

DOI: 10.1070/QEL16056

Influence of asymmetric injection of laser radiation into capillary waveguides on wake acceleration of electrons possessing various injection energies

M.E. Veisman, S.V. Kuznetsov, N.E. Andreev

Abstract. **Laser wakefield acceleration of electron bunches possessing various initial injection energies in capillary waveguides is studied in the conditions of an asymmetric input of laser radiation into a waveguide (the propagation direction of laser radiation deviates from the capillary axis or the laser spot is not symmetric). The factors determining the critical angle of the laser radiation input into the capillary, within which the wake acceleration of electrons is close to optimal, have been found. It is shown that in acceleration stages where electron energies are high, the requirements to angular concentricity of the capillary axis and laser radiation focusing are substantially weaker due to the relativistic 'weighting' of the electron mass.**

Keywords: *wakefield acceleration of electrons, capillary waveguides, laser radiation.*

1. Introduction

Study of new methods for acceleration of charged particles based on the accelerating fields of very high intensity that can be excited in plasma by electromagnetic radiation, started more than half a century ago [1–3]. The way to practical implementation of this idea has been long and by no means easy. It is related, first of all, to the fact that the idea of acceleration by plasma methods had been born long before the most appropriate sources of high-intensity accelerating fields in plasma were created, which turned out to be lasers [4]. Then a considerable amount of time was spent to expand the laser technologies to a level [5] needed for obtaining ultrashort (femtosecond) laser pulses of high (relativistic) intensity optimal for generating high-intensity plasma fields.

By now, the development of plasma accelerating methods resulted in acceleration of an electron bunch in a gas-filled capillary on a path length of ~1 cm to the energy of 2 GeV by using a petawatt laser (with a pulse duration of 150 fs at a wavelength of 1.057 μm) [6]. In this case, the relative spread of the electron energy was about 5%–10%. Only one year later, even higher energies of accelerated electrons were obtained: it was reported [7] about an energy of electron bunches of up to 4 GeV with an rms energy deviations of 6%.

The achieved energy of accelerated electrons is impressive from the viewpoint of an increase in the rate of the average energy of electron bunches in each next created experimental installation. Still, it is not sufficient for employment in nuclear high-energy physics, where bunches of accelerated electrons with the energy of ~1 TeV are needed. To approach the required electron energies, a possibility of employing an ensemble of several hundred separate plasma accelerators is considered with electron acceleration to a required high energy in a series of separate acceleration stages. However, in such an acceleration scheme, rather high requirements are placed on the energy homogeneity of the accelerated electron bunches transferred from stage to stage; ideally, the relative energy spread of bunches should be tenths of a percent [8, 9].

Qualitative characteristics (energy homogeneity, emittance) of an electron bunch after acceleration mainly depend on the method of injecting electrons into an accelerating wake field. Presently, in the most widely accepted experimental method of injecting electrons into a first acceleration stage, electrons are self-injected into the wake field generated by a laser pulse when the latter interacts with plasma in a strongly nonlinear, so-called bubble regime [10]. An advantage of this scheme of electron injection into a wake field is a simple construction of a laser-plasma accelerator, which does not require a special injector. However, self-injection of electrons in a bubble regime is inherently a nonstationary and difficult-to-control process: the evolution of a bubble determines the dynamics of electron self-injection into the bubble, which, in turn, determines the principal characteristics of the accelerated electron bunch, such as the energy spread of electrons in the bunch and angular divergence of the latter [6]. Up to now, there have been no theory which describes this process for controlling it.

In using an external injector of electrons having the parameters needed for perfect acceleration there are also problems related to symmetry breaking in the accelerating fields that are generated by a laser pulse in a waveguide structure. Actually, there are always both deviations in the angle of incidence of laser radiation at a waveguide input relative to the waveguide axis or deviations of the focusing point of laser pulse, and symmetry breaking in the intensity distribution over the focal spot. As a result, both the structure of electromagnetic field of a laser pulse and the structure of a wake wave along the whole path of the laser pulse propagation in a waveguide are broken [7, 11–14].

At some limiting values of the deviation of initial conditions of the laser pulse from ideal conditions (i.e. such that the axially symmetric incident laser pulse is exactly normal to the waveguide cross section and its point of focusing fits accurately the centre of a cylindrical circular waveguide) the wake filed generated by the laser pulse may become so bad that perfect acceleration of an electron bunch is impossible regardless of how ideal are the initial parameters of the injected bunch itself.

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Received 5 February 2016
Kvantovaya Elektronika **46** (4) 288–294 (2016)
Translated by N.A. Raspopov

The aim of the present work is to determine the limiting admissible deviations from the conditions of symmetric injection of a laser pulse into a capillary waveguide (it concerns both the angle between the axes of the waveguide and laser pulse and the degree of laser spot symmetry), at which perfect acceleration of an electron bunch is *a fortiori* impossible. In contrast to [13], in the present work we investigate an influence of the injection energy of electron bunches on the accuracy requirements to laser radiation focusing into capillary waveguides, which is of particular importance for the concept of multi-stage acceleration of electrons and positrons in a wake plasma wave, where the electrons accelerated at a previous stage are injected to next stage for further acceleration [15, 16].

