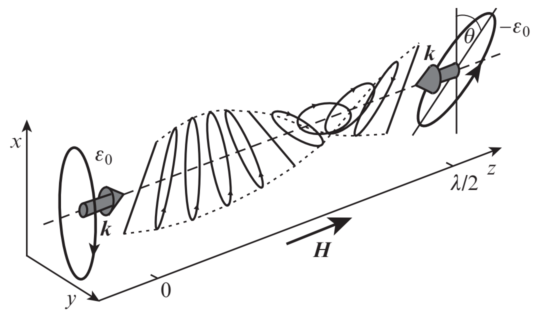

Figure 4. Light field in the $\varepsilon-\theta-\bar{\varepsilon}$ configuration, formed by the counterpropagating waves with opposite elliptical polarisations, which are determined by the ellipticity parameters ε_0 and $-\varepsilon_0$. Parameters $-\pi/4 < \varepsilon_0 < \pi/4$, $\varepsilon_0 = 0$ correspond to the linear polarisation of the wave, and $\varepsilon_0 = \pm\pi/4$ – to the left- and right-hand circular polarisations; θ is the angle of mutual orientation between the principal axes of the polarisation ellipses of the counterpropagating waves. To be specific, we can put $-\pi/2 \leqslant \theta \leqslant \pi/2$, with positive and negative angles corresponding to the clockwise and counterclockwise rotations of the polarisation ellipse, respectively.

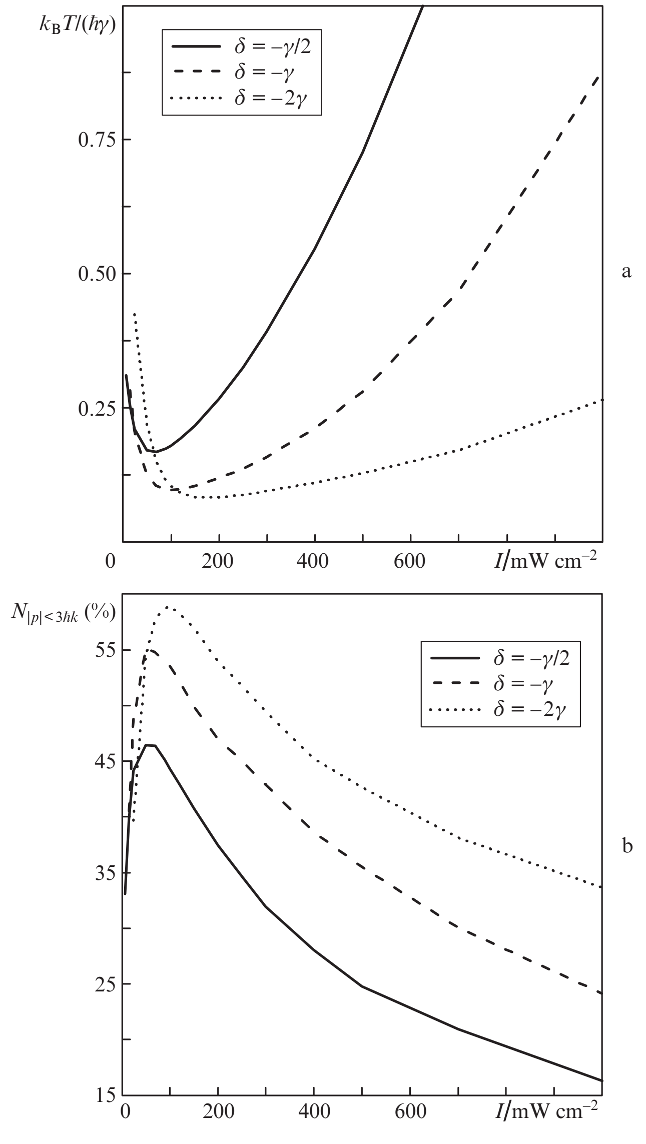

Figure 5. (a) Temperature of ^{24}Mg atoms in the $\varepsilon-\theta-\bar{\varepsilon}$ field at $\theta = -\pi/4$ and $\varepsilon_0 = 0$ and (b) fraction of cold atoms $N_{|p|<3\hbar k}$ as functions of the intensity of the light waves.

