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Quantum interference of biphotons with a Doppler frequency shift

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Abstract. **We report a theoretical study of transformation of a biphoton state of light under Bragg diffraction on a travelling sound wave in an acousto-optic modulator (AOM). It is shown that the diffraction of AOM biphotons emitted during the collinear parametric scattering of light leads to a shift of the carrier frequency of a biphoton wave packet, which exceeds twice the Doppler frequency shift for the classical field. A method is proposed for measuring the Doppler frequency shift of a biphoton, which is based on interference between independent biphotons.**

Keywords: *parametric scattering of light, acousto-optical interaction, Doppler frequency shift, interference of biphotons, wave packet.*

1. Introduction

In quantum optics, much attention is paid to issues related to the conversion of entangled states of photons as they pass through various optical systems. On the one hand, it is determined by the specific challenges posed by the preparation of the given states of light and optimisation of their characteristics, and on the other, by revealing specific features of quantum states that are not observed in classical optics.

One of the first discovered nonlocal quantum effects is nonlocal cancellation of dispersion of light during the propagation of a biphoton [1]. Specific quantum effects associated with an entangled state of two photons emerge upon diffraction of a biphoton by slits [2, 3] and by a diffraction grating [4]. The authors of Refs [2–4] showed that the entangled state of two photons with a wavelength of λ is similar to a single-photon state with a wavelength of $\lambda/2$.

Luo et al. [5] studied self-image replication of a transverse field structure (Talbot effect) for periodic objects illuminated by entangled photon pairs. It was shown that the quantum Talbot length, manifested in the second-order Talbot effect, is half the classical Talbot length.

In this paper, we present a new quantum effect, which arises upon diffraction of a biphoton by a moving grating of the refractive index of a medium, created by a travelling ultrasonic wave. It is known that the Bragg diffraction, in the case of the classical field, results in a Doppler frequency shift that is equal to the sound frequency Ω. As shown in this paper, in the diffraction of biphotons generated in the collinear parametric scattering of light, the Doppler frequency shift is equal to 2Ω. In this paper, we propose and study new opportunities for obtaining interference between independent spatially separated biphotons with a Doppler frequency shift.

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2. Diffraction of an electromagnetic waves by a moving grating

For classical electromagnetic fields, the Doppler frequency shift in reflection from a moving medium is a well known and widely used fundamental effect. Moving objects can be a diffraction grating that is produced by a travelling acoustic wave in an isotropic medium. Figure 1 shows an acousto-optic modulator (AOM), in which a plane sound wave with a wave vector $\boldsymbol{k}$ and an angular frequency Ω propagates. The dielectric constant of the medium $\varepsilon(\boldsymbol{r}, t)$, affected by the sound, changes as follows:

$$\varepsilon(\boldsymbol{r},t) = \varepsilon_0 + \varepsilon_1 \cos(kx - \Omega t), \tag{1}$$

where ε_1 is a factor proportional to the amplitude of the acoustic wave.

Let the classical electromagnetic field in the form of a quasi-monochromatic plane wave with a carrier frequency ω_0, slowly varying complex amplitude $E_0(t)$ and polarisation vector $\boldsymbol{e}_0$ be incident on the AOM:

$$\boldsymbol{e}_0 E_{\rm in}(t) = \boldsymbol{e}_0 E_0(t)\exp(-{\rm i}\omega_0 t). \tag{2}$$

In the case of the Bragg diffraction by a moving grating (1) formed by an acoustic wave, dominant are two diffraction peaks (zero and minus first). Inside the AOM, plane waves propagate in two directions (passing and diffracted waves, Fig. 1). At an angle of incidence $\vartheta_{\rm in}$, equal to the Bragg angle ($\vartheta_{\rm in} = \vartheta_{\rm B} = k/2k_0$, $k_0 = \sqrt{\varepsilon_0}\,\omega_0/c$), an electromagnetic field at the AOM output can be written in the form (see, e.g., [6]):

$$\boldsymbol{E}_{\rm out}(t) = E_{\rm in}(t - \sqrt{\varepsilon_0}\,l/c)(\boldsymbol{e}_0 T + \boldsymbol{e}_{-1} R), \tag{3}$$

where $T = \cos\gamma$; $R = {\rm i}\sin\gamma\exp[{\rm i}\Omega(t - \sqrt{\varepsilon_0}\,l/c)]$;

$$\gamma = \frac{\varepsilon_1 k_0 l}{4\varepsilon_0(\varepsilon_0 - \sin^2\vartheta_{\rm B})^{1/2}};$$

l is the length of the region of AO interaction in the cell; and $\boldsymbol{e}_0$ and $\boldsymbol{e}_{-1}$ are the unit vectors of polarisation of the transmitted and diffracted waves.