The study was conducted as a numerical simulation of electron acceleration in a wake wave generated by a laser pulse in a capillary waveguide filled with plasma of homogeneous density. In addition to the numerical simulation, with a simple physical model of laser pulse propagation in the waveguide in the approximation of geometrical optics it was shown that the characteristics of the accelerated electron bunch drastically worsen if the laser pulse experiences multiple reflections from capillary walls along the propagation path. It has been revealed that a direct reason for the breakdown of electron acceleration process is a change in a structure of the wake potential arising due to reflections of the laser pulse, generating this potential, from capillary walls. This results in strong defocusing forces, which act on electrons and push them to capillary walls. It has been shown that, to a certain degree, the process of pushing electrons to capillary walls is hindered by intertia of accelerated particles: for electrons, this effect arises only when they are accelerated to high energies and their relativistic mass substantially raises.

2. Formulation of the model

Asymmetric injection of laser power into a waveguide in installations on laser-plasma acceleration implies, generally, taking into account at least two main factors characterising distinction of this process from the ideal case, namely, broken concentricity between laser and waveguide axes and broken symmetry of a laser radiation spot. In turn, from the geometrical point of view, the broken concentricity of the waveguide and the laser pulse can be revealed in the form of a deviation of the laser spot centre from a waveguide centre, a nonzero angle between the laser and waveguide axes, and absent common plane in which these axes lie. From the violations of ideal injection of laser radiation into a waveguide mentioned above, only two are considered in the present study, i.e. an angle between laser and waveguide axes (under the condition that they lie in the same plane) and asymmetry of the laser spot.

In addition to asymmetric laser and wake fields, the efficiency of electron bunch acceleration is also affected by asymmetric injection of the bunch itself into the guiding structure [17]. However, this process is not considered in the present work. Note that the importance of taking into account asymmetric injection of the electron bunch, asymmetric features of the laser spot, and asymmetric injection of laser radiation into the guiding structure has been stressed in recent paper [18], where it was reported about obtaining electron bunches accelerated to an energy of 4.2 GeV.

Consider a laser pulse incident at a certain angle onto a face end of a capillary waveguide with an internal radius R. We will assume that the z axis coincides with the capillary axis, the capillary input resides at the point $z = 0$, the laser pulse propagates from left to right, and the angle $\theta_{\rm inc}$ between the propagation direction of laser pulse and the axis of capillary is, generally, distinct from zero. In describing propagation of a laser pulse through the capillary we take the dielectric function of plasma inside the capillary as a constant $\varepsilon = 1 - n_{\rm e,0}/n_{\rm cr}$ = const, $|1-\varepsilon| \ll 1$, where $n_{\rm e,0}$ is the density of background electrons in plasma; $n_{\rm cr} = m\omega_0^2/(4\pi e^2)$ is the critical density; ω_0 is the frequency of laser radiation; and m and e are the mass and charge of the electron ($e < 0$), respectively.

The assumption about spatial homogeneity and time independence of the dielectric function of plasma implies that such a nonlinear effect as self-focusing is small as compared to the main effect that affects the distribution of laser pulse fields in the capillary and depends on reflection of the radiation from capillary walls and on modification of the transverse envelope of the laser pulse in the capillary in cases of asymmetric injection of the laser power into the capillary of the asymmetric laser spot. The dependence of this main effect on the energy of injected electrons is studied in the present work.

The effect of laser self-focusing can be neglected for pulses of moderate intensity and in the case of low-density plasma, i.e., when the ratio of the laser pulse power to the critical power of relativistic self-focusing is small: $P < P_{\rm cr} = 0.017\gamma_{\rm ph}^2$ TW, or $I_0 < I_{\rm cr} = 4.3\times10^{19}$ (W cm^{-2}) $(k_{\rm p}r_0)^{-2}(\lambda_0/1~\mu{\rm m})^{-2}$, where $\gamma_{\rm ph} = \omega_0/\omega_{\rm p} = \sqrt{n_{\rm cr}/n_{\rm e,0}}$ is the relativistic γ-factor of the plasma wave; $\omega_{\rm p} = \sqrt{4\pi n_{\rm e,0}e^2/m}$ is the plasma frequency; $k_{\rm p} = \omega_{\rm p}/c$; c is the speed of light; I_0 is the peak intensity of laser radiation in vacuum at the capillary input; λ_0 is the laser radiation wavelength; and r_0 is the characteristic radius of the transverse cross section of the laser pulse in vacuum. Note that if plasma is produced by optical ionisation of gas atoms, which homogeneously fill the capillary, then the intensity of the laser pulse should be much above the threshold of field ionisation for gas atoms, which for the total ionisation of light gases, such as hydrogen or helium, is ususally about $10^{14}-10^{15}$ W cm^{-2}. Note also that a small, however, finite difference of the plasma dielectric function from unity inside the capillary determines a group velocity of the laser pulse and, consequently, the phase velocity of the wake wave, which is very important for correct description of electron acceleration.

