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***HOLOGRAPHIC MEMORY***

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# Detection of correlated fragments in a sequence of images by superimposed Fourier holograms

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***Abstract.* The problem of detecting correlated fragments in a sequence of images recorded by the superimposing holograms within the Fourier holography scheme with angular multiplication of a spatially modulated reference beam is considered. The approach to the solution of this problem is based on the properties of the variance of the image sum. It is shown that this problem can be solved by providing a constant distance between the signal and reference images when recording superimposed holograms and a partial mutual correlatedness of reference images. The detection efficiency is analysed from the point of view of estimated image data capacity, the degree of mutual correlation of reference images, and the hologram recording conditions. The results of a numerical experiment under the most complicated conditions (representation of images by realisations of homogeneous random fields) confirm the theoretical conclusions.**

**Keywords:** *superimposed holograms, multipole hologram, holographic memory, Fourier holography, data processing, correlation, sequence of images, detection of correlated fragments.*

## 1. Introduction

Superimposed holograms (SHs), i.e., holograms that are successively or simultaneously recorded is the same region of a holographic recording medium (HRM), are used to implement a holographic memory [1–7]. The SH method is based on dividing the dynamic range of the HRM exposure characteristic into subranges, each of which is designed for recording one SH via matching the exposure conditions for this hologram to the subrange size. The SHs recorded in a given HRM region produce a multiplexed hologram.

One of the urgent practical problems in both designing optical memory and processing optical data as a whole is to record and reconstruct not individual images but their sequences (including temporal ones) [8–12]. The term ‘image’ is used below as a synonym of the following concepts: ‘pattern’ and ‘complex amplitude field’. In this context, the term ‘sequence’ implies connectivity, i.e., partial correlation between the images forming this sequence (at least, between its neighbouring elements). In this case, the presentation of one arbitrarily chosen image from a stored series makes it possible to successively reconstruct all related images, in both forward and backward directions.

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The procedure of recording and reconstructing a sequence of images is often considered as a way to increase the capacity and operating speed of memory. At the same time, storage of sequences of images makes it possible to implement not only their reconstruction, but also more complex models of associative data processing. In particular, living systems use this technique to search for regularities and detect casual relationships in chains of events [13]. In turn, a key stage in solving these problems, which are also urgent for artificial information systems, is the search and detection of related (i.e., correlated) fragments of images within a sequence.

In view of the development of optical information technologies, it is of interest to analyse the potential of the SH method in detection of correlated fragments of images when recording and reconstructing image sequences. Note that the formulation of the problem in correlation terms suggests the absence of any other (except for the occurrence frequency) a priori criteria for assigning fragments to common (for the entire sequence) or individual ones. This statement complicates the problem, because it excludes from consideration the solution methods based on the difference in other characteristics of common and individual (different) fragments.

An approach to the solution of the aforementioned problem by the method of superimposed Fourier holograms was proposed and previously modelled in [14]. In this paper, the approach described in [14] is developed by considering the Fourier holography scheme in which SHs are recorded using angular multiplication of a spatially modulated reference beam. An estimate of the fragment detection efficiency is introduced through the ratio of the variances of the corresponding fragments of images in the field reconstructed by multiplexed hologram. Its dependence on the estimates of the data capacity of image fragments, mutual correlation of reference images, and hologram recording conditions when describing images as realisations of homogeneous random fields is analysed. The results of a numerical experiment confirming the theoretical conclusions are presented.

## 2. Optical scheme and model

### 2.1. Optical scheme and statement of the problem

Let us consider the 4*f* Fourier holography scheme, in which an SH is recorded using angular multiplication of a spatially modulated reference beam (Fig. 1). To simplify analytical expressions, the consideration will be performed (whenever possible) by describing images and, correspondingly, holograms as functions of one variable.

The scheme presented in Fig. 1 records a multiplexed Fourier hologram

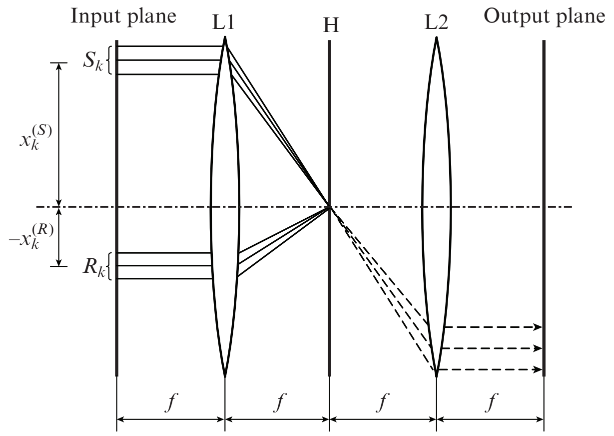


**Figure 1.** 4*f* Fourier holography scheme: $R_k$ and $S_k$ are images forming a pair, which are recorded in the hologram H; $x_k^{(R)}$ and $x_k^{(S)}$ are their shifts with respect to the principal optical axis; and L1 and L2 are Fourier transform lenses with a focal length *f*. The dashed lines show the paths of the rays reconstructing the image $S_k$ in the output plane when the image $R_k$ is presented to the hologram H.

