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Generation of quantum electrodynamic cascades by colliding laser pulses

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Abstract. **Quantum electrodynamic cascades in intense electromagnetic fields arise when the proper electron acceleration χ, expressed in Compton units, can attain values greater than or on the order of unity. For times $t \ll 1/\omega$, where ω is the carrier frequency of the field, we have derived a general formula for χ of an initially resting electron in an arbitrary electromagnetic field. Using this formula, we have found an optimal configuration of colliding laser pulses, which provides a significant reduction in the threshold intensity of occurrence of cascades up to a level of $\sim 10^{23}$ W cm^{-2}.**

Keywords: *quantum electrodynamic cascades, intense electromagnetic field, colliding laser pulses, ultra-relativistic electrons.*

1. Introduction

Quantum electrodynamic (QED) cascades [1–15] have recently attracted increasing attention of researchers due to a remarkable progress in laser technology. By now, the maximum laser intensity has been increased to 2×10^{22} W cm^{-2} [16], and several laser facilities, the radiation intensity of which will exceed 10^{23} W cm^{-2}, are being constructed [17, 18] in the world. Furthermore, we should mention the XCELS project [19] aimed at reaching an intensity of 10^{25} – 10^{26} W cm^{-2}.

According to numerical calculations [1, 2, 7, 11, 14], QED cascades will develop at a laser field intensity of $\sim10^{24}$ W cm^{-2}. Thus, probably in the next few years it will be possible to experimentally study this phenomenon.

QED cascades can be described as follows. Suppose that in the field of an intense electromagnetic wave there is a charged particle. It will be accelerated by the field and emit hard photons, which, interacting with the field, will produce an electron–positron pair. New particles will be accelerated again by the field and emit hard photons and so on. This avalanche process will terminate when all the particles leave the region of the intense field.

Note that other different schemes of generation of QED cascades are also possible. For example, a beam of high-energy photons can serve as a source of first electrons and positrons in the intense field [11]. Furthermore, cascades may be formed in collisions of fast electrons with laser pulses [4, 5, 9, 12]. In this formulation, the cascade development requires even less laser intensity due to increased field strength in the reference frame associated with the electron beam. Mironov et al. [12] showed that it is possible to select the parameters of the electron beam and the laser pulse so that to implement two schemes of QED cascade generation in a single experiment. In the first stage, the cascade will develop due to deceleration of electrons in collisions with the laser pulse. In this case, the electrons will penetrate into the region of the intense field, and then the cascade will continue to develop due to acceleration of the electrons by the field in the transverse direction.

In this paper, we focus on the study of cascade generation by initially resting charged particles. It was shown in [14] that the threshold field intensity required for the cascade production can be substantially reduced if use is made of multiple colliding pulses for a given total power of laser radiation. This is due to the fact that as a result of the coherent addition of pulses with correctly selected polarisation, the proper electron acceleration, expressed in Compton units, increases rapidly with time:

$$\chi = \frac{e\hbar}{m^3c^5}\mathcal{E}F_{\perp}, \tag{1}$$

where m and e are the mass and charge of the electron; c is the speed of light; $\mathcal{E}$ is the particle energy; and $e\boldsymbol{F}_{\perp}$ is the Lorentz force component $e\boldsymbol{F} = e(\boldsymbol{E} + [\boldsymbol{v}\times\boldsymbol{B}])$, perpendicular to the particle velocity $\boldsymbol{v}$. In the approximation of a locally constant field χ determines the probability of emission of a photon by a charged particle (W_γ) and the creation of a pair by the photon ($W_{\rm e}$) [20, 21]:

$$W_{{\rm e},\gamma}(\chi_{\gamma,{\rm e}} \gg 1) \sim \frac{e^2m^2c^4}{\hbar^2c^2\mathcal{E}_{{\rm e},\gamma}}\chi_{\gamma,{\rm e}}^{2/3},\ W_{\rm e}(\chi_\gamma \lesssim 1) \sim \exp\left(-\frac{8}{3\chi_\gamma}\right). \tag{2}$$