According to [11, 12], the slowly varying complex amplitudes $\boldsymbol{E}$ and $\boldsymbol{B}$ of the laser fields, $\tilde{\boldsymbol{E}} = {}^1/_2 A_{\max}\boldsymbol{E}\exp[{\rm i}k_0(z-ct)]$ + c.c. and $\tilde{\boldsymbol{B}} = {}^1/_2 A_{\max}\boldsymbol{B}\exp[{\rm i}k_0(z-ct)]$ + c.c. inside the capillary (where $A_{\max} = \sqrt{8\pi I_0/c}$ is the maximal amplitude of the electric field intensity of a laser pulse at the capillary input, $k_0 = \omega_0/c$), can be presented in the form:

$$\boldsymbol{E} = \sum_{l=-L}^{L}\exp({\rm i}l\varphi)\sum_{\sigma=\pm1}\sum_{n}\mathfrak{C}_{l\sigma n}(\xi,\zeta)\boldsymbol{E}_{l\sigma n}(\rho) \quad (1)$$

(the expression for $\boldsymbol{B}$ is similar), where $\boldsymbol{E}_{l\sigma n}$ and $\boldsymbol{B}_{l\sigma n}$ are the radial modes of electromagnetic fields inside the capillary; $\mathfrak{C}_{l\sigma n}$ are the mode coefficients [12, 19] (see Appendix); $\xi = k_{\rm p}(z-ct)$; $\zeta = k_{\rm p}z$; $\rho = k_{\rm p}r$ are dimensionless variables; r is the transverse coordinate; and φ is the azimuth angle (here, cylindrical coordinates are used with the z axis directed along the capillary axis and it is assumed that the laser radiation is linearly polarised).

In order to investigate the process of relativistic electron acceleration in the wake field, generated by a laser pulse propagating in the capillary, we need the expression for the wake force $\boldsymbol{F}_{\rm w}$, which acts on electrons moving at a velocity $v \approx c$ along the capillary axis OZ. In the cylindrical coordinates, the components of this force (in what follows, the force is normalised to $m\omega_{\mathrm{p}}c$) can be expressed in terms of the dimensionless components of electric $\boldsymbol{a}_{\mathrm{w}} = e\boldsymbol{E}_{\mathrm{w}}/(m\omega_{\mathrm{p}}c)$ and magnetic $\boldsymbol{b}_{\mathrm{w}} = e\boldsymbol{B}_{\mathrm{w}}/(m\omega_{\mathrm{p}}c)$ wake fields in the form

$$F_{\mathrm{w},z} = a_{\mathrm{w},z}, \quad F_{\mathrm{w},r} = a_{\mathrm{w},r} - b_{\mathrm{w},\varphi}, \quad F_{\mathrm{w},\varphi} = a_{\mathrm{w},\varphi} + b_{\mathrm{w},r}, \tag{2}$$

where $\boldsymbol{E}_{\mathrm{w}}$ and $\boldsymbol{B}_{\mathrm{w}}$ are the intensities of electric and magnetic wake fields, respectively, which slowly vary on the scales of the laser period and laser radiation wavelength.

The quasi-static approximation $\partial/\partial\zeta \ll \partial/\partial\xi$ [20–22] will be used hereafter. In this approximation, by using the Maxwell equations one may show the existence of a scalar wakefield potential ϕ (dimensionless in terms of mc^2/e) through which the components of wake fields can be expressed as

$$a_{\mathrm{w},z} = \partial_\xi\phi, \quad a_{\mathrm{w},r} - b_{\mathrm{w},\varphi} = \partial_\rho\phi, \quad a_{\mathrm{w},\varphi} + b_{\mathrm{w},r} = \rho^{-1}\partial_\varphi\phi \tag{3}$$

(notations like ∂x denote $\partial/\partial x$). From (2) and (3) it follows that the force $\boldsymbol{F}_{\mathrm{w}}$ mentioned above can be written as

$$\boldsymbol{F}_{\mathrm{w}} = k_{\mathrm{p}}^{-1}\nabla\phi. \tag{4}$$

According to (4), the equations of electron motion in a wake field with the allowance made for a possible simultaneous action of an electromagnetic field of the laser pulse, written in the ponderomotive approximation, can be presented in the form [23]:

$$\mathrm{d}P_z/\mathrm{d}\tau = \partial_\xi\phi - \gamma_{\mathrm{e}}^{-1}\partial_\xi|\boldsymbol{a}_\perp|^2/4, \tag{5}$$

$$\mathrm{d}P_r/\mathrm{d}\tau = \partial_\rho\phi - \gamma_{\mathrm{e}}^{-1}\partial_\rho|\boldsymbol{a}_\perp|^2/4, \tag{6}$$

$$\mathrm{d}P_\varphi/\mathrm{d}\tau = \rho^{-1}\partial_\varphi\phi - \rho^{-1}\gamma_{\mathrm{e}}^{-1}\partial_\varphi|\boldsymbol{a}_\perp|^2/4, \tag{7}$$

$$\mathrm{d}\xi/\mathrm{d}\tau = \gamma_{\mathrm{e}}^{-1}P_z - 1, \tag{8}$$

$$\mathrm{d}\rho/\mathrm{d}\tau = \gamma_{\mathrm{e}}^{-1}P_r, \tag{9}$$

$$\rho\mathrm{d}\varphi/\mathrm{d}\tau = \gamma_{\mathrm{e}}^{-1}P_\varphi, \tag{10}$$

