

EXTREME LASER RADIATION: PHYSICS AND FUNDAMENTAL APPLICATIONS DOI: 10.1070/QEL16068

Acceleration of electrons by a laser pulse at its output onto an optical surface of the vacuum–transparent medium interface. Laser synchrotron

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Abstract. **We consider the electron dynamics in the field of an electromagnetic wave produced at the vacuum–transparent medium interface upon reflection from the boundary, close to total internal reflection. The propagation velocity of a constant phase of the electromagnetic wave along the interface can vary from *c*/*n* to infinity (*c* is the speed of light in vacuum, and *n* is the refractive index of the medium at the interface). In this case, there emerge regions of positive and negative phases of the field with wavelengths, approximately equal to half the wavelength of the original laser beam, which can propagate at a speed close to that of light in vacuum. If a beam of relativistic electrons propagates along the surface, they can gain energy and accelerate, as well as radiate. With closed trajectories of electron motion, a laser synchrotron will be implemented as a result of many acceleration cycles.**

Keywords: *laser acceleration of electrons, relativistic electrons, laser synchrotron.*

1. Introduction

The idea of ‘direct’ acceleration of charged particles by a laser field, i.e. an electric field of propagating laser radiation, is attractive for several reasons. Firstly, this method is potentially much more efficient than the already implemented acceleration techniques (by a wakefield of a high-power laser pulse [1, 2], by a ‘bubble’ of positive ions [3, 4]), which are essentially nonlinear, although one of the acceleration regimes (by a wakefield) is referred to as ‘linear’ [5] (nevertheless, the achieved electron energies already reach 4.2 GeV [6]). Secondly, acting on accelerated particles the electric force is limited only by the laser field intensity, and accelerating fields, resulting from the use of the above nonlinear methods of acceleration, will always be less than this field.

On the other hand, known is the idea of ‘dynamical traps’ [7, 8], when, together with the particle, there propagates a corresponding medium (e.g., chemical [7] or plasma [8]), which transforms the particle in a desired way. It is also known that an electromagnetic wave in vacuum does not integrally accelerate charged particles, but changes their location [9]. The new idea of acceleration of charged particles in vacuum at the intersection of tilted laser fronts [10, 11] requires nonperpendicularity of the phase front of a laser wave relative to the vector of its propagation; the possibility of the generation of such electromagnetic fields is now only being discussed [12].

Nevertheless, dynamical traps of electromagnetic radiation, propagating with a speed close to the speed c of light in vacuum, can be implemented. One should consider the electron dynamics in the field of an electromagnetic wave generated at the vacuum–transparent medium interface upon close-to-total internal reflection from the interface. It is easy to see that the propagation velocity of the constant phase of this electromagnetic wave along the interface varies with decreasing angle of incidence from c/n to infinity (n is the refractive index of the medium at the interface). Thus, there emerge regions of positive and negative phases of the field with wavelengths, approximately equal to half the wavelength of the original laser beam, which can propagate at a speed close to the speed of light in vacuum. If in this case a beam of relativistic electrons propagates along the surface, the latter can gain energy and accelerate.

The propagation velocities of the constant phase of the field, close to c, are achieved at the output of the phase front of the laser pulse from an optically denser medium at an angle close to that of total internal reflection. When radiation is polarised in the incidence plane, the electric vector is perpendicular to the interface, and the acceleration is not possible. With polarisation in a plane perpendicular to the incidence plane, the electric vector of laser radiation in a less optically dense medium near the interface is parallel to the interface, but perpendicular to the propagation direction of the constant phase of the field. Nevertheless, in the latter configuration acceleration of relativistic electrons is possible. An initial electron beam is injected near the interface along the propagation direction of the constant phase, and it should keep up with (or lag behind a bit, see below) the propagating region of the constant electric field. Thus, the beam picks up speed in the direction perpendicular to the propagation vector. The energy of an electron in this case increases, i.e. it accelerates.

Obviously, with this acceleration the trajectory of the electron is curved, and it radiates. If, after several cycles of acceleration, the trajectory becomes closed (or nearly closed) and the energy acquired by the electron as a result of this acceleration is compared with the energy spent on emission, a laser synchrotron is implemented in such an experiment because the number of acceleration cycles can be large. Since it is obvious that the effective radius of the mentioned ring may be very small (on the order of tens of centimetres), an entire so-called synchrotron can be placed on a laboratory optical table. The estimates show that the maximum of the synchrotron radiation can lie in the range of photon energies from units and tens of keV, and more (see below).