$$H(\nu_x) = \sum_{k=1}^{n} H_k(\nu_x), \tag{1}$$

formed (on the assumption of recording within the linear portion of the dynamic range of the HRM exposure characteristic) by superposition of $n$ SHs of image pairs $S_k(x) \leftrightarrow R_k(x)$, where

$$H_k(\nu_x) = F(S_k(x))F^*(R_k(x))\exp[-\mathrm{j}\omega(x_k^{(R)} + x_k^{(S)})]; \tag{2}$$

$S_k(x)$ and $R_k(x)$ are, respectively, the signal and reference images; $F$ is the Fourier transform symbol; $\nu_x$ is the spatial frequency; $\omega_x = 2\pi\nu_x$; $x_k^{(R)}$ and $x_k^{(S)}$ are the coordinates describing the spatial position of the corresponding images (their shift) with respect to the principal optical axis in the input plane; and j is the imaginary unit. The sequences of signal and reference images will be denoted as $\{S_k\}_{k=1}^n$ and $\{R_k\}_{k=1}^n$, respectively.

If the $k$th image $R_k(x)$ is presented to the multiplexed hologram (1) recorded in the input plane, directly behind the hologram, in the +1st diffraction order (corresponding to the propagation direction of the signal beam $F(S_k(x)) \times \exp(-\mathrm{j}\omega x_k^{(S)})$ during hologram recording), we have the complex amplitude field

$$F(R_k(x))H(\nu_x) = F(R_k(x))F(S_k(x))F^*(R_k)\exp[-\mathrm{j}\omega(x_k^{(S)})]$$
$$+\sum_{k\neq l} F(R_k(x))F(S_l(x))F^*(R_l(x))$$
$$\times\exp[-\mathrm{j}\omega(x_k^{(R)} - x_l^{(R)} + x_l^{(S)})]. \tag{3}$$

In the rear focal plane of second Fourier transform lens L2, taking into account the coordinate inversion (because of the unrealisability of the inverse Fourier transform) when choosing signs, we obtain the complex amplitude field

$$E_{\mathrm{out}}(x) = F(F(R_k(x))H(\nu_x))$$
$$= S_k(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)] +$$
$$+\sum_{k\neq l} S_l(x + x_k^{(R)} - x_l^{(R)} + x_l^{(S)})*[R_k(x)\otimes R_l(x)], \tag{4}$$

where $*$ and $\otimes$ are the convolution and correlation symbols, respectively, and the terms in square brackets are pulsed responses, which describe the diffraction spread of points.

If hologram (1) is recorded in a thin HRM (according to the criterion of angular invariance) and the condition

$$\forall k,l \in [0,n]: x_k^{(R)} - x_l^{(R)} = x_k^{(S)} - x_l^{(S)} \tag{5}$$

is satisfied during recording (i.e., the distance between the signal and reference images in the input plane is retained the same when recording all SHs), all fields reconstructed by individual SHs and described by the terms in the right-hand side of expression (4) spatially coincide in the output plane in Fig. 1, i.e., are superimposed.

With allowance for the requirement of the absence of any other (except for the occurrence frequency in the image sequence) criteria for assigning image fragments to common (see Introduction), we take that all images are described as realisations of a homogeneous random field, on the assumption that the ergodic hypothesis holds true [15].

Let us consider the problem of detecting correlated fragments in a sequence of signal images $\{S_k\}_{k=1}^n$. Signal images are presented in the form

$$S_k(x) = S_k^{\mathrm{c}}(x) + S_k^{\mathrm{u}}(x),$$

where $S_k^{\mathrm{c}}(x)$ is a fragment correlated with other images, and $S_k^{\mathrm{u}}(x)$ is an uncorrelated fragment. Fragments $S_k^{\mathrm{c}}(x)$ and $S_k^{\mathrm{u}}(x)$ may spatially coincide, i.e., be completely or partially superimposed. Then, expression (4) takes the form

$$E_{\mathrm{out}}(x) = \sum_n [E_{\mathrm{out}}^{\mathrm{c}}(x) + E_{\mathrm{out}}^{\mathrm{u}}(x)]$$
$$= \sum_n S_k^{\mathrm{c}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]$$
$$+\Big\{S_k^{\mathrm{u}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]$$
$$+\sum_{k\neq l} S_l(x + x_k^{(S)})*[R_k(x)\otimes R_l(x)]\Big\}. \tag{6}$$

To solve the problem of detecting correlated fragments, one needs a criterion making it possible to distinguish the first term in (6) from the term in braces and a physical mechanism implementing this criterion. Below we will show that variance (as an integral estimate of the degree of modulation (contrast) of fragments) can play a role of a real criterion and evaluate the dependence of the ratio of the variances of common and individual fragments on the SH number and the correlation coefficient of reference images.