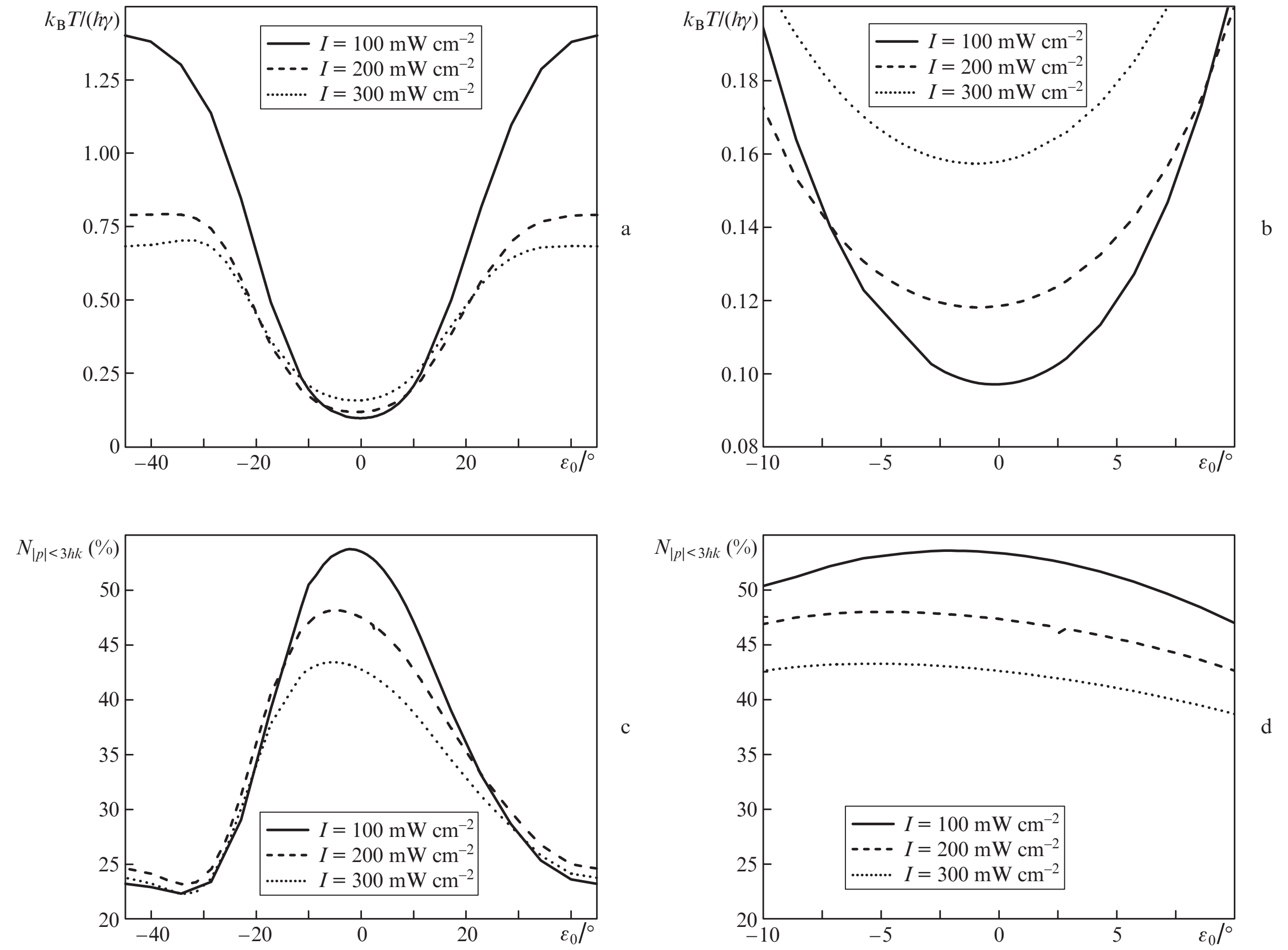

Figure 6. (a) Temperature of ^{24}Mg atoms in the $\varepsilon-\theta-\bar{\varepsilon}$ field at $\theta = -\pi/4$ and (c) fraction of cold atoms $N_{|p|<3\hbar k}$ as functions of the ellipticity parameter of the light waves; (b, d) dependences of the temperature and the fraction of cold atoms $N_{|p|<3\hbar k}$ in the region of small parameters of ellipticity ε_0 at a detuning $\delta = -\gamma$.

tions are small, and the temperature of the cooled atoms reaches a minimum value when the wave polarisations are close to linear: $\varepsilon_0 \approx -0.28°$ at an intensity of each wave I = 100 mW cm^{-2}, $\varepsilon_0 \approx -0.86°$ at I = 200 mW cm^{-2} and $\varepsilon_0 \approx -1.15°$ at I = 300 mW cm^{-2}. The maximum fraction of cold atoms $N_{|p|<3\hbar k}$ is achieved for $\varepsilon_0 \approx -2°$ at I = 100 mW cm^{-2}, for $\varepsilon_0 \approx -5.16°$ at I = 200 mW cm^{-2} and for $\varepsilon_0 \approx -5.26°$ at I = 300 mW cm^{-2}.

Table 1 shows the optimal values of the ellipticity parameters and their corresponding minimum temperatures of ^{24}Mg atoms at different intensities of the counterpropagating waves forming the field of $\varepsilon-\theta-\bar{\varepsilon}$ configuration at $\theta = -\pi/4$ and detuning $\delta = -\gamma$.

Table 1. Optimal values of ellipticity parameters and corresponding minimum temperatures of laser-cooled ^{24}Mg atoms, as well as optimal values of ellipticity parameters for trapping the largest fraction of cold atoms $N_{|p|<3\hbar k}$ at different intensities of the counterpropagating waves forming the $\varepsilon-\theta-\bar{\varepsilon}$ field at $\theta = -\pi/4$ and a detuning $\delta = -\gamma$.