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The desired effect of total internal reflection occurs at the interface between an optical transparent medium (glass, fused silica, etc.) and vacuum, and so the intensities of the laser beam are limited from above by the radiation strength of the transparent medium. Moreover, the laser pulses used are dispersed in the transparent medium, and the positive half-wave of the red frequency edge of the pulse can catch up with the blue negative half-wave, which limits the total length of acceleration. These and other limiting factors are also discussed below.

2. Dynamics of the electromagnetic field of the laser pulse at its output to the optical interface

Schematically, the output of a spatially limited laser pulse onto the surface of an optically transparent medium–vacuum interface is shown in Fig. 1. It is assumed that laser radiation is linearly polarised in the plane perpendicular to the plane of incidence. If the propagation vector of the constant phase of the field is directed along the positive z axis, as in Fig. 1, then the positive electric vector $\boldsymbol{E}$ has only a component E_x, and the magnetic vector $\boldsymbol{H}$ has components H_y and H_z (see, e.g., [13]). The angle is close to the angle of total internal reflection. The propagation velocity of the constant phase at the interface decreases with increasing angle θ. Since the material body (in the acceleration scheme in question it is an electron) always propagates with a speed lower than c, the acceleration scheme is implemented in the regime of total internal reflection (see also below).

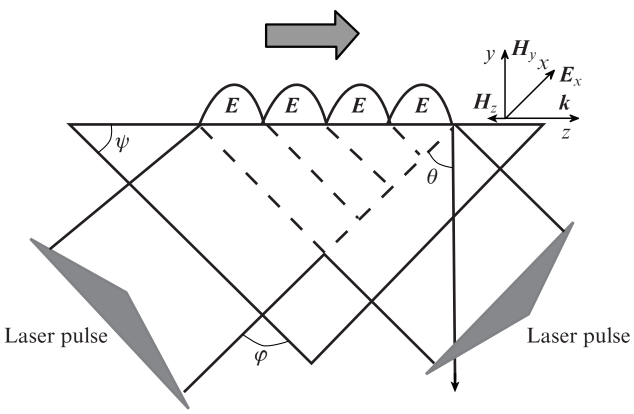

Figure 1. Schematic of an output of a spatially limited laser pulse onto the surface of an optically transparent medium–vacuum interface. The dashed lines, perpendicular to the propagation direction (wider arrows), show the surfaces of the constant phase. The bold arrow shows the direction of propagation of the regions of the electric field of one sign (i.e. the sought-for dynamical traps). The electric field is directed along the x axis, and the z axis lies in the plane of the figure.

Note that the scheme shown in Fig. 1 is used to generate a surface electromagnetic wave (SEW) (see, e.g., [14]). However, this wave 'starts' after the output of the electromagnetic wave to the interface. The SEW has a different propagation velocity, c/n [14], and amplitude. We do not consider here the SEW as a mechanism of acceleration of charged particles. At the same time, such a scheme has been proposed for longitudinal acceleration of charged particles (see [15] and references therein), unlike perpendicular acceleration considered in this paper.

The amplitude of the electromagnetic field in vacuum near the interface is known for a monochromatic wave [13]:

$$E(y,z,t) = E\exp[-k|y|\sqrt{n^2\sin^2\theta - 1} + \mathrm{i}(kzn\sin\theta - \omega t)]. \quad (1)$$

Here k is the amplitude of the wave vector of laser radiation in vacuum; and ω is the laser radiation frequency. The phase velocity of this wave in vacuum is $v_{\mathrm{ph}} = c/(n\cos\theta)$, and it is always less than c, because $n\cos\theta > 1$. Since the laser pulse is limited in time and space, the value of n and θ should be understood as central with respect to temporary and spatial spectra. Note that this is the standard approach when addressing the issue of total internal reflection of short laser pulses [16]. We will not consider all the attendant effects such as the appearance of precursors and others [16]; we note only those that are necessary for our acceleration, and those that can really limit it. Firstly, the refractive index n is frequency dependent. This means that the angle of total internal reflection for the different parts of the temporal spectrum of the laser pulse is different, i.e. if there is total internal reflection for the blue spectral edge, then it is absent for the red edge, and part of radiation enters into vacuum at an angle, close to a right angle. This effect does not prevent acceleration directly. However, if the intensity of the refracted red radiation is high, the total amplitudes of the electric and magnetic vectors converge and the acceleration effect is reduced.

