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Amplified spontaneous emission spectrum at the output of a diode amplifier saturated by an input monochromatic wave

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Abstract. **Expressions for the amplitudes of amplified spontaneous emission waves in a diode amplifier near the frequency ω_0 of a 'strong' input monochromatic wave have been derived in terms of a random function of a stationary Gaussian process. We have found expressions for the spectral density of the amplitudes and shown that, on the red side of the spectrum with respect to frequency ω_0, spontaneous emission waves obtain additional nonlinear gain, induced by the strong wave, whereas on the blue side of the spectrum an additional loss is induced. Such behaviour of the amplitudes of amplified waves agrees with previous results.**

Keywords: *diode amplifier, amplified spontaneous emission.*

1. Introduction

Diode optical amplifiers are quantum electronic devices that have gained a good reputation in practice and whose potentialities are far from being exhausted. For example, a diode amplifier can be used as a key component in creating devices for optical communication systems, such as optical beam modulators with speeds in the gigahertz range [1, 2] and watt-level modulated beam power amplifiers [3, 4]. Moreover, capable of maintaining high output optical beam quality and coherence, diode amplifiers can be used as building blocks of a large, kilowatt-level laser system with coherent summation of a large number of optical beams. The purpose of this work is to analyse such promising high-power laser systems.

Even though previous results [3] make it possible to model such a high-power diode laser system, it is worth noting however that, in previous work, spontaneous emission – an intrinsic, irremovable source of inherent noise in amplifiers – was left out of consideration. Clearly, increasing the number of channels in such a system will reduce the fraction of the master oscillator power that arrives at each amplifier channel as an input signal, whereas the spontaneous emission intensity will persist at a constant level or even rise. Accordingly, the output signal-to-noise ratio, related to spontaneous emission, will decrease. The extent to which the decrease in this ratio is acceptable in a particular application of the system determines the possible increase in the number of amplifier channels and, hence, the maximum output power of the system.

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In the light of the above, it was of interest to analyse amplified spontaneous emission in an amplifier highly saturated by an input beam. Such an analysis is the main purpose of this work.

2. Physical model

Consider a diode amplifier based on a single-transverse-mode ridge-waveguide laser diode similar to those described e.g. in Refs [4, 5]. The key distinction of an amplifier from a laser is that the former has no cavity mirrors. In the case under consideration, input light is amplified in a single pass through an amplifier of length L along its optical axis z (Fig. 1). The optical waveguide of the amplifier diode supports only one waveguide mode with a strictly constant transverse amplitude distribution, $v(x,y)$. The amplitude of a monochromatic wave in the bulk of the amplifier can then be represented in the following form:

$$\mathcal{E}_0(\bar{r},t) = \frac{1}{2}v(x,y)E_0(z)\exp(\mathrm{i}k_0z - \mathrm{i}\omega_0t) + \text{c.c.}, \tag{1}$$

where $\bar{r} = x,y,z$; $v(x,y)$ is a complex function normalised to its value on the optical axis [$v(0,0) = 1$]; $E_0(z)$ is a slowly varying complex amplitude; $k_0 = (\omega_0/c)\mathrm{Re}\sqrt{\varepsilon} = \omega_0 n/c$ is the waveguide propagation constant (the real part of the complex propagation constant); c is the speed of light; n is the waveguide effective refractive index; and ε is the complex dielectric permittivity of the waveguide, which should be found together with $v(x,y)$ by solving the waveguide problem (like e.g. in Popovichev et al. [5]). As in Ref. [2], the permittivity is represented in the form

$$\varepsilon(\omega,N) = \varepsilon_0(\omega) + \mathrm{i}\frac{\alpha\varepsilon_0}{k_0} - \mathrm{i}\frac{\varepsilon_0}{k_0}\Gamma G(N)(1-\mathrm{i}R), \tag{2}$$