$$\gamma_{\mathrm{e}} = \sqrt{1 + |\boldsymbol{P}|^2 + |\boldsymbol{a}|^2}, \tag{11}$$

where $P_{z,r,\varphi} = p_{z,r,\varphi}/(mc)$ are the dimensionless components of the electron momentum; γ_{e} is the gamma-factor of the electron; $|\boldsymbol{P}|$ is the dimensionless absolute value of its momentum; $\tau = \omega_{\mathrm{p}}t$; second summands in the right parts of (5)–(7) are related with the ponderomotive action of laser radiation; and $\boldsymbol{a}_\perp = e\boldsymbol{E}_\perp/(m\omega_0 c)$ is the dimensionless transverse (in the plane normal to the propagation axis) component of the electric field of a laser pulse.

Since the amplitude of a laser field (1) and square of its modulus comprise angular harmonics, the wakefield potential also comprises angular harmonics:

$$\phi = a_{\max}^2 \sum_{l=0,\pm1,\pm2,\dots} \phi_l \exp(\mathrm{i}l\varphi), \tag{12}$$

where $a_{\max} = eA_{\max}/(m\omega_0 c)$.

A combined system of equations for harmonics ϕ_l of the wake potential in the case of weak relativism ($a_{\max} < 1$) and a linear regime of the wake wave ($|\phi| < 1$) was presented in [13]. In the case of homogeneous plasma inside a capillary, this system reduces to a single equation

$$\left(\Delta_\rho - \frac{l^2}{\rho^2} - 1\right)\left(\frac{\partial^2}{\partial\xi^2} + 1\right)\phi_l = \left(\Delta_\rho - \frac{l^2}{\rho^2} - 1\right)\frac{|\boldsymbol{a}_\perp|_l^2}{4}, \tag{13}$$

where $|\boldsymbol{a}_\perp|_l^2$ is the lth angular harmonic of a square modulus of the dimensionless transverse component of the electric field; and $\Delta_\rho = \rho^{-1}\partial_\rho + \partial_\rho^2$.

In the case of symmetric propagation of the laser pulse in a capillary, all angular harmonics $|\boldsymbol{a}_\perp|$ with nonzero l are equal to zero ($|\boldsymbol{a}_\perp|_{l\neq0}^2 = 0$). Equation (13) for the component with $l = 0$ (in the case of a homogeneous density of background plasma) coincides with the equation obtained in [24, 25] and in [26], where the process of wake field generation inside plasma channels and capillaries by the laser pulse having a cylindrical symmetry and propagating exactly along the axis of a channel/capillary was studied.

The boundary conditions for equation (13) at the axis of a capillary waveguide and on its wall are written in the form $\phi_l(r \to 0) = O(1)$ and $\phi_l(r = R) = 0$ in accordance with [26]. In this case, a solution of equation (13) is

$$\phi_l(\xi,\rho) = \phi_{l,0}(\xi,\rho) - \phi_{l,0}(\xi,\mathcal{R})I_l(\rho)/I_l(\mathcal{R}), \tag{14}$$

$$\phi_{l,0}(\xi,\rho) = \frac{1}{4}\int_\infty^\xi \sin(\xi - \xi')\,|\boldsymbol{a}_\perp|_l^2(\xi',\rho)\,\mathrm{d}\xi', \tag{15}$$

where I_l is the modified l-order Bessel function; and $\phi_{l,0}$ is a solution of equation (13) under the condition $\Delta_\rho = 0$, $\mathcal{R} = k_p R$.

By using the expressions for the modes of electromagnetic fields inside a capillary obtained in [12] one can show that a zero order on the parameter $\Re^{-1} = (k_0 R)^{-1} \ll 1$ the values of $|\boldsymbol{a}_\perp|_l^2$ in (13) are defined as follows:

$$\begin{aligned}
|\boldsymbol{a}_\perp|_{l=0}^2 &= \left|\sum_n \tilde{\mathfrak{C}}_{0-1n} J_1(u_{1,n}\rho/\mathcal{R})\right|^2 \\
&\quad + 2\sum_{p\neq0}\left|\sum_{\sigma=\pm1}\sum_n \tilde{\mathfrak{C}}_{p\sigma n} J_{p-\sigma}(u_{p-\sigma,n}\rho/\mathcal{R})\right|^2, \\
|\boldsymbol{a}_\perp|_{l\neq0}^2 &= \sum_p \left|\sum_{\sigma=\pm1}\sum_n \sigma\tilde{\mathfrak{C}}_{l+p,\sigma n} J_{l+p-\sigma}(u_{l+p-\sigma,n}\rho/\mathcal{R})\right| \\
&\quad \times \left|\sum_{\sigma_2=\pm1}\sum_{n_2} \sigma_2\tilde{\mathfrak{C}}^*_{p\sigma_2 n_2} J_{p-\sigma_2}(u_{p-\sigma_2,n_2}\rho/\mathcal{R})\right| \\
&\quad + \sum_p \left|\sum_{\sigma=\pm1}\sum_n \tilde{\mathfrak{C}}_{l+p,\sigma n} J_{l+p-\sigma}(u_{l+p-\sigma,n}\rho/\mathcal{R})\right| \\
&\quad \times \left|\sum_{\sigma_2=\pm1}\sum_{n_2} \tilde{\mathfrak{C}}^*_{p\sigma_2 n_2} J_{p-\sigma_2}(u_{p-\sigma_2,n_2}\rho/\mathcal{R})\right|,
\end{aligned} \tag{16}$$

where p are natural numbers; n and n_2 are positive natural numbers; J_k are the kth-order Bessel functions of the first kind; and $u_{k,n}$ is the nth root of function J_k.