The main competing dispersion effect will be the mentioned output of different parts of the radiation spectrum at different angles to the interface. If the transverse size D of the beam and its possible total acceleration length $L = D/\cos\theta$ are large enough, the positive half-wave of blue light can coincide with the negative half-wave of red light, and acceleration will stop. The condition for termination of acceleration has the form:

$$N\lambda_{\mathrm{r}} = (N + {}^1/_2)\lambda_{\mathrm{b}}.$$

Here λ_{b} and λ_{r} are the wavelengths of blue and red edges of the pulse spectrum, respectively; and N is the maximum number of positive (or negative) half-waves of an electromagnetic field (suitable for acceleration) released on the surface. This will limit the total acceleration length L by the quantity L_{cr}:

$$L \leqslant L_{\mathrm{cr}} \approx \frac{\lambda n^2}{2\Delta n}, \quad (2)$$

where Δn is the complete change in the refractive index of the transparent medium from the blue edge of the pulse spectrum to the red edge. In the linear approximation, $\Delta n \sim \Delta\lambda$. Since $\Delta\lambda \approx \lambda^2/(c\tau_{\mathrm{p}})$, then

$$\Delta n \simeq \frac{\mathrm{d}n}{\mathrm{d}\lambda}\Delta\lambda \approx \frac{\mathrm{d}n}{\mathrm{d}\lambda}\frac{\lambda^2}{\tau_{\mathrm{p}}},$$

which gives

$$L \leqslant L_{\mathrm{cr}} \approx \frac{cn^2\tau_{\mathrm{p}}}{2\lambda(\mathrm{d}n/\mathrm{d}\lambda)}. \quad (2a)$$

For example, for fused silica [17] and a centre wavelength of 1.06 μm, we have $\mathrm{d}n/\mathrm{d}\lambda \sim 0.012$ μm^{-1}, $n \sim 1.45$ and $L_{\mathrm{cr}} \sim$ 2.5 cm at a pulse duration $\tau_{\mathrm{p}} \sim 1$ ps. Note that this estimate was obtained for the case of direct propagation of a conventional laser pulse through a fused silica prism. The process optimisation (introduction of the corresponding frequency chirp of the laser pulse, etc.) can increase L_{cr}.

3. Electron acceleration in a transverse electromagnetic field

The equations of motion of an electron in an electromagnetic field in the laboratory system of coordinates are written in the usual way:

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = -eE_x - \frac{e}{c}v_y H_z + \frac{e}{c}v_z H_y,$$

$$\frac{\mathrm{d}p_y}{\mathrm{d}t} = \frac{e}{c}v_x H_z, \tag{3}$$

$$\frac{\mathrm{d}p_z}{\mathrm{d}t} = -\frac{e}{c}v_x H_y.$$

Here $p_{x,y,z}$ and $v_{x,y,z}$ are the components of the momentum and electron velocity along the axes x, y and z, respectively; and e is the electron charge. The electric field vector is directed along the x axis, and the magnetic field vector lies in the yz plane (see Fig. 1). Without loss of generality, we choose the following initial conditions: $p_x = 0$, $p_y = 0$, $p_z = p_{z0}$, $x = 0$, $y = 0$ and $z = 0$ at $t = 0$.

If E and H are the functions of coordinates and time, as is the case of the laser pulse output at the interface, system (3) only allows a numerical solution (which will be obtained elsewhere). We restrict ourselves to the case of constant E_x, H_y and H_z, making the necessary reservations for the analysis of possible experimental realisation of acceleration in the proposed scheme.

It is known that system (3) has an exact solution at constant fields, since the transition to the corresponding coordinate system makes it possible to exclude one of the fields (electric or magnetic), and the solution of system (3), consisting of only one field, is very simple [9]. However, we must solve the system in the laboratory frame.