Here, the subscripts e denote quantities related to electrons and positrons, and the subscripts γ – quantities related to photons. Since at $\chi_\gamma \lesssim 1$ the probability of the pair creation is exponentially suppressed and the particle with $\chi_{\rm e} \lesssim 1$ cannot emit a photon with $\chi_\gamma \gtrsim 1$ by virtue of the conservation laws, the task is to find a laser field configuration, which provides the fastest growth of $\chi_{\rm e}(t)$. Below, instead of $\chi_{\rm e}$ we will use the notation χ.

In this work, the results obtained in [13, 14] are expanded in two directions. Firstly, unlike papers [13, 14] where an even number of colliding pulses was considered, we examine the dependence of $\chi(t)$ in an arbitrary electromagnetic field (in particular, for a single pulse). Secondly, it was assumed in [13, 14] that all pulses are located in one plane; however, we will show that by rejecting this restriction it is possible to

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lower even more the threshold of the QED cascade production.

2. Parameter $\chi(t)$ in an arbitrary electromagnetic field

Since the probability of QED processes in the cascade is determined by (1), to lower the threshold of the field intensity necessary for the production of the cascade, it is required to find such a field configuration which provides a maximum growth rate of $\chi(t)$ [14]. We consider a charged particle at rest at time $t = 0$ in the focus of an arbitrary electromagnetic field and calculate $\chi(t)$ of a particle moving near the focus, i.e. assuming that $t \ll 1/\omega$, where ω is the frequency of the field. In addition, we assume that the field is intense, and the particle becomes ultra-relativistic shortly after the start of acceleration ($eEt_{\rm acc} \gg mc$, where $t_{\rm acc} \ll 1/\omega$ is the acceleration time), but it is still considerably weaker than the QED critical field ($E \ll E_{\rm cr} = m^2c^3/e\hbar = 1.3\times10^{16}$ V cm^{-1}). Near the focus, the electric and magnetic fields can be expanded into a Taylor series with respect to coordinates and time $x^k = \{ct, \boldsymbol{x}\}$:

$$\boldsymbol{E}(x) \approx \boldsymbol{E}_0 + \boldsymbol{E}_{,k}(0)x^k,\ \boldsymbol{B}(x) \approx \boldsymbol{B}_0 + \boldsymbol{B}_{,k}(0)x^k. \tag{3}$$

We will seek a solution to the equation of motion of the particle

$$\dot{\boldsymbol{p}} = \mathrm{e}\boldsymbol{F} \tag{4}$$

as a series $\boldsymbol{p} = \boldsymbol{p}^{(0)} + \boldsymbol{p}^{(1)} + \ldots$, where $\boldsymbol{p}^{(k)} = O((\omega t)^k)$.

Let us first assume that the electric and magnetic fields in the focus at the initial instant of time are parallel to each other, i.e., $\boldsymbol{E}_0 \parallel \boldsymbol{B}_0$. In this case, within the zero order in ωt, the momentum, energy and radius vector of the particle have the form

$$\boldsymbol{p}^{(0)} = e\boldsymbol{E}_0 t,\ \mathcal{E}^{(0)} = eE_0 t,\ \boldsymbol{r}^{(0)} = \boldsymbol{v}^{(0)}t = \frac{\boldsymbol{E}_0}{E_0}t, \tag{5}$$

which takes into account that $t \gg m/eE_0$ (the system of units $\hbar = c = 1$ is used hereinafter). Expressions for fields up to first order in ωt take the form $\boldsymbol{E}(x) = \boldsymbol{E}_0 + \boldsymbol{E}'t$, $\boldsymbol{B}(x) = \boldsymbol{B}_0 + \boldsymbol{B}'t$, where

$$\boldsymbol{E}' = \left\{\frac{\partial}{\partial t} + \boldsymbol{v}^{(0)}\nabla\right\}\boldsymbol{E}(0),\ \boldsymbol{B}' = \left\{\frac{\partial}{\partial t} + \boldsymbol{v}^{(0)}\nabla\right\}\boldsymbol{B}(0). \tag{6}$$