## 2.2. Model for detecting correlated fragments

One of possible instrumental (and visual) methods for discriminating image fragments is based on the difference between the integral (over a fragment) estimates of the degree of modulation (contrast). The degree of modulation of images will be estimated using their second central moment: variance. It is necessary to find the dependence of the variance of a reconstructed fragment on the SH number, characteristics of recorded images, and the recording conditions of a multiplexed hologram. Taking into account the possibility of holographic recording of reconstructed images, we will estimate the variances of amplitudes. In this case, there are no fundamental difficulties in passing to the estimation of variances of image intensities when registering them by a quadratic detector.

We apply the well-known property of the correlation function of a sum of random processes [16] to random fields:

$$K\left(\sum_{k=1}^{n} X_k(x,y)\right) = \sum_{k=1}^{n} K_{kk}(X_k(x,y)) + \sum_{k\neq l}^{n} K_{kl}(X_k(x,y))X_l(x,y))$$

$$= nK_{kk}(X_k(x,y)) + \sum_{k\neq l}^{n} K_{kl}(X_k(x,y))X_l(x,y)), \quad (7)$$

where $X_k(x,y)$ is the $k$th image as the $k$th realisation of a homogeneous random field, and $K_{kl}$ is the cross-correlation function of the $k$th and $l$th images. The value of the autocorrelation function at the reference point $K_{kk}(0,0)$, normalised to the image area $L_x L_y$ ($L_x$ and $L_y$ are image geometric sizes (aperture) over corresponding coordinates on the assumption that the image is a rectangle), yields the variance of the image reconstructed by hologram: $D_{\rm out} = K_{kk}(0,0)/(L_x L_y)$.

The problem of detecting correlated fragments can be solved both instrumentally and visually if the variance of the first term in (6) significantly differs from the variance of the second term.

The reconstructed field of correlated fragments [the first term in (6)] will be presented in the form

$$E^{\rm c}_{\rm out}(x) = S^{\rm c}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+\sum_{k\neq l} S^{\rm c}_l(x + x^{(S)}_k)*[R_k(x)\otimes R_l(x)], \quad (8)$$

and the reconstructed field of uncorrelated fragments will be written as

$$E^{\rm u}_{\rm out}(x) = S^{\rm u}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+\sum_{k\neq l} S^{\rm u}_l(x + x^{(S)}_k)*[R_k(x)\otimes R_l(x)]. \quad (9)$$

We assume that the reference images within the sequence $\{R_k\}^n_{k=1}$ are partially correlated; i.e., they can be written as

$$R_k(x) = R^{\rm c}_k(x) + R^{\rm u}_k(x),$$
$$R^{\rm c}_k(x) = mR_k(x), \quad (10)$$
$$R^{\rm u}_k(x) = (1-m)R_k(x),$$

where coefficient $m \in [0,1]$ describes the specific weight of a correlated fragment of the image, both in area and amplitude. Correspondingly, the cross-correlation coefficient of reference images is $\rho^{(R)}_{kl} = m^2$ [16]. Then, if the $k$th reference image $R_k$ is presented to multiplexed hologram (1), the expression describing the reconstructed field of correlated fragments (8) takes the form

$$E^{\rm c}_{\rm out}(x) = S^{\rm c}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+\sum_{k\neq l} S^{\rm c}_l(x + x^{(S)}_k)*[R^{\rm c}_k(x)\otimes R^{\rm c}_l(x)]$$
$$+\sum_{k\neq l} S^{\rm c}_l(x + x^{(S)}_k)*[R^{\rm u}_k(x)\otimes R^{\rm u}_l(x)]$$
$$= \Big\{S^{\rm c}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+m^2\sum_{k\neq l} S^{\rm c}_l(x + x^{(S)}_l)*[R_k(x)\otimes R_k(x)]\Big\}$$
$$+\sum_{k\neq l} S^{\rm c}_l(x + x^{(S)}_k)*[R^{\rm u}_k(x)\otimes R^{\rm u}_l(x)]$$
$$= [1+m^2(n-1)]\{S^{\rm c}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]\}$$
$$+\sum_{k\neq l} S^{\rm c}_l(x + x^{(S)}_k)*[R^{\rm u}_k(x)\otimes R^{\rm u}_l(x)]. \quad (11)$$