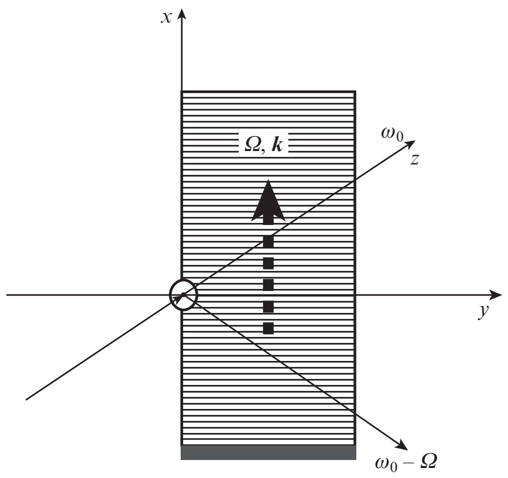

Figure 1. Bragg diffraction in the AOM by a travelling sound wave.

In the Bragg diffraction regime, the AOM is similar to a beam splitter with a transmittance T and reflectance R. Reflection (diffraction) causes a Doppler shift of the frequency by a value equal to the frequency of the sound wave Ω: the frequency of the reflected wave is equal to $\omega_0 - \Omega$.

3. Doppler shift of the carrier frequency of the biphoton wave packet

Let us now consider the Doppler frequency shift by a moving grating in the case of a quantum biphoton wave packet. Such a package is produced at the output of a nonlinear crystal NLC (Fig. 2) as a result of collinear parametric scattering of light.

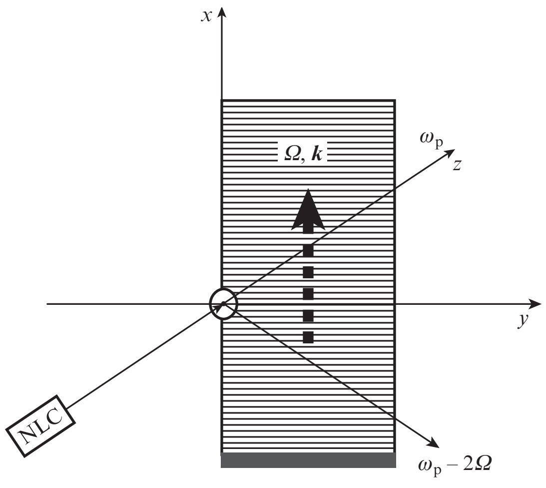

Figure 2. Diffraction of a biphoton in the AOM.

The pump field is considered classical and can be written in the form of a plane monochromatic wave

$$E_{\rm p}(z,t) = E_0 \exp[-{\rm i}(\omega_{\rm p} t - k_{\rm p} z)]. \quad (4)$$

In the case of degenerate parametric scattering of light, the pump quantum splits within the NLC crystal into two quanta propagating parallel to $\boldsymbol{k}_{\rm p}$ (in the direction of z). Creation operators $a_j^+(t,z_0)$ of idle (j = i) and signal (j = s) photons at the output end of the nonlinear crystal have the form of wave packets with a carrier frequency $\omega_{\rm p}/2$:

$$a_j^+(t,z_0) = a_{j0}^+ F(t) \exp\{{\rm i}[\omega_{\rm p} t - \varphi_{\rm p}(z_0)]/2\}. \quad (5)$$

Here, $\varphi_{\rm p}(z_0) = k_{\rm p}(z_0 + n_{\rm p} L)$ is the phase of the pump wave, depending on the position of the nonlinear crystal, which is located in the region $z_0 < z < z_0 + L$; L is the thickness of the crystal; $n_{\rm p}$ is the refractive index of the crystal for the pump wave; and a_{j0}^+ is the time-independent creation operator of a photon with a frequency $\omega_{\rm p}/2$. The shape of the wave packet is determined by a slowly varying time function $F(t)$, which is not an operator.

At a low coefficient of parametric amplification Γ, the wave function $|\psi\rangle$ at the output of the nonlinear crystal can be written as a superposition of the vacuum ($|0\rangle$) and biphoton states [7]:

$$|\psi\rangle = C|0\rangle + \frac{\Gamma}{2} a_{\rm s}^+(t_1,z_0) a_{\rm i}^+(t_2,z_0)|0\rangle, \quad (6)$$

where the constant C is close to unity (we will assume below that $C = 1$).