Then, for $\boldsymbol{p}^{(1)}$ we obtain the equation

$$\dot{\boldsymbol{p}}^{(1)} = e\left[\boldsymbol{f}^{(1)}t + \frac{[\boldsymbol{p}^{(1)}\times\boldsymbol{B}_0]}{\mathcal{E}^{(0)}}\right], \tag{7}$$

where

$$\boldsymbol{f}^{(1)} = \boldsymbol{E}' + [\boldsymbol{v}^{(0)}\times\boldsymbol{B}']. \tag{8}$$

The solution to equation (7) has the form

$$\boldsymbol{p}^{(1)} = et^2\left(\frac{\boldsymbol{f}_{\parallel}^{(1)}}{2} + \frac{2E_0^2\boldsymbol{f}_{\perp}^{(1)} + E_0[\boldsymbol{f}^{(1)}\times\boldsymbol{B}_0]}{4E_0^2 + B_0^2}\right). \tag{9}$$

Here, $\boldsymbol{f}_{\parallel}^{(1)}$ and $\boldsymbol{f}_{\perp}^{(1)}$ are the components of $\boldsymbol{f}^{(1)}$ that are parallel and perpendicular to the initial direction of the electric and magnetic fields $\boldsymbol{E}_0$ and $\boldsymbol{B}_0$.

The modulus of the Lorentz force component, perpendicular to the velocity of the particle,

$$eF_{\perp} = |[e\boldsymbol{F}\times\boldsymbol{v}]| = eE_0 f_{\perp}^{(1)}t/\sqrt{4E_0^2 + B_0^2},$$

and for $\chi(t)$ we obtain the expression

$$\chi(t) = \frac{e^2E_0^2f_{\perp}^{(1)}}{m^3\sqrt{4E_0^2 + B_0^2}}t^2, \tag{10}$$

which coincides with the result of [14] in the limit $B_0 = 0$.

If the electric and magnetic fields at the initial instant of time are not parallel, there is, as is known, a reference system $\tilde{K}$, in which the fields are parallel, or one of them is zero. An exception is the case when the electric and magnetic fields are perpendicular and equal in magnitude; this situation occurs in particular in a plane wave, where the cascades are not produced. Thus, using the Lorentz invariance of χ, we can calculate it in the reference frame $\tilde{K}$ by formula (10). Note that in the reference frame $\tilde{K}$ the initial momentum of the particle $p_{\rm i} = \gamma mV$ (here, V is the velocity of $\tilde{K}$ relative to the laboratory reference frame, $\gamma = 1/\sqrt{1 - V^2}$)) will not be equal to zero, and we can use formula (10) only for the case $p_{\rm i} \ll p_{\perp}$, where $p_{\perp}$ is the momentum in the direction that is transverse to that of the initial electric field. This momentum is gained by the particle as a result of interaction with the field. It follows from (9) and (10) that $p_{\perp} \sim \chi m^3/e\tilde{E}_0$, where $\tilde{E}_0$ is the electric field strength at a starting point in $\tilde{K}$. Given that $E_0 \sim \gamma\tilde{E}_0$, we obtain the condition of applicability of our approach for the calculation of $\chi(t)$ in an arbitrary electromagnetic field:

$$E_0 \ll \chi E_{\rm cr}. \tag{11}$$

3. Parameter $\chi(t)$ in a focused field

Consider a realistic model of a focused laser field, proposed in [22] (see also [13]):

$$\boldsymbol{A} = \mathrm{i}\frac{\sqrt{P}f(\xi)\exp(-\mathrm{i}\omega t)}{\omega^3|\boldsymbol{\varepsilon}|}[\hat{\boldsymbol{n}}\times\nabla_{\perp}]([\boldsymbol{\varepsilon}\times\hat{\boldsymbol{n}}]\cdot\nabla_{\perp})\frac{\sin(\omega\mathcal{R})}{\mathcal{R}}, \tag{12}$$