The reconstructed field as a whole, including both correlated and uncorrelated fragments, will be described by the expression

$$E_{\rm out}(x) = [1+\rho^{(R)}_{kl}(n-1)]\{S^{\rm c}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]\}$$
$$+\sum_{k\neq l} S^{\rm u}_l(x + x^{(S)}_k)*[R^{\rm u}_k(x)\otimes R^{\rm u}_l(x)]$$
$$+S^{\rm u}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+\sum_{k\neq l} S^{\rm u}_l(x + x^{(S)}_k)*[R_k(x)\otimes R_l(x)]$$
$$= [1+\rho^{(R)}_{kl}(n-1)]\{S^{\rm c}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]\}$$
$$+\sum_{k\neq l} S^{\rm c}_l(x + x^{(S)}_k)*[R^{\rm u}_k(x)\otimes R^{\rm u}_l(x)]$$
$$+S^{\rm u}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+m^2\sum_{k\neq l} S^{\rm u}_l(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+\sum_{k\neq l} S^{\rm u}_l(x + x^{(S)}_k)*[R^{\rm u}_k(x)\otimes R^{\rm u}_l(x)]$$
$$= [1+\rho^{(R)}_{kl}(n-1)]\{S^{\rm c}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]\}$$
$$+S^{\rm u}_k(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+m^2\sum_{k\neq l} S^{\rm u}_l(x + x^{(S)}_k)*[R_k(x)\otimes R_k(x)]$$
$$+\sum_{k\neq l} S_l(x + x^{(S)}_k)*[R^{\rm u}_k(x)\otimes R^{\rm u}_l(x)]. \quad (12)$$

The first term in (12) is the image required to solve the problem: correlated fragment of signal images $\{S_k\}_{k=1}^{n}$ reconstructed by multiplexed hologram (1). The second term

$$S_k^{\mathrm{u}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]$$

describes the reconstructed uncorrelated fragment, i.e., plays the role of interference independent of the SH number. Let us show that the third and fourth terms, which depend on the SH number, reduce the influence of this interference. To this end, we will use expression (7) to estimate the dependence of the variances of the first and all other terms in (12) on the SH number $n$.

The variance of the image necessary to solve the problem, described by the first term in (12), depends [according to (7)] on the SH number $n$ and correlation coefficient of reference images $\rho_{kl}^{(R)}$ [16]:

$$D_{\Sigma}^{\mathrm{c}} = D([1 + \rho_{kl}^{(R)}(n-1)]\{S_k^{\mathrm{c}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]\})$$

$$= [1 + \rho_{kl}^{(R)}(n-1)]^2 D(S_k^{\mathrm{c}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)])$$

$$= [1 + \rho_{kl}^{(R)}(n-1)]^2 D_{k\,\mathrm{out}}^{\mathrm{c}}, \qquad (13)$$

where $D_{k\,\mathrm{out}}^{\mathrm{c}}$ is the variance of a fragment reconstructed by one SH, i.e., with allowance for its change with respect to the variance of the reference fragment $S_k^{\mathrm{c}}$ as a result of filtering on the hologram, which is described by the pulsed response of the scheme $[R_k(x)\otimes R_k(x)]$. At $\rho_{kl}^{(R)} = 1$, i.e., when the reference images are completely correlated, $D_{\Sigma}^{\mathrm{c}}$ quadratically depends on the SH number $n$.

The expression for the variance of the interference [the field described by the other terms in (12)] takes the form

$$D_{\Sigma}^{\mathrm{n}} = D\Big(S_k^{\mathrm{u}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]$$

$$+ m^2 \sum_{k\neq l} S_l^{\mathrm{u}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]$$

$$+ \sum_{k\neq l} S_l(x + x_k^{(S)})*[R_k^{\mathrm{u}}(x)\otimes R_l^{\mathrm{u}}(x)]\Big). \qquad (14)$$

A detailed analysis of (14) leads to occurrence of a large number of covariance terms, which make the final expression highly cumbersome. Therefore, we leave it beyond the scope of this study and restrict ourselves to two extreme cases: completely correlated and completely uncorrelated reference images.

For completely correlated reference images, expression (14) takes the form

$$D_{\Sigma}^{\mathrm{n}} = D\Big(S_k^{\mathrm{u}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]$$

$$+ \sum_{k\neq l} S_l^{\mathrm{u}}(x + x_l^{(S)})*[R_l(x)\otimes R_l(x)]\Big)$$

$$= D\Big(\sum_{k} S_k^{\mathrm{u}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]\Big)$$

$$= nD_{k\,\mathrm{out}}^{\mathrm{u}}\left[1 + (n-1)\frac{D_{kl\,\mathrm{out}}^{\mathrm{u}}}{D_{k\,\mathrm{out}}^{\mathrm{u}}}\right], \qquad (15)$$

where $D(S_k^{\mathrm{u}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)]) = D_{k\,\mathrm{out}}^{\mathrm{u}}$ and $D_{kl\,\mathrm{out}}^{\mathrm{u}}$ is the covariance of the $k$th and $l$th signal images.