The biphoton state is usually described by the amplitude of this state (biphoton amplitude) $A(t_1, t_2)$, which is given by

$$A(t_1,t_2) = \langle 0|a_{\rm s0} a_{\rm i0}|\psi\rangle. \quad (7)$$

Taking into account (5)–(7), the biphoton amplitude can be written in the form (see, e.g., [7]):

$$A(t_1,t_2,z_0) = F(\tau)\exp\{-{\rm i}[\omega_{\rm p} t - \varphi_{\rm p}(z_0)]\}, \quad (8)$$

where $\tau = t_1 - t_2$; $t = (t_1 + t_2)/2$. In accordance with (8), the biphoton amplitude has the form of a wave packet with a carrier frequency equal to the frequency of the pump field $\omega_{\rm p}$.

Let us consider now how the biphoton amplitude changes under diffraction by a moving grating of the refractive index. Coming out of the crystal, the signal and idler waves fall on the AOM at the Bragg angle $\vartheta_{\rm in} = \vartheta_{\rm B}^0 = k/2k_{0\rm p}$, where$k_{0\rm p} = \sqrt{\varepsilon_0}\,\omega_{\rm p}/2c$. Each of the wave incident on the AOM (signal and idler, j = o, e), in the Bragg diffraction regime, splits into the transmitted and the diffracted waves. For simplicity we consider only the case when the amplitude of the sound wave and the length of the AO interaction l are chosen such that $\gamma = \pi/2$. Under this condition, the signal and idler waves are totally reflected by the grating, and the transmitted wave at the AOM output is absent. According to (3), at $\gamma = \pi/2$ the annihilation operators $b_j(t)$ of the photons of idler (j = i) and signal (j = s) waves at the AOM output are related with the respective operators $a_j(t)$ at the input by the expression:

$$b_j(t) = -{\rm i}\exp[{\rm i}\Omega(t-\tau_0)]a_j(t-\tau_0), \quad (9)$$

where $\tau_0 = \sqrt{\varepsilon_0}\,l/c$ is the time delay during the passage through the AOM.

Taking into account (8) and (9), we obtain, for the biphoton amplitude after passing through the AOM, the expression:

$$B(t_1,t_2,z_0) = F(\tau)\exp\{-{\rm i}[(\omega_{\rm p} - 2\Omega)t - \varphi_{\rm p}(z_0) - \varphi_0]\}, \quad (10)$$

where $\varphi_0 = 2\Omega\tau_0$.

One can see from (10) that as a result of the AO interaction, the carrier frequency of the biphoton packet is shifted by 2Ω, whereas for the classical field the frequency shift in the diffraction of a travelling sound wave is equal to the frequency of sound Ω.

Spatial and temporal characteristics of the biphoton state are usually measured by counting the number of coincidences R_c of photon counts on two detectors. Fields of the signal and idler waves pass through optical systems into two photodetector. The rate of coincidence R_c is proportional to the square of the amplitude biphoton modulus. Because $|A(t_1, t_2)|^2$ and $|B(t_1, t_2)|^2$ are independent of the carrier frequency of the biphoton packet, in experiments with biphotons produced in a nonlinear crystal, it is impossible to detect the Doppler frequency shift of the biphoton wave packet. Below, we describe a method for measuring this shift in the interference of biphotons emerging from two crystals.

4. Interference of biphotons

4.1. Interference in the absence of Doppler frequency shifts

Interference of biphotons emitted independently in two nonlinear crystals was investigated in a number of papers (see, e.g., [8, 9]). First, we will consider a conventional scheme (Fig. 3) for observing the fourth-order interference between two biphotons emitted during degenerate collinear parametric scattering of light [9].

As shown in Fig. 3, a monochromatic pump wave passing through a beam splitter BS1 is incident on nonlinear crystals α and β. The coordinates of the input end faces of these crystals z_α, z_β are assumed equal to their respective distances from the end faces to the beam splitter BS1. In a system with two nonlinear crystals, biphotons are generated independently in each crystal, and the wave function $|\psi\rangle$ can be written as

$$|\psi\rangle = |\psi\rangle_\alpha |\psi\rangle_\beta, \quad (11)$$

where $|\psi\rangle_{\alpha,\beta}$ are the wave functions for each crystal.

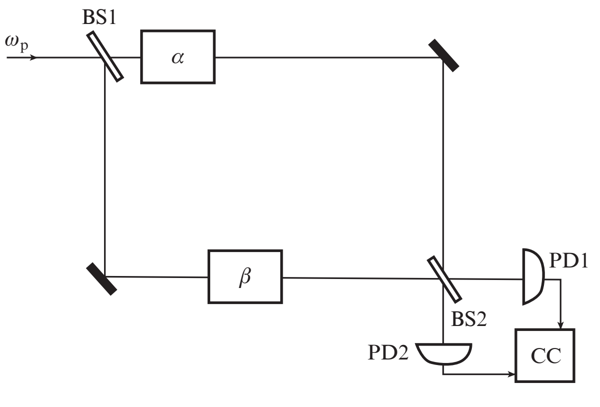

Figure 3. Interference of biphotons in the scheme of a Mach–Zehnder interferometer [(α, β) nonlinear crystals; (BS1, BS2) beam splitters; (CC) coincidence circuit; (PD1, PD2) photodetectors].