which is an exact solution of Maxwell's equations (for convenience we have introduced an additional factor i in comparison with [13, 22]). Here, $\mathcal{R} = (r_{\parallel} - \mathrm{i}b)\sqrt{1 + r_{\perp}^2/(r_{\parallel} - \mathrm{i}b)^2}$; $\hat{\boldsymbol{n}}$ is the unit vector along the axis of the laser beam; $\boldsymbol{r}_{\parallel}$ and $\boldsymbol{r}_{\perp}$ are the radius vector components that are parallel and perpendicular to $\hat{\boldsymbol{n}}$; $\boldsymbol{\varepsilon}$ is the polarisation vector; P is the beam power; b is the Rayleigh length; $\xi = b\omega$; and

$$f(\xi) = \frac{8\sqrt{\pi}\xi^{5/2}\exp\xi}{\sqrt{\exp(4\xi)(4\xi^2 - 6\xi + 3) - 4\xi^2 - 6\xi - 3}}. \tag{13}$$

At the initial instant of time in the focus, the electric and magnetic fields in model (12) are mutually perpendicular, with $E_0 > B_0$. Therefore, in a reference frame moving with velocity $\boldsymbol{V} = [\boldsymbol{E}_0 \times \boldsymbol{B}_0]/B_0$, the magnetic field is zero, and the parameter χ can be calculated using (10). Using again the original reference frame, we obtain

$$\chi_1(t) = \frac{e^2P\omega^3}{m^3}\frac{|\boldsymbol{\varepsilon}_1\boldsymbol{\varepsilon}_2|}{\varepsilon_1^2 + \varepsilon_2^2}g_1(\xi)t^2, \tag{14}$$

where $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$ are the components of the polarisation vector; $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_1 - \mathrm{i}\boldsymbol{\varepsilon}_2$, $\boldsymbol{\varepsilon}_1 \perp \boldsymbol{\varepsilon}_2$ and

$$g_1(t) = 4\sqrt{2}$$

$$\times \frac{\sqrt{2\xi^4 - 4\xi^2 + 2(2\xi^2+9)\xi\sinh(2\xi) - (14\xi^2+9)\cosh(2\xi) + 9}}{\xi^6(\sinh\xi - \xi\cosh\xi)^2((4\xi^2+3)\sinh 2\xi - 6\xi\cosh 2\xi)}$$

$$\times[\xi(-3\xi(72\xi^2+281)\sinh 3\xi + \xi(28\xi^4+36\xi^2-63)\sinh\xi$$

$$+ (34\xi^4 + 585\xi^2 + 648)\cosh 3\xi + (4\xi^6 - 118\xi^4$$

$$+ 279\xi^2 - 648)\cosh\xi) - 864\sinh^3\xi]. \quad (15)$$

The dependences $g(\xi)$ for one (g_1) and two (g_2) focused laser beams are shown in Fig. 1. It follows from (14) that the circular polarisation ($\boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon}_2$) provides the most rapid growth in $\chi(t)$ at a given focusing of the field and is therefore optimal for the development of the cascade from the particle, which begins its motion from the focus of the laser pulse. This is also true in the case of cascades, resulting from a collision of an intense laser pulse with a beam of high-energy photons [11].

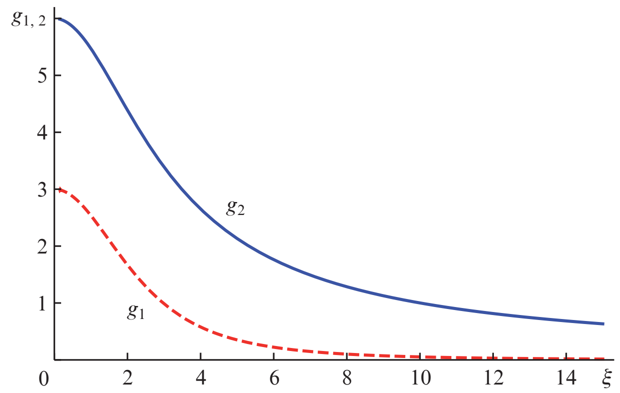

Figure 1. Functions $g_1(\xi)$ and $g_2(\xi)$ describing the dependence of the parameter χ on focusing for one or two laser beams, respectively.