To estimate the $D_{kl\,\mathrm{out}}^{\mathrm{u}}/D_{k\,\mathrm{out}}^{\mathrm{u}}$ ratio, we will use the results presented in [17]. We introduce into consideration the correlation estimate of the fragment data capacity:

$$\Omega^{\mathrm{u}} = \frac{L_x^{\mathrm{u}} L_y^{\mathrm{u}}}{\pi r_{\mathrm{out}}^2}, \qquad (16)$$

where $L_x^{\mathrm{u}}$ and $L_y^{\mathrm{u}}$ are the sizes of the uncorrelated fragment (which is assumed to be rectangular), and $r_{\mathrm{out}}$ is the correlation length of the field reconstructed by the hologram (on the assumption that this field is isotropic). Then the aforementioned ratio, according to [17], can be is estimated by the formula

$$\frac{D_{kl\,\mathrm{out}}^{\mathrm{u}}}{D_{k\,\mathrm{out}}^{\mathrm{u}}} \approx \sqrt{\frac{\Omega^{\mathrm{u}}}{2\kappa}},$$

where $\kappa$ is a coefficient dependent on the field correlation function. Hence, expression (15) takes the form

$$D_{\Sigma}^{\mathrm{n}} = nD_{k\,\mathrm{out}}^{\mathrm{u}}\left(1 + (n-1)\sqrt{\frac{2\kappa}{\Omega^{\mathrm{u}}}}\right). \qquad (17)$$

Thus, the efficiency of selecting correlated fragments can be presented as the ratio of the variances of reconstructed fragments, which depends on the data capacity of uncorrelated fragments:

$$V(n) = \frac{D_{\Sigma}^{\mathrm{c}}}{D_{\Sigma}^{\mathrm{n}}} = \frac{D_{k\,\mathrm{out}}^{\mathrm{c}}}{D_{k\,\mathrm{out}}^{\mathrm{u}}} \frac{[1 + (n-1)]^2}{n\left[1 + (n-1)\sqrt{\dfrac{2\kappa}{\Omega^{\mathrm{u}}}}\right]}. \qquad (18)$$

It can be seen that dependence (18) of the efficiency of selecting correlated fragments (against the background of uncorrelated ones) on the SH number $n$ becomes linear with an increase in the estimated data capacity of uncorrelated fragments $\Omega^{\mathrm{u}}$.

In the case of completely uncorrelated reference images $D_{\Sigma}^{\mathrm{c}}$, dispersion (13) is independent of the SH number:

$$D_{\Sigma}^{\mathrm{c}} = D_{k\,\mathrm{out}}^{\mathrm{c}},$$

whereas $D_{\Sigma}^{\mathrm{n}}$ (14), on the contrary, retains this dependence, because (14) takes the form

$$D_{\Sigma}^{\mathrm{n}} = D\Big(S_k^{\mathrm{u}}(x + x_k^{(S)})*[R_k(x)\otimes R_k(x)] +$$

$$+ \sum_{k\neq l} S_l(x + x_k^{(S)})*[R_k(x)\otimes R_l(x)]\Big)$$

$$= D_{k\,\mathrm{out}}^{\mathrm{u}} + D\Big(\sum_{k\neq l} S_l(x + x_k^{(S)})*[R_k(x)\otimes R_l(x)]\Big) + 2D_{13}, \qquad (19)$$

where $D_{13}$ is the covariance of the first and second terms, and the subscript 13 is taken with allowance for the position of the terms in the initial expression (14). Since the number of local maxima of the cross-correlation function that contribute to the $S_k(x + x_k^{(S)})*[R_k(x)\otimes R_l(x)]$ value is $\Omega^{\mathrm{u}}$, one can assume in the first approximation that

$$D(S_k(x + x_k^{(S)}) * [R_k(x) \otimes R_l(x)]) \approx D^{\mathrm{u}}_{k\,\mathrm{out}}.$$

Hence, it can be seen that the estimated selection efficiency at $n = 1$ is

$$V(n) = \frac{D^{\mathrm{c}}_{\Sigma}}{D^{\mathrm{n}}_{\Sigma}} = \frac{D^{\mathrm{c}}_{k\,\mathrm{out}}}{D^{\mathrm{u}}_{k\,\mathrm{out}}},$$

whereas at $n = 2$, even with the covariance term $2D_{13}$ disregarded,

$$V(n) \approx \frac{D^{\mathrm{c}}_{k\,\mathrm{out}}}{2D^{\mathrm{u}}_{k\,\mathrm{out}}},$$

i.e., the estimate decreases. A further increase in the SH number $n$, in view of property (7), should somewhat increase this ratio; however, it will be smaller than unity at any $n$. Thus, the problem of selecting correlated fragments of signal images by the method of SHs cannot be solved within the scheme under study in the case of uncorrelated reference images.

## 3. Numerical simulation

We simulated the recording of volume holograms and formation of an image in the output plane of the scheme presented in Fig. 1 for the images described as functions of only one coordinate. Images were modelled by realisations of one stationary random process with an exponential spectrum of amplitudes and a random spectrum of phases with zero mathematical expectation and a variance equal to $2\pi$ and with different correlation lengths; the length of all realisations, both signal and reference, was 1024 references (pixels). Signal images $S_i$ were modelled for the following two cases:

(1) spatial separation of fragments $S_k^{\mathrm{c}}$ and $S_k^{\mathrm{u}}$ in image $S_k$, with lengths of both fragments equal to 512 references;

(2) spatial superposition of fragments $S_k^{\mathrm{c}}$ and $S_k^{\mathrm{u}}$ with lengths of 512 and 1024 references, respectively.