3. Calculation results

At a first stage of the calculations, the linearly polarised laser pulse is assumed axisymmetric relative to its axis (the propagation direction), the axes of the laser pulse and of the waveguide are in the same plane with an angle θ_{inc} relative to each other. The time-domain and space-domain presentations of the laser pulse had Gaussian envelopes with the HWHM width of the spot intensity amplitude profile $r_0 = 50$ μm in the transverse cross section and the FWHM pulse duration $\tau_{\mathrm{L}} =$

56 fs; the wavelength of high-frequency radiation was λ_0 = 0.8 μm. It was assumed that the capillary waveguide was made of silica with the internal radius R = 82 μm. The dimensionless peak intensity of the laser pulse was taken a_{max} = 0.5 to ensure not very large steeping of the wake wave front and other nonlinear effects when the intensity of laser pulse increases [13]. At the prescribed wavelength, the peak intensity of the laser pulse $I_0 = 7.3\times10^{17}$ W cm^{-2} corresponds to this value of a_{max}.

In contrast to the laser pulse, an axisymmetric electron bunch is injected into the waveguide ideally, i.e., exactly along the waveguide axis. Electron bunches were injected in the first period of the plasma wave behind the laser pulse to a point z_{inj} along the capillary length ($k_p z_{inj}$ = 800, $z_{inj}/L_{ph} \approx 0.02$, where L_{ph} is the dephasing length) with various initial electron energies E_{inj}. In this case, the phase of bunch injection relative to the wake wave ξ_{inj} (the position of the bunch in the wake wave) corresponded to the phase of the maximal accelerating force ξ_*, which for a linear plasma wave coincides with a boundary between a focusing and defocusing domains of the wake field. Such an injection phase provides the fastest energy acquisition by electrons without worsening the quality of the accelerated bunch [13].

The initial distribution of electrons in the accelerated bunch was taken Gaussian in all spatial coordinates (along the OZ axis and in the transverse directions), $n_b(\xi, r, z = z_{inj}) = n_{b,0}\exp[-0.5(\xi - \xi_{inj})^2/\sigma_z^2 - 0.5r^2/\sigma_r^2]$, with the characteristic dimensions $k_p\sigma_r = 0.3 \ll k_p r_0$ and $k_p\sigma_z = 0.1 \ll 1$, respectively.

For modelling the process of electron acceleration in the wake wave generated by this laser pulse, the electron bunch was presented as an ensemble of macro-particles. The initial number of macro-particles in the bunch was N_d^3 (N_d^2 layers in the radial direction and N_d layers in the longitudinal direction; N_d was chosen from 41 to 131). The weight (mass and charge) of each macro-particle was determined from its initial position at the instant of injection and from the density of the electron spatial distribution in the bunch injected. The spatial and energy characteristics of electrons in a bunch were determined by solving numerically equations (5)–(10) for each particle.

Characteristics of bunches of accelerated electrons (the average energy, rms energy spread, the number of trapped and accelerated particles) are shown in Fig. 1 versus the angle θ between the axes of laser pulse and capillary waveguide at various injection energies for the acceleration length z/L_{ph} = 0.5073 ($k_p z$ = 20400). This length corresponds to the average energy of the electron bunch, close to the maximal value. Here, the dephasing length is $L_{ph} = \lambda_0\gamma_{ph}^3$ [27] (in the conditions of our calculations $k_p L_{ph}$ = 40960), the relativistic gamma factor γ_{ph} was calculated from the phase velocity of the plasma wake wave propagating in the waveguide. In all calculations we used γ_{ph} = 80, which at the considered wavelength of laser radiation corresponded to the background concentration of plasma electrons $n_{e,0} = 2.73\times10^{17}$ cm^{-3}.

Analysis of the set of characteristics presented in Fig. 1 shows, first of all, that the process of acceleration is substantially influenced by a mass of accelerated particles. In the considered case, by the mass of particles should be meant the relativistic mass of electrons $m\gamma_e$, which initially depends on the energy of electron injection into the capillary. In the considered calculation examples the relativistic mass varies within several orders of magnitude.