Now let us consider two identical head-on colliding beams, described by model (12) (see also [13, 14]). We assume the total power P of the laser field to be fixed and replace P by $P/2$ in (12). The magnetic field at the focus of two identical colliding beams is zero, and for χ we obtain the expression

$$\chi_2(t) = \frac{e^2 P\omega^3}{m^3}\frac{|\boldsymbol{\varepsilon}_1\boldsymbol{\varepsilon}_2|}{\varepsilon_1^2+\varepsilon_2^2}g_2(\xi)t^2, \quad (16)$$

where

$$g_2(\xi) = 128\exp(2\xi)$$

$$\times\frac{(\xi\cosh\xi - \sinh\xi)[3(\xi^2+4)\sinh\xi + \xi(\xi^2-12)\cosh\xi]}{\xi^3[-4\xi^2 + \exp(4\xi)(4\xi^2 - 6\xi + 3) - 6\xi - 3]}. \quad (17)$$

Note that in the case of a weakly focused field ($\xi \gg 1$), $g_2(\xi) \sim \xi^{-1}$, and $g_1(\xi) \sim \xi^{-7/2}$; hence $g_2(\xi) \gg g_1(\xi)$. This is one of the reasons why the scheme with two colliding pulses is more efficient for the observation of the cascades, than the scheme with a single pulse of total power [1, 2].

4. Threshold for the QED cascade production by an initially resting particle

In [13, 14] we have shown that the growth rate of the parameter χ can be increased (and the threshold of the cascade production respectively reduced) if at a fixed total power of the laser field use is made of several pairs of colliding beams (instead of one pair). At the same time, there is an optimal polarisation of the beams

$$\boldsymbol{\varepsilon}_j = \{\mathrm{i}\sqrt{2}\sin\varphi_j\cos\varphi_j, -\mathrm{i}\sqrt{2}\cos^2\varphi_j, 1\}, \quad (18)$$

where $\varphi_j = 2\pi(j-1)/n$ is the angle between the axis of the jth beam and the x axis, which provides the fastest growth of the parameter χ. In this optimal case it is proportional to the number n of beams:

$$\chi_n^{(\mathrm{plane})}(t) = \frac{e^2\omega^3 P g_2(\xi)}{2\sqrt{2}m^3}nt^2. \quad (19)$$

It is assumed that the axes of all the beams are coplanar (Fig. 2a). It follows from formula (19) and Fig. 1 that $\chi(t)$ increases rapidly with increasing degree of beam focusing. In modern laser systems [16] the degree of focusing is such that in the focal plane half of the laser power passes through the cross section with a diameter equal to the wavelength; in this case, the parameter $\xi \approx 8$ [13, 14]. The aperture of the beam is $0.216\pi > \pi/5$, and so a maximum of eight beams can be arranged in a plane (the number n of beams must be an even number).

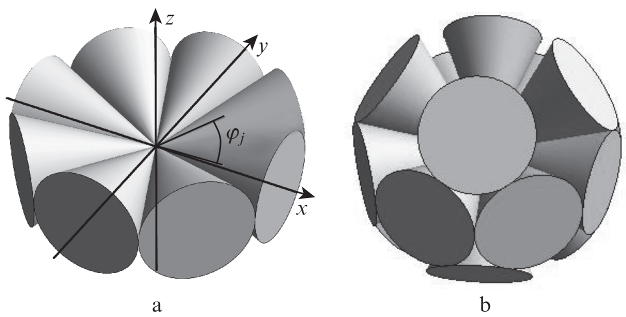

Figure 2. (a) Position of laser beams colliding in the same plane, and (b) configuration of 16 colliding laser beams; angle $\varphi_j = 2\pi(j-1)/n$.