Fragment $S_k^{\mathrm{c}}$ always occupied the lateral regions of the signal image $S_k$ (from zero to the 255th and from the 768th to the 1024th reference), while fragment $S_k^{\mathrm{u}}$ occupied (in the absence of superposition) the central region (from the 256th to the 767th reference).

We modelled the recording of a linear hologram and a hologram with reduced intensity of image spectra:

$$F(R_k)F^*(S_k) = \mathrm{const},$$
$$F(R_k)F^*(R_k) = \mathrm{const}. \qquad (20)$$

The reduction of intensity (20) was used to minimise the influence of the pulsed response of the system by reducing this response to the $\delta$ function: $[R_k(x) \otimes R_k(x)] = \delta(x)$.

Figure 2 shows examples of images that confirm the conclusions drawn in the previous section: the first reference image $S_1$ and the images obtained under certain conditions for recording and reconstructing of a multiplexed hologram formed by 15 SHs. The multiplexed hologram was reconstructed by the first reference image $R_1$. The amplitudes of both reference and reconstructed images were normalised to the maximum value in the region of $S_k^{\mathrm{c}}$ (and $E^{\mathrm{c}}_{\mathrm{out}}$, respectively); in some realisations the amplitudes in the region of $S_k^{\mathrm{u}}$ ($E^{\mathrm{u}}_{\mathrm{out}}$) could exceed the amplitudes in the region of $S_k^{\mathrm{c}}$ ($E^{\mathrm{c}}_{\mathrm{out}}$), i.e., emerge beyond the boundaries of the interval [1, –1].

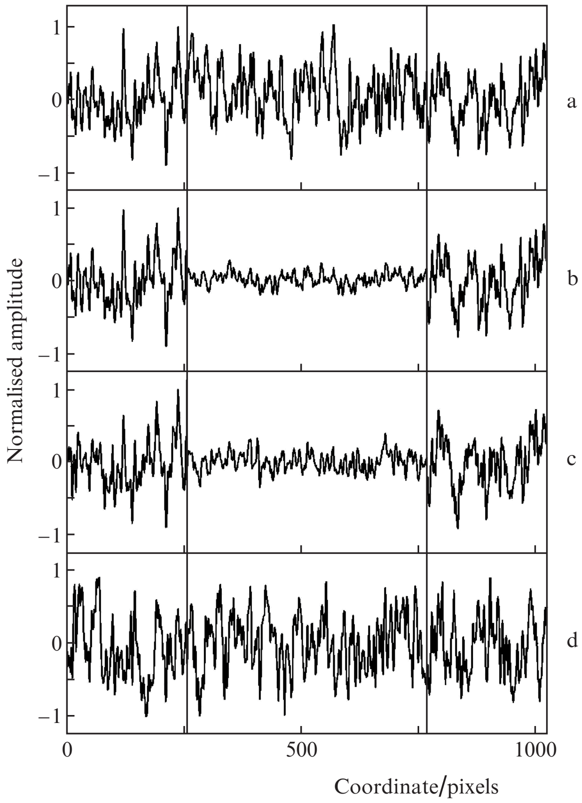


**Figure 2.** Examples of images used in the numerical experiment: (a) signal image $S_1$ (the first SH in the multiplexed hologram recording) in the case of spatial spacing of fragments $S_k^{\mathrm{c}}$ and $S_k^{\mathrm{u}}$ when estimating data capacity $\Omega^{\mathrm{c}} = \Omega^{\mathrm{u}} \approx 95$ and (b–d) images $E_{\mathrm{out}}$, reconstructed in the cases of (b) complete ($\rho_{kl}^{(R)} = 1$) and (c) partial ($\rho_{kl}^{(R)} = 0.25$) correlatedness of reference images and (d) for completely uncorrelated reference images ($\rho_{kl}^{(R)} = 0$). The boundaries of the fragments are shown by vertical lines.

The efficiency of the method is clearly illustrated in Fig. 2:

(i) in the case of completely correlated reference images (Fig. 2b), both the degree of modulation and the realisation form barely change in the range of correlated fragments of the reconstructed image $E^{\mathrm{c}}_{\mathrm{out}}$ (from zero to the 255th and from the 768th to the 1024th reference); i.e., image $E^{\mathrm{c}}_{\mathrm{out}}$ is the undistorted reference image $E^{\mathrm{c}}_{\mathrm{out}} = S_1^{\mathrm{c}}$. At the same time, in the range of uncorrelated fragments $E^{\mathrm{u}}_{\mathrm{out}}$ (from the 256th to the 767th reference), the degree of modulation significantly decreases, the maximum and minimum values of the amplitude approach the mean value, and the reconstructed image $E^{\mathrm{u}}_{\mathrm{out}}$ resembles none of the reference images of the set $\{S_k^{\mathrm{u}}\}_{k=1}^{n}$;