where $G(N) = \sigma(N - N_{\mathrm{tr}})$ is the gain coefficient in the active region; α is the background loss coefficient; ε_0 is the real part of permittivity at the inversion threshold; Γ is the optical confinement factor; R is the amplitude–phase coupling factor; σ is the differential gain (stimulated recombination cross section); $N(z)$ is the carrier concentration on the optical axis (at $x = y = 0$); and N_{tr} is the carrier concentration at the inversion threshold (transparency carrier concentration). We assume that it is the input wave of amplitude $E_0(z)$ which saturates the amplifier, so the carrier concentration $N(z)$ is only determined by pumping and the wave intensity $\backsim|E_0|^2$. The problem of finding $E_0(z)$ and $N(z)$ in the model under consideration was repeatedly solved in previous work (see e.g. Refs [2, 3, 6]). Here, we use known solutions, so in what follows these func-

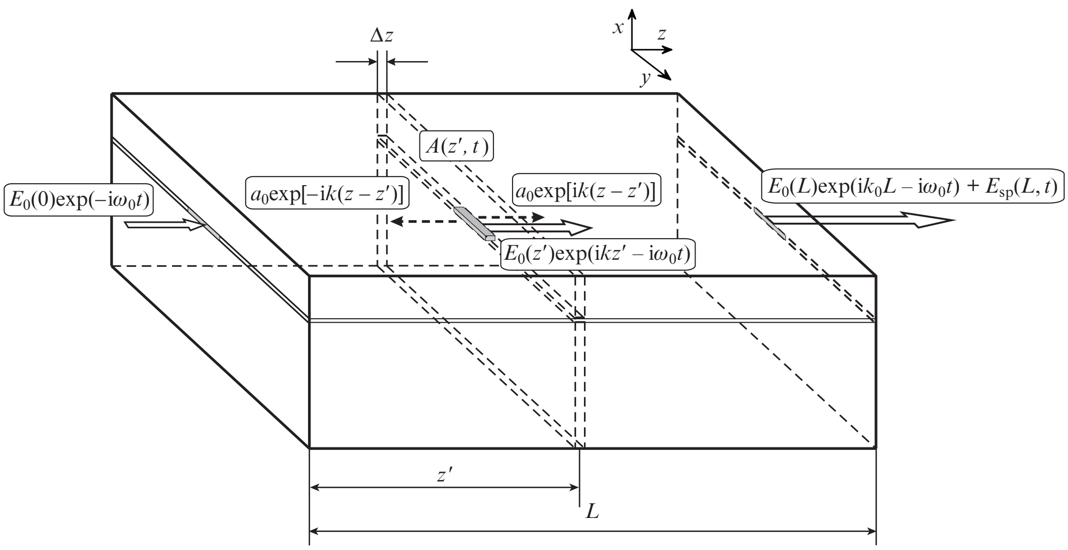

Figure 1. Schematic illustrating the physical model used in calculating amplified spontaneous emission in an amplifier saturated by monochromatic light.

tions are thought to be known. The corresponding formulas are given in the Appendix section.

Spontaneous emission will be represented by stochastic sources distributed along the length of the amplifier. Our purpose is to assess the contribution of the sources to the waveguide mode with a transverse amplitude distribution, $v(x,y)$, identical to that of the monochromatic field of a 'strong' wave.

To determine the contribution of the spontaneous emission sources, we consider them as dipoles located in a rather thin layer (of thickness Δz) of the active region, which is parallel to the (x,y) plane and contains the point with the coordinate z'. The layer can be thought of as physically thin if the following condition is fulfilled:

$$k_0\Delta z \leqslant 1. \quad (3)$$

The number of oscillating dipoles in such a layer, $\tilde{N}$, can be estimated in the following way:

$$\tilde{N} = \int N(\bar{r})\,\mathrm{d}x\mathrm{d}y\Delta z = N(z')d_{\mathrm{a}}W_N\Delta z, \quad (4)$$