I/mW cm^{-2}	$\varepsilon_0/°$	$k_BT/(\hbar\gamma)$	$\varepsilon_0/°$	$N_{|p|<3\hbar k}$ (%)
100	−0.28	0.097	−2	53.74
200	−0.86	0.118	−5.16	48.16
300	−1.15	0.157	−5.16	43.43

4. Magneto-optical potential in the $\varepsilon-\theta-\bar{\varepsilon}$ field of the MOT

The nonlinear dependence of the magneto-optical force on the magnetic field in units of the ratio of the Zeeman shift Ω_H of the lower state sublevels $|J_g = 2, \mu_g = 1\rangle$ to γ is shown in Fig. 7a for the $\sigma_+-\sigma_-$ configuration of the light field and in Fig. 8a for the $\varepsilon-\theta-\bar{\varepsilon}$ configuration at $\theta = -\pi/4$ and the polarisation of the light waves close to the linear. Here, the detuning $\delta = -\gamma$, and the intensity of the light waves I = 100 mW cm^{-2} correspond to the optimum parameters of the light fields in Fig. 5. The Zeeman shift of the ground state $\Omega_H = \gamma$ corresponds to the magnetic field H = 12.7 Gs.

A standard MOT configuration is formed by three pairs of light waves counterpropagating along the axes x, y, z, and ensuring the trapping of atoms in all three directions in the presence of a static magnetic field of quadrupole configuration [21]. The magnetic field in the beam intersection point (x = 0, y = 0, z = 0) is zero and increases in absolute value upon a displacement from this point.