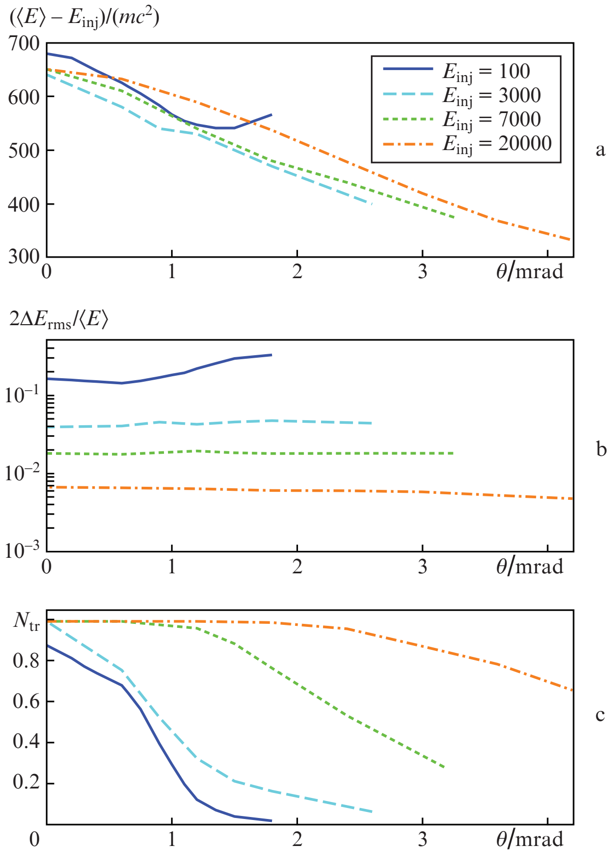

Figure 1. Characteristics of an electron bunch after acceleration: (a) increase in the average energy of the electron bunch, (b) rms relative spread of the electron energy in the bunch and (c) the number of trapped and accelerated particles vs. the angle θ between the direction of laser pulse injection into the waveguide and its axis for the energies of injected electrons in the bunch (in terms of mc^2) 100, 3000, 7000, 20000.

At the considered intensities of the laser pulse, which generates a wake wave, and at comparatively small [less than 1.5 GeV ($E_{inj} \leqslant 3000mc^2$)] energies of injected electrons, the efficient laser-plasma acceleration of electrons can be reached if the main factor limiting the admissible deviation of the input angle of laser pulse injection into the waveguide is excluded. This factor is the process of electron departure to a wall due to the catastrophic changes in the structure of a wake field [13], occurring at the instant of laser pulse reflection from the capillary wall. From the viewpoint of geometrical optics, this change is observed at such a length L of laser pulse propagation along the capillary of radius R that with the chosen angle of laser pulse input to the capillary θ the light ray touches the capillary wall, $\theta = R/L$.

The gradients of a wake field arising in this case in the transverse direction push most of electrons of the accelerated bunch to a wall so that they drop out of the acceleration process (Fig. 1c). For this capillary, at the chosen acceleration length, the critical angle of the laser pulse input is estimated approximately as 0.6 mrad.

At an energy of injected electrons above ~3.5 GeV (E_{inj} > $7000mc^2$), which corresponds to next stages of a laser-plasma accelerator, another factor limits the angle θ, namely, the fall of the rate of acceleration (energy acquisition) with an increase in the angle. Physically it means that although structure distortions of the wake wave cannot deflect a 'heavy' electron from the trajectory along the capillary axis, they, nevertheless, substantially reduce the accelerating force of the wake field. Finally, the rate of acceleration falls and the increase in the electron energy of accelerated bunch reduces (Fig. 1a). However, the calculations performed show that at a large injection energy, requirements to concentricity of the waveguide and laser pulse are substantially weaker, and for $E_{\text{inj}} = 20000mc^2 = 10$ GeV the critical angle for the same capillary is approximately 3 mrad.

Note that within the limits of admissible angles θ, the rate of energy acquisition $(E - E_{\text{inj}})/(mc^2)$ is approximately the same for various E_{inj}. At a greater θ, the relative energy spread $\Delta E/E$ for the particular value of z/L_{ph} increases in initial acceleration stages (at small E_{inj}) and actually is constant in the stages with a high value of E_{inj} (Fig. 1b).

The dependence of the rms energy spread and the number of trapped and accelerated particles on the energy E_{inj} is shown in Fig. 2 for $\theta = 1.8$ mrad. Conclusions that can be made from the dependences shown in Fig. 2 are similar to those of Fig. 1: with an increase in E_{inj}, the number of accelerated particles in the case of inaccurate focusing ($\theta > 1$ mrad) raises, and the relative energy spread substantially reduces.