In this case, the system has an integral of motion, which can be written as

$$\gamma = \gamma_0 - \frac{eEx}{mc^2}. \tag{4}$$

Here, $\gamma = 1/\sqrt{1-(v_x^2+v_y^2+v_z^2)/c^2}$ is the relativistic energy factor; γ_0 is its initial value; x is the current value of the transverse coordinate; and m is the electron mass. It is considered in (4) that motion occurs in the negative direction of the x axis; otherwise, the particle decelerates. Consider some possibilities and prospects for the experimental implementation of the proposed acceleration scheme and its requirements.

From (4) it is clear that when a particle propagates a distance of at least ~1 cm in the transverse direction x, the final energy of the accelerated electron may differ from the initial energy by the value $\gamma - \gamma_0$, which is now determined only by the magnitude of the electric field. Obviously, the field limiting factor in the proposed scheme is the radiation resistance of the transparent medium, which for picosecond pulses does not allow one to achieve a laser pulse intensity exceeding 10^{13} W cm^{-2} for even the most radiation-resistant fused silica. For femtosecond laser pulses, this value is apparently greater [18]. The above limitation will determine the maximum possible amplitude $E_x \sim 3\times10^5$ CGSE units, and the upper bound of actually achievable energy increment per acceleration cycle $\gamma - \gamma_0$ is ~170, i.e. about 87 MeV. Since the cross-sectional diameter of the laser beam in this case is no more than 1 cm, the maximum required power of the laser pulse is 10^{13} W. For the shortest pulse duration (100 fs), it will give a necessary laser pulse energy of 1 J. Such an energy is produced by sufficiently compact (table-top) laser systems, which determines the prospects of this acceleration scheme (see below).

System (3) obviously integrates once, and then by substituting the second and third equations into the first one, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}[v_x(x_1 - x)] = -c^2\delta - c^2\frac{H^2}{E_x^2}\frac{x}{x_1 - x}. \tag{5}$$

Here $x_1 = \gamma_0 mc^2/(eE_x)$; $\delta = 1 - v_{z0}H_y/(cE_x)$; and $H^2 = H_y^2 + H_z^2$. At characteristic values of the parameters $E_x \sim 3\times10^5$ CGSE units and $\gamma_0 \sim 10$, $x_1 \sim 0.06$ cm. For two polar conditions, $x \gg x_1$ and $x \ll x_1$, we obtain apparent expansions corresponding to different scenarios of the experiments. In the second case,

$$x \approx -\delta\frac{c^2}{2x_1}t^2. \tag{6}$$

Here the time at $x \ll x_1$ for fused silica with the above values of E_x and γ_0 should be much less than 10 ps. If we consider the output of the laser pulse onto the surface of the interface, the time is limited by the transverse size of the beam, i.e. it should be much less than 0.06 cm. All the experiments at large (more than 100) γ_0 also correspond to the same case, $x \ll x_1$.

Using (4), for the energy increment in this case we have

$$\Delta\gamma = \gamma - \gamma_0 = \frac{\delta e^2 E_x^2}{2\gamma_0 m^2 c^2}t^2. \tag{6a}$$

Recall once again that for real experimental conditions the maximum time t is actually the time of the laser pulse output to the interface, which is equal to L/c, rather than the laser pulse duration.

For the opposite situation, $x \gg x_1$, we should double integrate equation (5). This can be done exactly, but the solution under the condition $x \gg x_1$ and even $x \sim x_1$ has no physical sense (see below).

Let us find the electron path length Λ at which it still resides in the accelerating half-wave of radiation outgoing to the surface:

$$\Lambda = \int_0^t v_z \mathrm{d}t \leqslant v_{\mathrm{ph}}t - \frac{\lambda}{2}. \tag{7}$$

From (7) it follows that for large x the electron always falls out of the accelerating half-wave in the absence of optimisation, which is beyond the scope of the problem in question.

We still need to determine the change in the direction of the accelerated electron. Obviously, this is the angle of rotation α, which can be found as follows:

$$\tan\alpha = \left|\frac{v_x}{v_z}\right|. \tag{8}$$

In the case of $x \ll x_1$

$$\tan\alpha \simeq \frac{\delta c^2 t}{v_{z0}x_1} \approx \frac{\delta eE_x t}{\gamma_0 mc}, \tag{8a}$$

because $v_{z0} \approx c$.