Assuming the growth of the magnetic field in the region of trapping of atoms to be linear, the magneto-optical potential depth can be estimated as

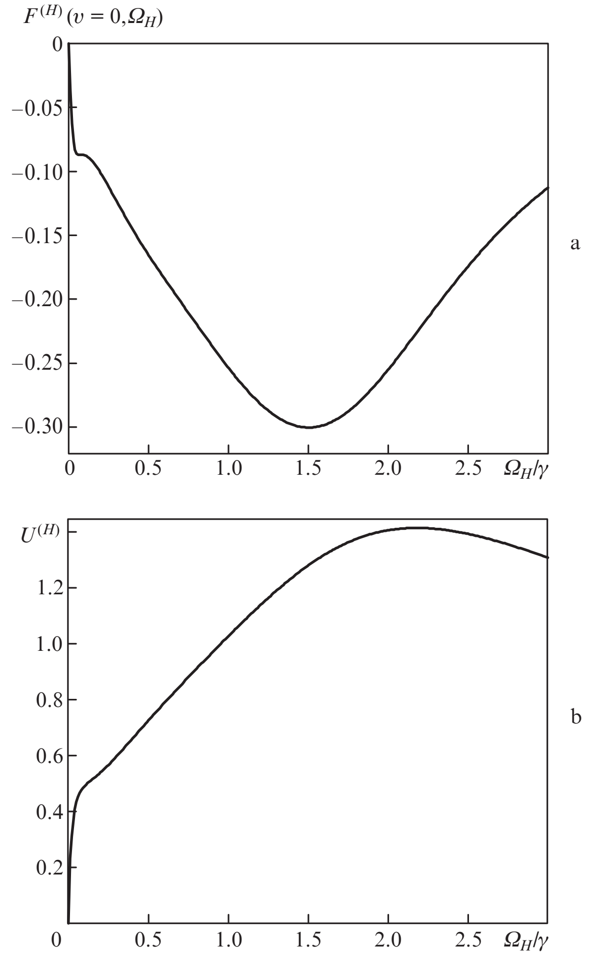

Figure 7. Dependences of the magneto-optical forces in $\hbar k\gamma$ units, acting on ^{24}Mg atoms (a), and the depth of the magneto-optical potential in $\hbar\gamma R_w/\lambda$ units (b) on the value of the magnetic field at the boundary of the trapping region in the Ω_H/γ units in the $\sigma_+ - \sigma_-$ field ($I = 100$ W cm^{-2}, $\delta = -\gamma$).