The amplitudes $|\psi\rangle_{\alpha,\beta}$ of biphotons produced in the crystals α and β are determined by the expressions:

$$A_{\alpha,\beta}(t_1,t_2) = F_{\alpha,\beta}(\tau)\exp[-\mathrm{i}(\omega_p t - \varphi_{\alpha,\beta})], \quad (12)$$

where

$$\varphi_{\alpha,\beta} = k_p(z_{\alpha,\beta} + n_p L_{\alpha,\beta} + z'_{\alpha,\beta}); \quad (13)$$

n_p is the refractive index for the pump wave in the crystal; $L_{\alpha,\beta}$ are the thickness of the crystals; and $z'_{\alpha,\beta}$ are the distances from the output end faces of the crystals to the beam splitter BS2.

As shown in [9], the rate of coincidence R_c of photon counts on photodetectors PD1 and PD2 (Fig. 3) is given by

$$R_c = K[|F_\alpha(\tau)|^2 + |F_\beta(\tau)|^2 - 2\,\mathrm{Re}(F_\alpha(\tau)F^*_\beta(\tau))\mathrm{e}^{\mathrm{i}\Phi}], \quad (14)$$

where $\Phi = \varphi_\alpha - \varphi_\beta$; and K is the proportionality factor.

It follows from (14) that the rate of coincidence oscillates when changing Φ with a period equal to the pump wavelength λ_p. These oscillations are due to the interference between biphotons generated in crystals α and β. The visibility of the interference pattern does not depend on the position of the crystals inside the interferometer.

4.2. Influence of the Doppler frequency shifts on interference

Let us now place the AOM, onto which signal and idler photons are incident at the Bragg angle, behind the nonlinear crystal (Fig. 4). The amplitude of the sound wave and the length of the AO interaction l are chosen such that $\gamma = \pi/2$ (in this case, the transmitted wave at the AOM output is absent). The scheme in Fig. 4 is a simplified: we do not show here a change in the direction of the wave reflected in the AOM. Figure 4 shows two AOMs that are in both arms of a Mach–Zehnder interferometer.

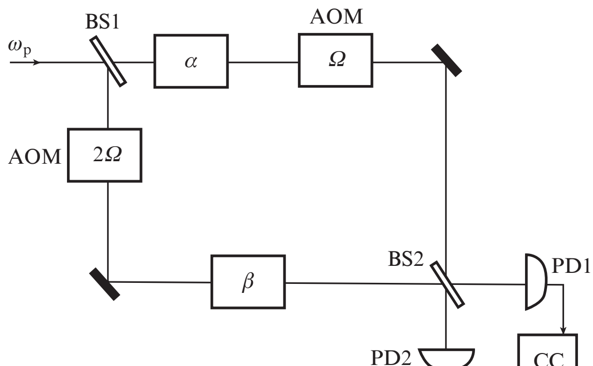

Figure 4. Interference of biphotons in the scheme of a Mach–Zehnder interferometer with two AOMs; notations are the same as in Fig. 3.

Let us first assume that there is only one AOM (in the arm containing the crystal α). In this case, the coincidence rate of photon counts R_c on photodetectors PD1 and PD2 is determined by the formula:

$$R_c = K\{|F_\alpha(\tau)|^2 + |F_\beta(\tau)|^2 - 2\,\mathrm{Re}(F_\alpha(\tau)F^*_\beta(\tau)) \times \exp[\mathrm{i}(\Phi + \varphi_{0\alpha} - 2\Omega t)]\}, \quad (15)$$

where $\varphi_{0\alpha} = 2\Omega\tau_{0\alpha}$ is the phase shift of the biphoton packet in the AOM located in the interferometer arm with the crystal α.

In accordance with (15) the rate of coincidence oscillates in time with a frequency of 2Ω. In experiments, one usually measures the average coincidence rate of photon counts. The interference term in (15), when averaged over time, vanishes. Thus, by using only one AOM in the scheme shown in Fig. 4, interference of biphotons disappears due to the Doppler frequency shift of the biphoton in the AOM located in the interferometer arm with the crystal α.