If we do not limit our consideration to a single plane, the number of colliding laser beams may be increased to 16 (Fig. 2b), by adding eight beams whose axes are inclined to xy plane at angles $\pm\arcsin(16/25)$ to eight beams in the xy plane. For this configuration, the dependence of χ on the polarisation of the beams becomes much more complicated than in a flat plane scenario, but it allows one to numerically determine the optimal polarisation, which will provide the most rapid growth in $\chi(t)$. The details of the optimal configuration are given in the Appendix. Thus, $\chi_{16}(t)/\chi_8^{(\mathrm{plane})}(t) \approx 1.8$, which for a given power leads to a more intense development of the cascade in the case of 16 beams (see below).

The QED cascade threshold can be found numerically. We have used the same numerical technique, as in [12–14], where it is assumed that electrons and positrons between the acts of photon emission move along classical trajectories, and the emission and creation of pairs are modelled by an event generator, similar to those described in [6, 23]. In addition, we selected (12) as a field model, limiting the time of calculation to the five laser periods, which corresponds to the characteristic duration of laser pulses for advanced facilities (see, e.g., [16]). The focusing parameter $\xi = 8$ is selected according to the reasons given above, and the wavelength $\lambda = 1.24$ μm corresponds the photon energy of 1 eV. As the seed particles, we used $N_0 = 1000$ electrons that were initially in the focus of the field, $r = 0$.

As a condition of cascade production it is assumed that during the passage of pulses each seed particle generates on average one electron–positron pair. In our calculations for the optimal configuration of 16 laser pulses, the threshold total power of cascade production was P = 6.2 PW, which corresponds to the total intensity $I = 4.4\times10^{23}$ W cm^{-2}. These values are lower than P = 7.9 PW and $I = 5.6\times10^{23}$ W cm^{-2}, which have been obtained in [13, 14] for eight pulses colliding in the same plane. Figure 3 shows time dependences of the number of electrons for eight pulses colliding in the plane [13, 14], and for 16 pulses at the same total power P = 6.2 PW.

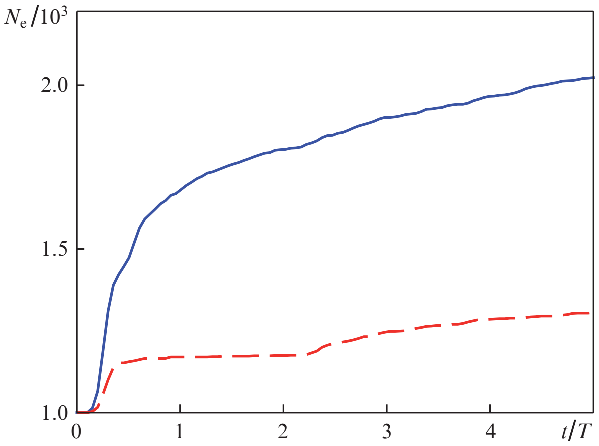

Figure 3. Time dependences of the number of electrons for eight optimally polarised pulses colliding in the same plane (dashed curve) and for the optimal configuration of 16 pulses (solid curve). In both cases, the total power of the laser field is P = 6.2 PW, $\xi = 8$, ω = 1 eV; T is the laser period.

5. Conclusions

We have investigated laser field configurations optimal for observing QED cascades. For this purpose, we have derived an analytical formula for calculating the QED parameter χ, which determines the probability of emission of a photon by a particle and creation of an electron–positron pair by a photon in an arbitrary external electromagnetic field. Applying this formula to the case of a focused laser beam, we have shown that for an electron in the field of two colliding beams $\chi(t)$ grows much faster than in the field of a single beam of total power. This leads to a more efficient use of the fields of a standing-type wave for the observation of the cascades.