where $N(\bar{r}) = N(z)f(x,y)$; $f(0,0) = 1$; $f(x,y)$ is the transverse carrier concentration distribution normalised to its value on the optical axis; d_{a} is the thickness of the quantum well active layer (the total thickness of the layers if there are several ones); and W_N is the effective width of the carrier concentration distribution in the active layer along the y axis. At N values above the transparency carrier concentration $N_{\mathrm{tr}} \approx 2 \times 10^{18}$ cm^{-3}, $d_{\mathrm{a}} = 7$ nm, $W_N = 5$ μm and a layer thickness $\Delta z \leqslant 20$ nm, meeting condition (3), we obtain $\tilde{N} \geqslant 3 \times 10^3$. This means that the wave amplitude corresponding to spontaneous emission from such a thin layer is the result of the summation of the amplitudes of more than 3×10^3 simultaneously emitting independent emitters. The emitters are independent because the electromagnetic field of each individual elementary emitter originates from the internal intrinsic motion of its charge. The motion is not correlated with that of the charge of the other emitters, so the emission field amplitudes of the individual emitters are not correlated with each other and, hence, the conditions of the de Moivre–Laplace theorem are satisfied with good accuracy [7]. Because of this, in what follows we use a normal distribution law for the amplitude of the spontaneous emission field propagating in a waveguide. The resultant amplitude of all waves emitted into a waveguide in opposite directions from a physically thin layer will be denoted as $A(z',t)$ (Fig. 1).

Since the waveguide under consideration is single-mode, the transverse distribution of these amplitudes is identical to the distribution for a strong wave being amplified, $v(x,y)$. For this reason, the function $v(x,y)$ will appear in all relations as the same factor, which will be omitted in most expressions for the amplitude of the total field. Using spectral decompositions of a random source function, $A(z',t)$ can be written in the form

$$A(z',t) = \frac{1}{2}\int_{-\infty}^{+\infty} a_0(z',\omega)\exp(-\mathrm{i}\omega t)\,\mathrm{d}\omega, \quad (5)$$

where $a_0(z',\omega)$ is the spectral amplitude of the sources. The well-known fundamental solution to the one-dimensional wave equation for a monochromatic point source at the $z = z'$ point has the following form:

$$\mathcal{E}^0_{\mathrm{sp}}(\omega,z,t) = \frac{1}{2}\{a_0(z',\omega)\exp(\mathrm{i}k|z - z'| - \mathrm{i}\omega t) + \mathrm{c.c.} \quad (6)$$

Here $k = \omega n_{\mathrm{gr}}/c$, where n_{gr} is the waveguide group index of refraction. This solution represents two waves propagating in opposite directions from the $z = z'$ plane. In what follows, we will be interested only in waves propagating in one (positive) direction, in which the main strong wave propagates at frequency ω_0. Given the above, the following equation is valid for the amplitude $a_0(z',\omega)$:

$$\langle a_0(z',\omega)a_0^*(z'',\omega')\rangle = \rho_0(z',\omega)\delta(\omega - \omega')\delta(z' - z''), \quad (7)$$

where the angle brackets denote the ensemble average, as is common in analysis of random processes. The function $\rho_0(z',\omega)$ (the spectral density of the random field amplitude of spontaneous emission, $\mathcal{E}^0_{\mathrm{sp}}$) is related to the power of this emission, $P_{\rm sp}^{0}$, from a layer of thickness Δz in one transverse mode per unit spectral interval $\delta\omega$ by

$$P_{\rm sp}^{0} = S\frac{cn}{8\pi}\rho_0(z',\omega)\frac{H(\omega)}{\Delta\omega}\delta\omega\Delta z = S\beta\frac{\hbar\omega}{\tau}N(z')\frac{H(\omega)}{\Delta\omega}\delta\omega\Delta z\,. \quad (8)$$