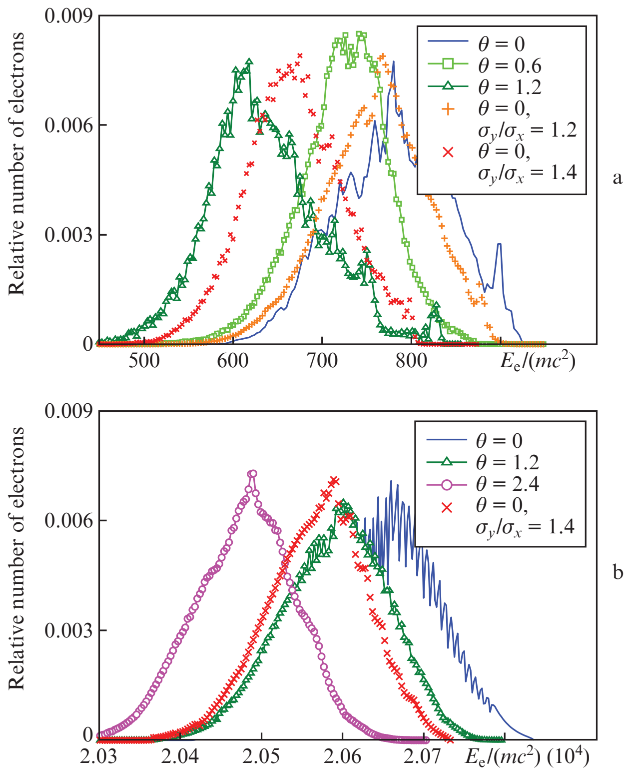

Figure 3. Energy spectrum of electrons for the acceleration length $z/L_{\text{ph}} = 0.497$ ($k_pz = 20000$) for the injection energies $E_{\text{inj}}/(mc^2)$ = (a) 100 and (b) 20000. The spectra are shown for angles $\theta = 0.06$, 1.2, and 2.4 mrad and also for various focusing spots at $\theta = 0$ with $\sigma_y/\sigma_x = 1.2$ and 1.4.

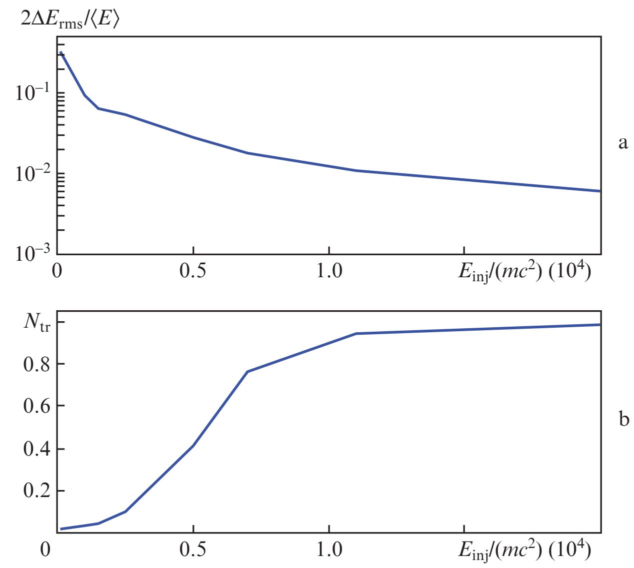

Figure 2. Characteristics of the electron bunch vs. the injection energy: (a) rms relative deviation of energies and (b) the number of trapped and accelerated particles at the angle between the capillary axis and the axis of injected laser pulse $\theta = 1.8$ mrad.

Energy spectra of accelerated electrons (at an instant close to the maximal acceleration energy) are shown in Fig. 3 for low ($E_{\text{inj}} = 50$ MeV) and high ($E_{\text{inj}} = 10$ GeV) injection energies at various input angles of laser radiation θ and laser pulses with asymmetric focusing spots. The envelope of the electric field intensity of the laser pulses can be written in the form $E(r) = E_0\exp(-x^2/\sigma_x^2 - y^2/\sigma_y^2)$, at $\sigma_x = 50$ µm, $\sigma_y/\sigma_x = 1.2$, $(\sigma_y^2/2 + \sigma_x^2/2)^{1/2} = 50$ ('+' marks) and at $\sigma_y/\sigma_x = 1.4$, $\sigma_y = 56$, $\sigma_x = 40$ ('×' marks).

From Fig. 3 one can see that the spectrum shapes negligibly vary with changes in the conditions of focusing. With deterioration of accuracy of focusing by the angle of incidence or by the laser spot shape, the spectral curves shift to lower energies along the energy scale, in accordance with results of Fig. 1. In a case of imperfect focusing, when the relative part of trapped and accelerated particles is substantially decreased, the spectrum shape becomes asymmetric relative to the energy corresponding to the spectrum maximum, with a greater number of trapped electrons corresponding to higher energies (see the curve corresponding to $\theta = 1.2$ in Fig. 3a).

In the considered acceleration conditions, an imperfect shape of a laser spot has a weaker effect on the quality of accelerated electron bunches than a nonzero angle between the axes of the laser pulse and capillary waveguide. By comparing the solid curve ($\theta = 0$, $\sigma_y/\sigma_x = 1$) with the curves for $\theta = 0$ and $\sigma_y/\sigma_x = 1.2$ and 1.4 one may conclude that deformation of the laser spot exerts a substantial influence only for $\sigma_y/\sigma_x > 1.2$.

Note that under calculation conditions of Fig. 3, the relative part of trapped and accelerated electrons N_{tr} for the length $z/L_{\text{ph}} = 0.497$ near the maximum of the acceleration energy, at $E_{\text{inj}} = 50$ MeV [$E_{\text{inj}}/(mc^2) = 100$, Fig. 3a] was 0.88 for the completely symmetric focusing and symmetric laser spot. For the asymmetric spot with $\sigma_y/\sigma_x = 1.2$ it was 0.87 and for the asymmetric spot with $\sigma_y/\sigma_x = 1.4$ it was 0.55.

The relative shift of the electron spectrum to lower energies at $\sigma_y/\sigma_x = 1.4$ in the case of $E_{\text{inj}} = 10$ GeV (Fig. 3b) is approximately the same as in the case of $E_{\text{inj}} = 50$ MeV (Fig. 3a). However, at higher injection energies ($E_{\text{inj}} = 10$ GeV), almost all electrons are trapped: $N_{\text{tr}} = 0.99$ for both the completely symmetric case and the case of $\sigma_y/\sigma_x = 1.4$.