Data (6)–(8) are sufficient to discuss possible experiments on acceleration of electrons at the output of the laser pulse to the surface of the transparent medium–vacuum interface.

4. Discussion of possible experiments on electron acceleration at the output of the laser pulse to the interface

Thus, when a laser pulse polarised perpendicular to the plane of incidence (see Fig. 1) reaches the surface of the interface, half-waves of positive and negative electric fields will run along this surface. For the acceleration process to continue long enough (ideally, the total time of the laser pulse output onto the surface, $t = L/c$), several requirements should be met.

The first of them has already been mentioned: the competing dispersion effect will be the output of different parts of the radiation spectrum to the interface at different angles. If the acceleration length L is long enough, the positive half-wave of blue light can coincide with the negative half-wave of red light, and the acceleration will stop. The maximum acceleration length $L_{\rm cr}$ (2a) of fused silica at $\tau_{\rm p} = 1$ ps and $E_0 \sim 3\times 10^5$ CGSE units is about 2.5 cm. Accordingly, for shorter pulses this length (if an ultrashort laser pulse is not chirped on purpose), which is proportional to $\tau_{\rm p}$, will cause a shorter length of the total possible acceleration. Note that from the energy point of view, the shorter the pulse the better; however, there is a limitation in this case due to dispersion.

It is possible to optimise the experiment by using a thin wedge instead of a fairly thick prism, as in Fig. 1. The incidence angle φ on the input face of the prism in this case is not as straight as in Fig. 1. It is easy to see that the propagation of the positive half-wave of the electric field along the interface at a speed approximately equal to the speed of light can be achieved by the following relationship between the wedge angle ψ and the angles θ and φ:

$$\sin\varphi \approx \frac{\cos(\theta - \psi) - n\sin\psi}{\cos\theta}.$$

For example, for fused silica at $\psi = 11°$, the angle of incidence is $\varphi = 61°$, i.e. without a special AR coating of this face most of the laser light is reflected from it. The acceleration length L in this scheme can be increased by several times (by approximately three times in the case of fused silica at the above-considered angles) with decreasing dispersion because the laser pulse path length decreases in a refractive dispersive medium.

The second restriction is also evident. It is easy to see that in the case of acceleration along the x axis the electron slows down along the z axis. This leads to the fact that the electron leaves the accelerating half-wave of the electric field of the laser pulse and starts moving in the decelerating wave, whereas the accelerating wave runs along the interface at a speed close to c. There may arise more complicated effects, such as the concentration of electrons in a certain phase of the accelerating half-wave [10, 11], which should be investigated numerically. Thus, the electron path length can differ at maximum by approximately $\lambda/2$ from the path length of the accelerating half-wave of the laser pulse at the interface. It is also obvious that the transverse size of the laser beam should also be sufficiently large to provide acceleration for all x.

The third limitation is the limitation of the interface with respect to the radiation strength; therefore, in our estimates we will not use electric fields greater than 3×10^5 CGSE units [18].

We will estimate the conditions of possible experiments for fused silica as a medium with the highest radiation resistance. Moreover, we consider the so-called demonstration experiment, when the increment $\Delta\gamma$ is small compared to γ_0, and the experiment when it is relatively large.

Consider system (3) for this case, when all fields are generated exclusively by a laser pulse emerging at the interface between fused silica and vacuum. From paper [15] [see in it formulas (31) and (33) at an azimuth angle of 90°] we will use the relations between the components of the fields in vacuum: $H_y = E_x n\sin\theta$, $H_z = -{\rm i}E_x \times (n^2\sin^2\theta - 1)^{1/2}$, i.e. $H_y^2 + H_z^2 = H^2 = E_x^2$. The value of $v_{z0} < v_{\rm ph}$ in (6) must be positive; otherwise, acceleration is absent. This is only achieved at $v_{z0} < v_{\rm ph}$, and so $v_{\rm ph}$ should be very close to the speed of light, and in this case, $\delta \approx 1/2\gamma_0^2$. The considered case $x \ll x_1$ (6) corresponds to the demonstration experiment. The angle θ of incidence is close to a total internal reflection angle of 43.6° (the refractive index of fused silica at a wavelength of 1.06 μm is 1.45). If we have a normal beam of diameter $D \sim 200$ μm, the condition $x \ll x_1$ is satisfied. For fused silica with the above value of E we will have $\Delta\gamma \approx 0.07/\gamma_0$, while the electron path length along the z axis 'inside' the accelerating half-wave (it determines the total 'fall out' of a particle from the phase of the wave) will be more than 300 μm, and along the x axis it will be equal to 8 μm/γ_0. For $\gamma_0 = 10$ it will give $\Delta\gamma \approx 3.5$ keV and the path of less than 1 μm along the x axis. The angle of deviation of the accelerated electron beam in this case is extremely small.