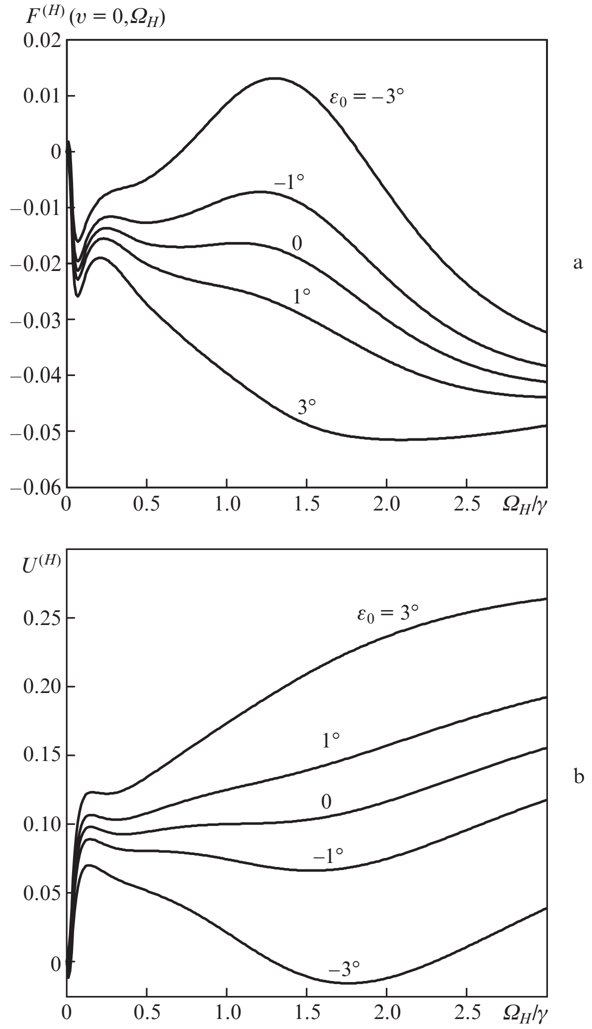

Figure 8. Dependences of the magneto-optical forces in $\hbar k\gamma$ units, acting on ^{24}Mg atoms (a), and the depth of the magneto-optical potential in $\hbar\gamma R_w/\lambda$ units (b) on the value of the magnetic field at the boundary of the trapping region in the Ω_H/γ units in the $\varepsilon - \theta - \bar{\varepsilon}$ field for different parameters of ellipticity ε_0 ($\theta = -\pi/4$, $I = 100$ mW cm^{-2}, $\delta = -\gamma$).

$$U^{(H)} = -\frac{R_w}{\Omega_H(R_w)}\int_0^{\Omega_H(R_w)}\langle F^{(H)}(v = 0, \Omega_H)\rangle \mathrm{d}\Omega_H, \tag{3}$$

where R_w is the size of the atom trapping region determined by the radii of the light beams; and $\Omega_H(R_w)$ is the Zeeman shift at the boundary of the trapping region. The depth of the magneto-optical potential as a function of the magnetic field at this boundary is shown in Fig. 7b for the $\sigma_+ - \sigma_-$ configuration of the light field and in Fig. 8b for the $\varepsilon - \theta - \bar{\varepsilon}$ configuration at a linear increase in the magnetic field within the atom trapping region (3).

The depth of the magneto-optical potential in the $\varepsilon - \theta - \bar{\varepsilon}$ field is considerably smaller than in the $\sigma_+ - \sigma_-$ field, but nonetheless, it remains significantly higher than the temperature of cold atoms. Thus, for example, in the $\varepsilon - \theta - \bar{\varepsilon}$ fields with $R_w = 0.5$ cm and a magnetic field gradient $\partial H/\partial z = 12.7$ Gs cm^{-1} (corresponding to $\Omega_H/\gamma \approx 0.5$ at the boundary of the trapping region), $U^{(H)} = 0.094\hbar\gamma R_w\lambda^{-1} \approx 1.56K$ (Fig. 8b), which is much greater than the temperature of cold atoms, $T = 124$ mK, at the trap parameters under consideration.

The ellipticity of the light fields, leading to a decrease in the temperature of cooled atoms in the $\varepsilon - \theta - \bar{\varepsilon}$ field as a result of abnormal contributions to the frictional force at $\varepsilon_0 < 0$ and $\theta = -\pi/4$, reduces the depth of the magneto-optical potential (Fig. 8b). Thus, at $\varepsilon_0 = -3°$ the magneto-optical force changes its sign when the magnetic field reaches a critical value $H_c = 9$ Gs, which corresponds to the Zeeman shift $\Omega_H \approx 0.71\gamma$ (Fig. 8a).

In assessing the number N_c of atoms trapped in a MOT, it is necessary to take into account the nonlinear dependence of the magneto-optical force $F^{(H)}(v, \Omega_H)$ on the magnetic field values for atoms with nonzero velocities. The number of atoms trapped in a MOT is determined by the characteristic velocity v_c, at which atoms can be trapped by a magneto-optical force: $N_c \propto v_c^4$ [22]. For a magneto-optical trap formed by waves of $\sigma_+ - \sigma_-$ configuration, the critical velocity of trapping is $v_c \approx 3.5\gamma/k$ [22], which for a trap formed by waves with the beam radii $R_w = 0.5$ cm yields an estimate of $N_c \approx 7 \times 10^7$ atoms.