4. Conclusions

The study conducted has shown that an efficient laser-plasma acceleration of electrons (with obtaining high-quality accelerated electron bunches) can be realised in practice only if sufficiently severe requirements to adjustment of different elements of an experimental setup (in particular, the laser generating a wake wave in a waveguide and the waveguide itself) are fulfilled. Irrespective of all other reasons, broken concentricity between the laser and the waveguide (capillary) results in such substantial distortions of the wake field generated by a laser pulse at a certain propagation length in the capillary that, even under the ideal injection of electrons into the accelerator, such wavefields become inappropriate for quality acceleration of electron bunches.

An asymmetric feature of the laser spot is comparatively less important and becomes substantial for electron bunch acceleration only if the lengths of the axes of the laser spot ellipse differ by more than 20%.

The investigation has shown that the most sensitive to violation of concentricity are schemes of laser-plasma acceleration, in which weakly relativistic electron bunches are used. The same conclusion can be made with respect to first acceleration stages in multistage acceleration systems. This is explained by the fact that even relativistic electrons with the energy of hundreds MeV sufficiently easily leave the trajectory along the capillary axis as soon as the laser pulse touches the wall and the structure of the wake wave breaks.

More stable to wake field distortions are electrons with the energy of an order of GeV. In this case, the relativistic mass of accelerated electrons may be sufficiently ‘heavy’ to withstand, through inertia, their motion to a wall under the action of strong gradients of the distorted wake field, which are directed to the capillary wall, at the instant of laser pulse approaching the wall. The critical limitation to violation of concentricity in this case is related to the fact that multiple reflections of a laser pulse from walls reduce the rate of acceleration and make the acceleration process of electrons inefficient. However, in the acceleration stages with the acceleration energy of above several GeV, limitations to focusing accuracy (in the frameworks of the considered acceleration scheme with electron injection into the domain with a maximum accelerating force) can be weaken by several times.

Acknowledgements. The work was partially supported by the Presidium of the Russian Academy of Science (Programme No. 21 ‘Extreme Laser Radiation: Physics and Fundamental Applications’).

Appendix. Mode coefficients of the electromagnetic field

Mode coefficients are found from a solution of the wave equation for an electromagnetic field in a waveguide and are expressed as follows (see [11, 12]):

$$\tilde{\mathfrak{E}}_{l\sigma n}(t,z) = \mathfrak{E}_{l\sigma n} F_{||}(z - ct + \Phi_{ln}(z)) \exp(-\mathrm{i}k_0 \Phi_{ln}(z)),$$
$$\Phi_{ln} = k^2_{\perp l\sigma n} z/2, \tag{A1}$$

where the expression for Φ_{ln} is written for the case of a waveguide with wall characteristics that are constant along z;

$$k_{\perp 0-1n} = (u_{1,n}/\Re)(1 - \mathrm{i}\mu_B/\Re),$$
$$k_{\perp 01n} = (u_{1,n}/\Re)(1 - \mathrm{i}\mu_E/\Re), \tag{A2}$$
$$k_{\perp l\sigma n} = (u_{l-\sigma,n}/\Re)(1 - \mathrm{i}\mu_+/\Re), \quad l = \pm1, \pm2, \ldots$$

are the transverse wave vectors with the values determined by boundary conditions for electromagnetic fields on a waveguide wall;

$$\mu_+ = (\mu_E + \mu_B)/2, \quad \mu_B = 1/\sqrt{\varepsilon - 1}, \quad \mu_E = \varepsilon/\sqrt{\varepsilon - 1} \tag{A3}$$

are coefficients determined by a dielectric function ($\varepsilon > 1$) of the waveguide;

$$\mathfrak{E}_{0-1n} = N^{-1}_{1,n} \int_0^1 y F_1(y) J_1(u_{1,n} y)\,\mathrm{d}y,$$
$$\mathfrak{E}_{01n} = \eta \mathfrak{E}_{0-1n}, \quad y \equiv r/R, \tag{A4}$$

$$\mathfrak{E}_{l\sigma n} = \frac{1 + \sigma\eta}{2} N^{-1}_{l-\sigma,n} \int_0^1 y F_{l-\sigma}(y) J_{l-\sigma}(u_{l-\sigma,n} y)\,\mathrm{d}y,$$
$$l = \pm1, \pm2, \ldots \tag{A5}$$

are constant coefficients determined from the boundary conditions for electromagnetic fields at the input aperture of the waveguide;

$$F_l(r) = \frac{1}{2\pi} \int_0^{2\pi} \exp(-\mathrm{i}l\varphi) F_\perp(r,\varphi)\,\mathrm{d}\varphi \tag{A6}$$

are the angular harmonics of the transverse envelope of a laser pulse at the input of capillary;

$$N_{k,n} = \int_0^1 y J^2_k(u_{k,n} y)\,\mathrm{d}y$$

are normalisation factors; and $F_{||}(t)$ and $F_\perp(r,\varphi)$ are the longitudinal and transverse envelopes of the laser pulse, respectively.

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