The use of multiple pairs of colliding pulse allows one to reduce the intensity threshold required for the emergence of the cascade [13, 14]. However, the focusing limits the number of pulses that can fit in the space without overlapping away from the focal region. For focusing used in modern facilities, this number may not exceed 16. Using the analytical formula for the parameter χ, we have found a configuration of the field of 16 colliding pulses that is optimal for the development of the cascade. We have found that for the same total power in the case of 16 pulses about three times more pairs are created than in the case of the eight pulses (see Fig. 3). This is due to the fact that the interference-induced redistribution of the field of a given power in the focal region makes it possible to effectively accelerate particles located near the focus. In turn, this leads to an increase in the growth rate of the parameter χ and therefore an increase in the probability of emission of a hard photon and creation of electron–positron pairs by the photon.

For above-described optimal configuration of 16 colliding pulses the QED threshold power and intensity of the laser field are 6.2 PW and 4.4×10^{23} W cm^{-2}, respectively. It can be expected that these values will be attained in the near future by the laser devices under construction [17, 18].

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Appendix. Optimal configuration of the field for 16 colliding pulses

Consider eight pairs of laser beams (12), whose axes $\hat{\boldsymbol{n}}^j$ are located as follows: in the xy plane at angles 0, π/4, π/2, 3π/4 to the x axis at $1 \leqslant j \leqslant 4$ and at an angle arcsin(16/25) in the xy plane with projections on this plane at angles π/8, 5π/8, 9π/8, 13π/8 to the x axis at $5 \leqslant j \leqslant 8$ (see Fig. 2b). We denote the beam polarisation vectors by $\boldsymbol{\varepsilon}^j = \boldsymbol{\varepsilon}_1^j - \mathrm{i}\boldsymbol{\varepsilon}_2^j$, $\boldsymbol{\varepsilon}^j \perp \hat{\boldsymbol{n}}^j$, $\boldsymbol{\varepsilon}_1^j \perp \boldsymbol{\varepsilon}_2^j$. To fix the total power of all the beams we replace P in (12) by $P/(2\varepsilon^2)$, where $\varepsilon^2 = \sum_j [(\varepsilon_1^j)^2 + (\varepsilon_2^j)^2]$. Using formula (10) and putting $\xi = 8$, we can numerically find the polarisation vectors $\boldsymbol{\varepsilon}^j$, which maximise $\chi(t)$ at a given power. As a result we obtain

$$\begin{aligned}
&\boldsymbol{\varepsilon}^1 = \{0, -0.87\mathrm{i}, -0.84\}, \boldsymbol{\varepsilon}^2 = \{0.22\mathrm{i}, -0.22\mathrm{i}, -0.84\},\\
&\boldsymbol{\varepsilon}^3 = \{0.43\mathrm{i}, 0, -0.84\}, \boldsymbol{\varepsilon}^4 = \{0.65\mathrm{i}, 0.65\mathrm{i}, -0.84\},\\
&\boldsymbol{\varepsilon}^5 = \{0.51 + 0.21\mathrm{i},\ 0.09 - 0.66\mathrm{i},\ -0.61 + 0.08\mathrm{i}\},\\
&\boldsymbol{\varepsilon}^6 = \{0.37 - 0.11\mathrm{i},\ 0.37 + 0.04\mathrm{i},\ -0.24 - 0.1\mathrm{i}\},\\
&\boldsymbol{\varepsilon}^7 = \{-0.18 - 0.29\mathrm{i},\ 0.18 - 0.69\mathrm{i},\ -0.11 - 0.64\mathrm{i}\},\\
&\boldsymbol{\varepsilon}^8 = \{0.43 - 0.68\mathrm{i},\ -0.54 - 0.29\mathrm{i},\ -0.46 - 0.34\mathrm{i}\}.
\end{aligned} \tag{A1}$$

For this configuration, the parameter χ of the electron, beginning its motion from the focus at time $t = 0$, has the form

$$\chi(t) \approx 6.4\frac{e^2\omega^3 P}{m^3}t^2. \tag{A2}$$

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