For atoms in a MOT formed by the $\varepsilon - \theta - \bar{\varepsilon}$ light field, at some large values of the magnetic field the magneto-optical force changes its sign in some range of velocities for slow atoms (Fig. 9). Therefore, the atoms falling into this region of parameters (v, Ω_H) cannot be trapped by a magneto-optical force. The boundaries of these regions determine the critical

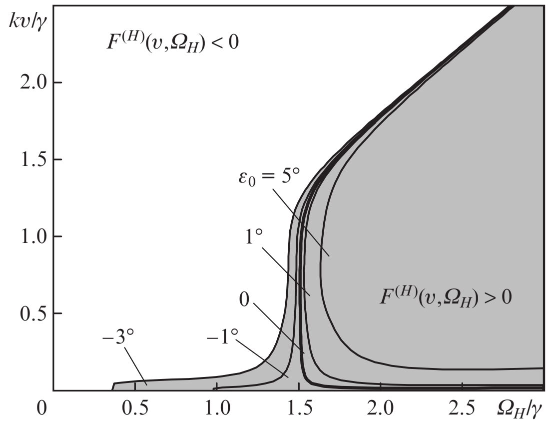

Figure 9. Range of parameters (v, Ω_H), in which the magneto-optical force becomes repulsive, in the $\varepsilon-\theta-\bar{\varepsilon}$ configuration ($\theta = -\pi/4$) at different ellipticities ε_0 of the light waves ($I = 100$ mW cm^{-2}, $\delta = -\gamma$).

velocity v_c of trapping at different values of the magnetic field. Thus, for the lin$-\theta-$lin configuration at $\theta = -\pi/4$ and sufficiently large magnetic fields ($\Omega_H/\gamma > 1.5$) the critical velocity of trapping falls down to $\sim 0.014\gamma/k$ (0.014 m s^{-1}), which reduces the number of atoms in a trap almost to zero.

Thus, the implementation of a stable magneto-optical trap in the $\varepsilon-\theta-\bar{\varepsilon}$ fields requires magnetic fields with a small spatial gradient such that the magnetic field in the trapping region does not reach a critical value at which the magneto-optical force for slow-moving atoms changes its sign.

Thus, for example, for stable operation of the trap at $I = 100$ mW cm^{-2}, $\delta = -\gamma$ and $\varepsilon_0 = -1°$, the magnetic field in the trap region $r < R_w$ should not exceed the critical value $\Omega_H/\gamma \approx 0.98$ ($H_c \approx 12.5$ Gs), and at $\varepsilon_0 = 5°$ it should not exceed $\Omega_H/\gamma \approx 1.63$ ($H_c \approx 20.8$ Gs) (Fig. 9).

Figure 10 shows the critical values of the magnetic field as functions of the intensity of the light waves for ^{24}Mg atoms. At such values of the fields the magneto-optical force ceases to trap the atoms moving at low velocities ($kv < \gamma$) in a MOT formed by the $\varepsilon-\theta-\bar{\varepsilon}$ field at different parameters of ε_0 (the orientation angle $\theta = -\pi/4$).

5. Conclusions

We report a MOT for ^{24}Mg atoms, working on a closed optical $^3P_2 \rightarrow {}^3D_3$ transition and formed by waves with elliptic polarisations in the $\varepsilon-\theta-\bar{\varepsilon}$ configuration. Within the framework of a one-dimensional model we have performed an analysis of the magneto-optical potential, temperature and fraction of cold atoms $N_{|p|<3\hbar k}$ as functions of the intensity of the light waves forming a MOT field and their polarisations. The analysis is based on a newly developed method [18, 19], which allows one to take into account the quantum recoil effects in the interaction of atoms with photons of the field, as well as to properly take into account the fraction of cold atoms trapped in an optical potential.