Here $S = \int|v(x,y)|^2{\rm d}x{\rm d}y$ is the effective cross-sectional area of the optical beam, with $v(0,0) = 1$;

$$\rho_0(z',\omega) = \frac{\hbar\omega}{\tau}\beta N(z')\frac{8\pi}{cn}H(\omega);$$

$$\beta = \frac{\pi\gamma\Gamma}{2n_{\rm a}nS}\left(\frac{c}{\omega_0}\right)^2 \quad (9)$$

is a dimensionless fraction of the power of emission from one elementary emitter into one waveguide mode;

$$\Gamma = \frac{\int f(x,y)\,|v(x,y)|^2{\rm d}x{\rm d}y}{S};$$

$H(\omega)$ is the form factor of the spectral line of spontaneous emission, normalised to unity at frequency ω_0: $H(\omega_0) = 1$;

$$\int_0^\infty H(\omega){\rm d}\omega = \Delta\omega;$$

$\Delta\omega$ is the effective spontaneous emission linewidth; τ is the spontaneous excited state lifetime; $n_{\rm a}$ is the refractive index of the active layer; and $\gamma \sim 1$ is a dimensionless parameter characterising dipole moment anisotropy along and across the quantum well active layer.

The above expression for the parameter β differs somewhat from analogous expressions reported by Petermann [8] and Newstein [9]. We believe that the expression for β used in this study is the most adequate to our model for a laser. Sequential vector analysis for β calculation which was used to obtain an approximate scalar expression for β in (9) will be presented in a subsequent communication [10].

In our model, the strong wave has no effect on the way in which spontaneous emission emerges in the layer with the coordinate z' but radically changes its gain during propagation through the active region $z' \leqslant z \leqslant L$. As shown earlier [11], travelling through the active region of the amplifier, $z' \leqslant z \leqslant L$, spontaneous emission waves generated in a layer of thickness Δz in the presence of a strong monochromatic wave will be involved in nonlinear interaction with it, thus experiencing further nonlinear gain on the red side of the spectrum and suppression on the blue side relative to the frequency of the strong wave. Calculation of the spontaneous emission spectrum at the amplifier output with allowance for such nonlinear interaction between waves is the main part of this study.

3. Calculation of the spectral amplitudes of coupled spontaneous emission waves at the amplifier output

To find the amplitude $\mathcal{E}_{\rm sp}(z,z',t)$ of a wave that was generated in the layer with the coordinate $z = z'$ and passed through the gain medium in the positive direction to a point at $z > z'$, we represent it in the following form with allowance for (6):

$$\mathcal{E}_{sp}(z,z',t) = \frac{1}{2}\Big\{\exp[-{\rm i}\omega_0 t + {\rm i}k_0(z-z')]$$
$$\times\int_{-\Omega_0}^{\Omega_0} a(z,z',\omega_0+\Omega)\exp[-{\rm i}\Omega t + {\rm i}q(z-z')]{\rm d}\Omega + {\rm c.c.}\Big\}, \quad (10)$$

where $q = \Omega n_{\rm gr}/c$. In (10), we take into account that the spectral density of spontaneous emission is limited to the spectral range $\omega_0 \pm \Omega_0$, where $\Omega_0 \ll \omega_0$. The wave vectors k_0 and q are expressed through the frequencies ω_0 and Ω. The spectral amplitude $a(z,z',\omega_0+\Omega)$ is the spontaneous emission field generated by the source $a_0(z',\omega_0+\Omega)$ and amplified over the length from z' to z.