(ii) similar results were obtained for the spatial superposition of fragments $S_k^{\mathrm{c}}$ and $S_k^{\mathrm{u}}$, with the only difference: the reconstructed fragment $E^{\mathrm{c}}_{\mathrm{out}}$ did not completely coincide with the reference fragment $S_1^{\mathrm{c}}$ (these results are omitted to avoid cumbersomeness);

(iii) in the case of partially correlated reference images (Fig. 2c), the efficiency of the method is somewhat lower: one can observe both a decrease in the variance ratio (17) and distortions of fragment $E^{\mathrm{c}}_{\mathrm{out}}$ with respect to reference the fragment $S_1^{\mathrm{c}}$;

(iv) the method is invalid (as follows from the theoretical analysis) for completely uncorrelated reference images (Fig. 2d).

The dependences of the measured variances on the SH number $n$ in the absence of spatial superposition of fragments $S_k^{\rm c}$ and $S_k^{\rm u}$ can be approximated by the formulas

$$D_\Sigma^{\rm c} = -0.56 + 0.29n + 25.157n^2,$$

$$D_\Sigma^{\rm n} = 14.77n + 0.25n^2,$$

i.e., the dependence is in fact quadratic for $E_{\rm out}^{\rm c}$ , according to (13), and practically linear for the region of uncorrelated fragments $E_{\rm out}^{\rm u}$, also in correspondence with theoretical estimate (17). For the case with superposition of $S_k^{\rm c}$ and $S_k^{\rm u}$, the dependence of the variance on $n$ retains to a great extent the quadratic character for the region of correlated fragments $E_{\rm out}^{\rm c}$ and is basically linear for $E_{\rm out}^{\rm u}$:

$$D_\Sigma^{\rm c} = -80.37 + 85.22n + 22.99n^2,$$

$$D_\Sigma^{\rm u} = -4.26 + 22.13n - 0.44n^2.$$

Figure 3 shows the dependences of measured variances of reconstructed images (with their amplitudes normalised to the maximum value) on the SH number $n$ for fragments $E_{\rm out}^{\rm c}$ and $E_{\rm out}^{\rm u}$ in the cases of spatial spacing and superposition of fragments $S_k^{\rm c}$ and $S_k^{\rm u}$ in signal images. The dependences were obtained for completely correlated reference images ($\rho_{kl}^{(R)} = 1$).

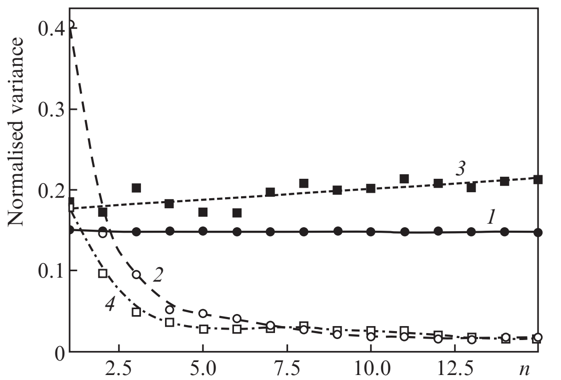


**Figure 3.** Dependences of the variances of normalised images reconstructed into the interval [–1, 1] on the number of SHs forming a multiplexed hologram, for fragments (*1*) $E_{\rm out}^{\rm c}$ and (*2*) $E_{\rm out}^{\rm u}$, under spatial separation of $S_k^{\rm c}$ and $S_k^{\rm u}$ in examples, and fragments (*3*) $E_{\rm out}^{\rm c}$ and (*4*) $E_{\rm out}^{\rm u}$, under spatial superposition of $S_k^{\rm c}$ and $S_k^{\rm u}$ in signal images.

It can be seen in Fig. 3 that, in the absence of superposition, the variance in the region of correlated fragments $E_{\rm out}^{\rm c}$ (i.e., the estimate of the degree of modulation of reconstructed image) remains constant. This is also evidenced by the data of Fig. 2b. In the case of spatial superposition of correlated ($S_i^{\rm c}$) and uncorrelated ($S_i^{\rm u}$) image fragments, the variance slightly changes with an increase in the SH number due to the superposition of attenuated uncorrelated fragments. On the contrary, the dispersion in the region of uncorrelated fragments $E_{\rm out}^{\rm u}$ decreases with an increase in the SH number in both cases.

Thus, the above-described mechanism of detecting correlated fragments is also valid for spatial superposition of fragments $S_k^{\rm c}$ and $S_k^{\rm u}$. The efficiency of selecting correlated fragments increases with an increase in the number of SHs constituting the multiplexed hologram.