The value of δ can be increased if the system is subjected to the action of an additional constant magnetic field ΔH (the authors of [19] apparently knew about it already) along the y axis (see Fig. 1). In this case, only one of the half-waves that came to the surface will be accelerating. Then, $\delta \approx \Delta H/E_x$. At present there are permanent magnets with a field of 30000 Gs, for which $\delta \approx 0.1$. At $\gamma_0 = 10$ this yields $\Delta\gamma \approx 1.4/\gamma_0 \approx 70$ keV and a path of length about 15 μm along the x axis, which corresponds to the angle of rotation of about 3° (8a). These parameters of energy and angle variation can be easily recorded experimentally. Since for the above assumption of proximity of $v_{\rm ph}$ to c the field H is very small compared with the others, it will not lead to the departure of the accelerated electrons from the acceleration region (slit, see below) in the vertical direction.

Note that in the proposed experiment, the value of γ_0 cannot be too small (on the order of unity), because in this case the speed of the accelerating half-wave along the surface should be much less than c, and the penetration depth of the electric field in vacuum, according to (1), becomes clearly insufficient. The phase velocity of the accelerating half-wave at the interface should be approximately equal to the input velocity of the electron, v_{z0}. Then

$$n\sin\theta = \frac{\gamma_0}{\sqrt{\gamma_0^2 - 1}}.$$

Thus, for the depth of the exponential penetration of the field into vacuum along the z axis from (1) we have

$$\frac{1}{k} = \frac{1}{k\sqrt{n^2\sin^2\theta - 1}} = \frac{\lambda}{2\pi}\sqrt{\gamma_0^2 - 1}.$$

Therefore, when $\gamma_0 < 10$, this depth is less than 2λ.

The demonstration experiment can be performed using a laser pulse with an energy of $\sim cE_0^2D^2\tau_{\rm p}/32$. The pulse duration can be very small, because the path length of the laser pulse in the prism can be made small enough (up to $D/\cos\theta \sim D$ for fused silica), which will give a minimum pulse duration $\tau_{\rm p} \sim 30$ fs and a minimum laser pulse energy of 0.1 mJ.

Let us now discuss a possible experiment with a larger relative increment of energy of the accelerated electron, and then – the way to optimise both experiments. It would seem that the main limitation on the conditions of their implementation is caused by dispersion: the acceleration length must be less than or about 1 cm. At large γ_0 (about 100) and $L \sim ct \sim$ 1 cm the value of $\gamma - \gamma_0$ can be ~100. In this case, $-x$ equals 1 cm. However, unless special measures are taken, the particle 'falls out' of the accelerating half-wave before it reaches a maximum energy. To avoid this, the accelerating half-wave must move with the particle. This is possible if the accelerating half-wave is cylindrically divergent (in a projection on the surface of the interface) rather than plane, the focus (imaginary) of the wave being located before radiation reaches the surface of the interface. The precise parameters of this optical system (either a cylindrical lens cut at an angle which provides a radiation output onto the surface at an angle close to the angle of total internal reflection, or a specially defocussed laser beam) must be calculated numerically. In these calculations it should also be taken into account that the electric vector of the radiation field is rotated relative to the moving particles not quite the way it is written for system (3). Apparently, in the case of a divergent accelerating cylindrical half-wave the situation will improve due to the rotation of the electric vector and deteriorate due to a decrease in the radiation intensity caused by divergence. System (3) does not describe all these factors.