It is shown that the use of elliptically polarised fields in the $\varepsilon-\theta-\bar{\varepsilon}$ configuration makes it possible to reach temperatures of laser-cooled ^{24}Mg atoms that are several orders of magnitude lower than those in a MOT formed by $\sigma_+ - \sigma_-$ waves. Thus, in the $\varepsilon-\theta-\bar{\varepsilon}$ field formed by the waves with elliptical polarisations, oriented at an angle of $\theta = -\pi/4$, and with close-to-linear polarisations, it is possible to reach a minimum temperature $T \approx 100$ mK in a sufficiently deep magneto-optical potential of the MOT with a fraction of ultracold atoms $N_{|p|<3\hbar k} \approx 52\%$ of the total number $N_c \approx 7 \times 10^7$ of trapped atoms. As compared to a MOT formed by the waves with circular polarisations ($\sigma_+ - \sigma_-$ configuration), our MOT with the $\varepsilon-\theta-\bar{\varepsilon}$ field has a lesser depth of the optical potential, but allows one to reach lower temperatures of laser-cooled ^{24}Mg atoms.

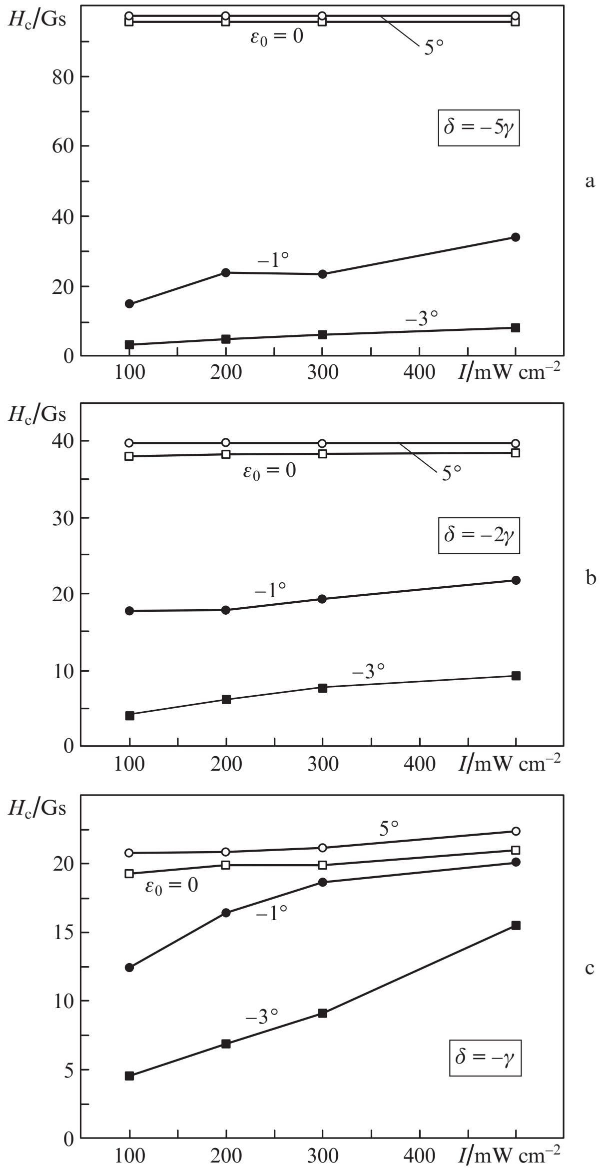

Figure 10. Critical values of the magnetic fields at which the magneto-optical force changes its sign, i.e., begins to push the atoms from the MOT formed in the $\varepsilon-\theta-\bar{\varepsilon}$ field, resonant to the closed optical $^3P_2 \rightarrow {}^3D_3$ transition of ^{24}Mg atoms. The orientation angle $\theta = -\pi/4$.

It is shown that the $\varepsilon-\theta-\bar{\varepsilon}$ MOT is more sensitive to the parameters of the light and magnetic fields, because with increasing magnetic field the magneto-optical force for insufficiently slow atoms can change its sign, which will limit the number of atoms trapped in the MOT. Therefore, stable operation of the $\varepsilon-\theta-\bar{\varepsilon}$ MOT requires a monitoring of the magnetic field, which should not exceed the critical values, depending on the parameters of the light fields. Parameters of the critical values of the magnetic field are also presented.

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