Figure 4 shows the experimental dependences of the variance ratio on the SH number $n$ for some values of the reference-image correlation coefficient $\rho_{kl}^{(R)}$ and their linear approximations for one realisation of a random process. It can be seen that the dependence of the estimated efficiency of selecting correlated fragments through the ratio of their variances (18) on the SH number is almost linear in these cases; i.e., it is in agreement with the theoretical predictions. The spread of experimental points with respect to the approximating straight lines is due to the poor validity of the accepted ergodic hypothesis for the given image sizes.

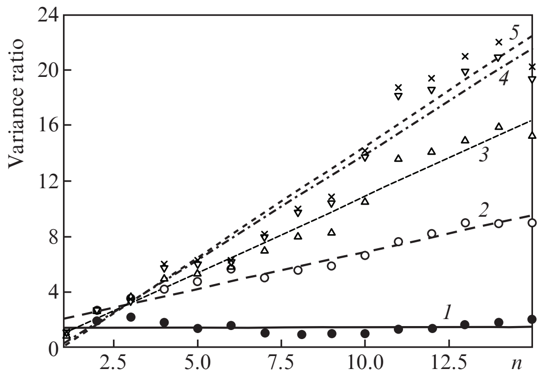


**Figure 4.** Experimental dependences (symbols) of the variance ratio for the reconstructed correlated ($E_{\rm out}^{\rm c}$) and uncorrelated ($E_{\rm out}^{\rm u}$) fragments on the SH number and their approximations (lines) for reference-image correlation coefficients $\rho_{kl}^{(R)}$ = (●, *1*) 0.01, (○, *2*) 0.25, (△, *3*) 0.49, (▽, *4*) 0.81, and (×, *5*) 1, obtained when estimating the data capacity of fragments for all dependences $\Omega^{\rm c} = \Omega^{\rm u} \approx 22$.

As an illustration, Fig. 5a shows the dependences (for 20 realisations) of the mathematical expectation of the slope of the straight lines approximating the experimental data presented in Fig. 4 on the reference-image correlation coefficient

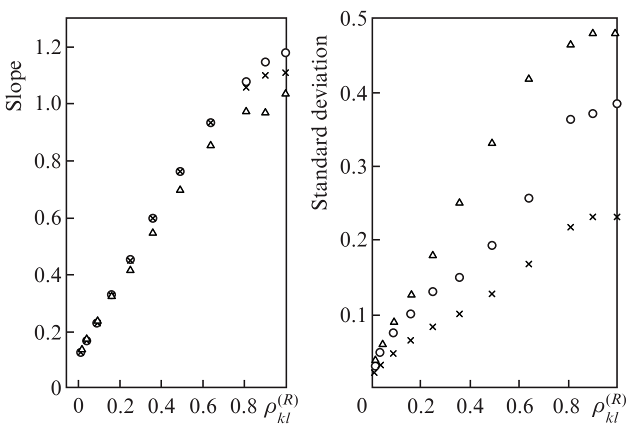


**Figure 5.** Dependences of the (a) mathematical expectation and (b) standard deviation of the slope of linear approximation of dependences on the variance ratio $n$ of the reconstructed correlated ($E_{\rm out}^{\rm c}$) and uncorrelated ($E_{\rm out}^{\rm u}$) fragments on the reference-image correlation coefficient for an ensemble of 20 realisations and estimates of the data capacity (△) $\Omega^{\rm c} = \Omega^{\rm u} \approx 22$, (○) $\Omega^{\rm c} = \Omega^{\rm u} \approx 46$, and (×) $\Omega^{\rm c} = \Omega^{\rm u} \approx 93$.

$\rho_{kl}^{(R)}$ for a number of estimates of image data capacity; the standard deviations of this expectation are presented in Fig. 5b. The slope of the dependence of the variance ratio on the SH number yields an estimate of the method sensitivity to the reference-image correlation coefficient.

Figure 5a demonstrates that the method sensitivity to the reference-image correlation coefficient is almost independent of the image data capacity. From the practical point of view, it is important that the data of Fig. 5b confirm the efficiency of the method even in the absence of ergodicity: a decrease in the estimated data capacity leads to an increase in the standard deviation from the mathematical expectation for the dependence of the method sensitivity to the correlation coefficient but does not change the character of this dependence.

## 4. Conclusions

The Fourier holography scheme with angular multiplication of the reference beam and SHs recorded for a sequence of image pairs makes it possible to solve the problem of detecting correlated fragments of signal images. A necessary condition for solving this problem is partial correlatednes of reference images. Thus, we demonstrated the new possibilities of this scheme in the framework of the development of optical information technologies, in particular, systems of on-line analysis of augmented archive databases [5–7]. The application of a unified element base, physical principles, and circuit designs jointly with memory systems is an urgent task of practical importance for instrumental unification of the computer memory and processor.

The above-described method does not impose any requirements on spatial or spectral differences in the common and individual signs of recorded images. At the same time, it has a limitation: the spatial location of correlated fragments, as well as the mutual arrangement of signal and reference images must be preserved in the entire stored sequence of images.

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