Let us now consider a scheme with two AOMs shown in Fig. 4. In the arm with the crystal β, the AOM is mounted in front of this nonlinear crystal, and the frequency of the sound wave in it is equal to 2Ω. Upon reflection from the moving grating in this AOM, the frequency of the pump wave is shifted by -2Ω.

In the presence of two AOMs, the coincidence rate R_c of photon counts on detectors PD1 and PD2 (see Fig. 4) is found from the expression:

$$R_c = K\{|F_\alpha(\tau)|^2 + |F'_\beta(\tau)|^2$$

$$-2\,\mathrm{Re}(F_\alpha(\tau)F'^{*}_\beta(\tau))\exp[\mathrm{i}(\Phi + \Phi_0)]\}, \qquad (16)$$

where $\Phi = \varphi_\alpha - \varphi_\beta$; $\Phi_0 = \varphi_{0\alpha} - \varphi_{0\beta}$; and $\varphi_{0\beta} = 2\Omega\tau_{0\beta}$ is the phase shift of the pump wave in the AOM located in the arm with the crystal β. The prime in $F'_\beta(\tau)$ indicates that, when the pump frequency changes from ω_p to $\omega_p' = \omega_p - 2\Omega$, the shape of the biphoton packet may change. At $\Omega \ll \Delta\omega$, where $\Delta\omega$ is the width of the biphoton spectrum, the change in the packet shape can be neglected, i.e. we can assume that $F'_\beta(\tau) = F_\beta(\tau)$.

It follows from (16) that when use is made of two AOMs (Fig. 4), one can restore the interference pattern of biphotons with Doppler frequency shifts. In accordance with (16) the coincidence rate oscillates when changing Φ having a period equal to the pump wavelength λ_p. The visibility of the interference pattern does not depend on the position of the crystals and the AOM inside the interferometer.

Recovery of the interference pattern of biphotons in the scheme with two AOMs only occurs when the pump frequency passing through the AOM in the arm with the crystal β is shifted by 2Ω. This difference in the frequencies of the sound waves in two AOMs confirms the fact that the Doppler frequency shift for the biphoton packet exceeds twice the frequency shift for the classical field.

Katamadze and Kulik [10] discussed and briefly described recently proposed and studied methods for controlling the biphoton field. The results obtained in this study showed that: 1) the use of the AO interaction allows one to control the carrier frequency of biphoton wave packets; 2) the use of the interference of quantum states allows one to measure the Doppler shift of the carrier frequency of a biphoton wave packet. Thus, using the AO interaction we can purposefully control optical quantum states.

5. Conclusions

We have investigated the Doppler frequency shift of a biphoton wave packet under Bragg diffraction by a travelling sound wave. It is shown that after diffraction in the AOM, the shift of the carrier frequency of a biphoton wave packet emitted during collinear parametric scattering of light exceeds twice the Doppler frequency shift of the classical field. A method has been proposed for measuring the Doppler frequency shift of a biphoton wave packet, based on the fourth-order interference between independent biphotons emitted during collinear parametric scattering of light in two crystals. The results of the research in this paper show that the acousto-optical interaction can be used as one of the methods for controlling the quantum states of light and their interference.

References

1. Franson J.D. *Phys. Rev. A*, **45**, 3126 (1992).
2. Boto A.N., Kok P., Abrams D.S., Braunstein S.L., Williams C.P., Dowling J.P. *Phys. Rev. Lett.*, **85**, 2733 (2000).
3. D'Angelo M., Chekhova M.V., Shih Y.H. *Phys. Rev. Lett.*, **87**, 013602 (2001)
4. Ostermeyer M., Puhlmann D., Korn D. *J. Opt. Soc. Am. B*, **26**, 2347 (2009).
5. Luo K.H., Wen J., Chen X.H., Liu Q., Xiao M., Wu L.A. *Phys. Rev. A*, **80**, 043820 (2009).
6. Tarn C.W. *Opt. Soc. Am. A*, **14**, 2231 (1997).
7. Rubin M.H., Klyshko D.N., Shih Y.H., Sergienko A.V. *Phys. Rev. A*, **50**, 5122 (1994).
8. Ou Z.Y., Wang L.J., Zou X.Y., Mandel L. *Phys. Rev. A*, **41**, 566 (1990).
9. Burlakov A.V., Klyshko D.N., Kulik S.P., Chekhova M.V. *Pis'ma Zh. Eksp. Teor. Fiz.*, **69**, 788 (1999) [*JETP Lett.*, **69**, 831 (1999)].
10. Katamadze K.G., Kulik S.P. *Zh. Eksp. Teor. Fiz.*, **139**, 26 (2011) [*JETP*, **112**, 20 (2011)].