The experiment can be also optimised through the use of two counterpropagating accelerating half-waves, each of which propagates along its interface (the second prism should be placed symmetrically to the prism in Fig. 1, with respect to the xz plane); the amplitudes of these waves are summed, i.e. they have the desired phase. Now the electron is accelerated in a narrow slit between these surfaces. Under acceleration along one surface the electric field, with increasing distance from it, falls proportionally to $\exp(-kz)$ (1). For this scheme the field in the slit in the transverse direction will be (in a very rough approximation) stable. At a slit width l and the field amplitude E on each surface, the field in the slit is $E_c \approx E(2 - \kappa l)$, i.e. generally, in the first approximation ($\kappa l \ll 1$) it will be constant. It is also possible to use laser pulses polarised not purely linearly in the plane perpendicular to the plane of incidence, but slightly elliptically; the rotation of counterpropagating pulses on different surfaces must be different. Then in a slit, together with a nearly uniform accelerating field along the surface, there will be a nonuniform field, propelling the particles toward the slit centre, and the angular divergence of the beam can be significantly reduced. Note that a typical transverse size of the accelerated bunch of particles in accelerators is ~1 μm; therefore, a slit with a width of 10 μm is sufficient for an optimal experiment.

5. Laser synchrotron

For the above-described acceleration, the electron radiates. As a result of several successive acceleration cycles, in each of which acceleration takes place along the input velocity of the electron, the electron (or the electron bunch) will finally turn by 360° and the process can be looped. Thus, a synchrotron will be implemented, which, according to the acceleration method, will be a laser synchrotron. Let us estimate the maximum possible frequency of synchrotron radiation and the possibility of principal implementation of the device.

The total radiation energy $\Delta\varepsilon$ of the particle in the field is defined in [9] (§73). Since the synchrotron is characterised by large values of γ_0, then in using $\Delta\varepsilon$ from [9] the solution to system (3) should be extended to the case $x \ll x_1$. The limiting value of γ_0 is easy to determine by equating the energy $mc^2\Delta\gamma$ acquired in one acceleration cycle to losses $\Delta\varepsilon$ in this cycle. Calculation using (6) yields

$$\Delta\varepsilon \approx \frac{2e^4E^2}{3m^2c^3}\delta^2\gamma_0^2 t = mc^2\Delta\gamma \approx \frac{\delta e^2E^2}{2\gamma_0 mc^2}t^2 \qquad (9)$$

and

$$\gamma_0 \leqslant \gamma_{\rm cr} = \sqrt[3]{\frac{3mc^3t}{4e^2\delta}}. \qquad (10)$$

At $\gamma_{\rm cr} \sim 10^4$ and $\delta = 0.1$ (estimates show that the scheme of the laser synchrotron without an additional magnetic field cannot be implemented), the equilibrium acceleration length $L \sim ct$ is about 3 cm. In this case, the rotation angle in one acceleration cycle is $\alpha = 0.053 \approx 3°$. If we imagine a synchrotron, consisting of sections of pairs of fused silica prisms, we need $360°/\alpha(°) \sim 120$ such pairs. The minimum radius R is around 50 cm, which gives the fundamental frequency of synchrotron radiation [20] (fundamental overtone)

$$\Omega = \frac{3c}{2R}\gamma_0^3 \sim 10^{21}\,{\rm s}^{-1}.$$

The energy of the emitted photons will thus be ~ 0.15 MeV.

Note that if acceleration is carried out by 'sections', consisting of two counter prisms, the radiation from such a system will always be coupled out along the interface. This radiation cannot create stray electrons that in a conventional synchrotron are knocked out by radiation from the walls of the tube, which can be an additional advantage of the laser synchrotron.

6. Conclusions

Obviously, to demonstrate the effect of acceleration of electrons at the output of the laser radiation onto the optical interface under total internal reflection it is sufficient to launch a relativistic electron beam with an initial electron energy of $\gamma_0 \geqslant 10$, parallel to the surface in the direction of the field phase propagation. Without an additional (vertical) magnetic field the transverse acceleration will occur in both positive and negative directions of the x axis. In this case, it will be possible to measure the electron deflection angle. Re-acceleration of an accelerated electron bunch can be implemented similarly, i.e. the process can be continued. In this case, we will obviously lose electrons, approximately half of them in each cycle, but the acceleration effect, including multiple-cycle acceleration, can be registered. In the presence of an additional magnetic field it will be necessary to control the phases of accelerating laser pulses. Of course, it is also necessary for the implementation of a synchrotron: here need to catch a bunch with a desired phase of the accelerating field in the prism slit, but this is a topic of a separate publication, because the slit dynamics in these traps requires a separate calculation (see, e.g., [10, 11]). Note that in a scheme with an additional magnetic field the input electron velocity may be somewhat greater than the field phase propagation velocity at the interface; in this case, acceleration is also implemented, but the calculation becomes more complex. Ideally, the initial velocity of the electron should exceed the phase velocity of the field, so that the electron first reached the opposite edge of the trap under continuous braking, which continues in the transversely accelerating phase up to a complete loss of synchronism. Then, the total acceleration length and synchronisation period can be increased, apparently, up to two times.

Thus, in launching an electron beam along an optical interface between vacuum and a transparent medium in the case of an output of a pulse of polarised laser light under total internal reflection, one can expect an increase in the particle energy due to transverse acceleration. The use of many cycles of successive electron acceleration and of a 'loopback' scheme makes it possible to implement a laser synchrotron. The physical limit of the frequency of the synchrotron radiation of such a device lies in the region of very hard gamma radiation. Apparently, without an additional magnetic field this scheme is not realisable.

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References

1. Leemans W.P., Nagler B., Gonsalves A.J., et al. *Nature Phys.*, **2**, 696 (2006).
2. Nakamura K., Nagler B., Tóth Cs., et al. *Phys. Plasmas*, **14**, 056708 (2007).
3. Mora P., Antonsen T.M. Jr. *Phys. Rev. E*, **53**, R2068 (1996).
4. Pukhov A., Meyer-ter-Vehn J. *Appl. Phys. B*, **74**, 355 (2002).
5. Korzhimanov A.V., Gonoskov A.A., Khazanov E.A., et al. *Usp. Fiz. Nauk*, **181**, 9 (2011).
6. Leemans W.P., Gonsalves A.J., Mao H.-S., et al. *Phys. Rev. Lett.*, **113**, 245002 (2014).
7. Cross M.C., Honenberg P.C. *Rev. Mod. Phys.*, **65**, 851 (1993).
8. Lubashevsky I.A., Manke R., Hajimahmoodzadeh M., et al. *Eur. Phys. J. B*, **44**, 63 (2005).
9. Landau L.D., Lifshitz E.M. *The Classical Theory of Fields* (Oxford: Pergamon Press, 1971).
10. Korobkin V.V., Romanovskiy M.Yu., Trofimov V.A., et al. *Laser Part. Beams*, **31**, 23 (2013).
11. Korobkin V.V., Romanovskiy M.Yu., Trofimov V.A., et al. *Kvantovaya Elektron.*, **43**, 232 (2013) [*Quantum Electron.*, **43**, 232 (2013)].
12. Balakin A.A., Fraiman G.M. *Pis'ma Zh. Eksp. Teor. Fiz.*, **99**, 443 (2014).
13. Born M., Wolf E. *Principles of Optics* (Oxford: Pergamon Press, 1970; Moscow: Nauka, 1973).
14. Libenson M.N. *Sorosovskii Obrazovatel'nyi Zh.*, (10), 92 (1996).
15. Fernow R.C. *Techn. Rep. Brookhaven Nat. Lab. BNL-52290* (New York: Upton, 1991).
16. Akhmanov S.A., Vysloukh V.A., Chirkin A.S. *Optics of Femtosecond Laser Pulses* (New York: American Institute of Physics, 1992; Moscow: Nauka, 1988).
17. Grigoriev I.S., Meilikhov E.Z. (Eds) *Handbook of Physical Quantities* (Boca Raton, NY: CRC Press, 1996; Moscow: Energoatomisdat, 1991).
18. Ivanov A.V. *Prochnost' opticheskikh materialov* (Strength of Optical Materials) (Leningrad: Mashinostroieniie, 1989).
19. Nagorsky H.A., Amatuni A.Ts., Harutiunian W.M. *Proc. 12th Intern. Conf. High Energy Accelerators* (Fermilab, USA, 1983) p. 488–490.
20. Ternov I.M., Mikhailin V.V., Khalilov V.R. *Synchrotron Radiation and Its Applications* (Amsterdam: Harwood Academic Pub., 1985; Moscow: Izd-vo MGU, 1980).