Since nonlinear interaction couples only waves with the frequencies $\omega_1 = \omega_0 + \Omega$ and $\omega_{-1} = \omega_0 - \Omega$, which are located symmetrically with respect to the frequency of the strong wave, ω_0, we can separately analyse each pair of amplitudes, $a(z,z',\omega_0+\Omega)$ and $a(z,z',\omega_0-\Omega)$, and then integrate the result over all frequencies ω in the spectral band of spontaneous emission. Thus, along with the strong field amplitude $E_0(z')$, the spectral amplitudes of spontaneous emission at the $z = z' + 0$ point can be treated as components of the input signal $\mathcal{E}(z,z',t)$ of an amplifier of length $L' = L - z'$, as schematised in Fig. 1. The expression for $\mathcal{E}(z,z',t)$ as a combination of waves with frequencies ω_0 and $\omega_0 \pm \Omega$ at a point with $z > z'$ can then be represented in the following form:

$$\mathcal{E}(z,z',t) = \frac{1}{2}\{E_0(z)\exp({\rm i}k_0 z - {\rm i}\omega_0 t) + \exp[-{\rm i}\omega_0 t + {\rm i}k_0(z-z')]$$
$$\times\{a(z,z',\omega_0+\Omega)\exp[-{\rm i}\Omega t + {\rm i}q(z-z')]$$
$$+ a(z,z',\omega_0-\Omega)\exp[{\rm i}\Omega t - {\rm i}q(z-z')]\} + {\rm c.c.}\}$$
$$= \frac{1}{2}\{E_0(z)\exp({\rm i}k_0 z - {\rm i}\omega_0 t)[1 + V_1(z,z')\exp({\rm i}qz - {\rm i}\Omega t)$$
$$+ V_{-1}(z,z')\exp(-{\rm i}qz + {\rm i}\Omega t)] + {\rm c.c.}\}, \quad (11)$$

where

$$V_1(z,z') = \frac{a(z,z',\omega_0+\Omega)}{E_0(z)}\exp(-{\rm i}k_+ z'); \quad (12a)$$

$$V_{-1}(z,z') = \frac{a(z,z',\omega_0-\Omega)}{E_0(z)}\exp(-{\rm i}k_- z'); \quad (12b)$$

and $k_\pm = k_0 \pm q$. The last term in (11) is identical to the analogous expression for the amplitude of the sum field in Ref. [3] [Eqn (2)]. Using previous results [3] and knowing V_1 and V_{-1} at the $z = z'$ point, which are determined by (12a) and (12b), we can find their values at the amplifier output, at the $z = L$ point. As a result, we obtain

$$\mathcal{E}(L,t) = \frac{1}{2}\{[E_0(L)\exp({\rm i}k_0 L - {\rm i}\omega_0 t) + E_{\rm sp}(L,t)] + {\rm c.c.}\}, \quad (13)$$

$$E_{\rm sp}(L,t) = \int_{\omega_0-\Omega_0}^{\omega_0+\Omega_0} E_{\rm sp}(\omega)\exp(-{\rm i}\omega t + {\rm i}kL){\rm d}\omega$$
$$= \exp({\rm i}k_0 L - {\rm i}\omega_0 t)\tilde{E}_{\rm sp}(L,t). \quad (13a)$$

For convenience, the relation for the amplitude of a spontaneous emission wave, $E_{\rm sp}(L,t)$, in (13a) is expressed through the slowly varying envelope $\tilde{E}_{\rm sp}(L,t)$:

$$\tilde{E}_{\rm sp}(L,t) = E_0(L,t)\int_{-\Omega_0}^{\Omega_0} a(\omega_0+\Omega)\exp(-{\rm i}\Omega t)\,{\rm d}\Omega, \qquad (13b)$$

$$a(\omega_0+\Omega) = \exp({\rm i}qL)\int_0^L [F_1(L,z',\Omega)a_0(z',\omega_0+\Omega)$$

$$+F_2(L,z',\Omega)a_0^*(z',\omega_0-\Omega)]\,{\rm d}z', \qquad (14)$$

$$F_1(L,z',\Omega) = \frac{\exp(-{\rm i}k_+z')[K(L,z',\Omega)(1-{\rm i}R)+(1+{\rm i}R)]}{2E_0(z')}, \qquad (14a)$$

$$F_2(L,z',\Omega) = \frac{\exp({\rm i}k_-z')[K(L,z',\Omega)-1](1-{\rm i}R)}{2E_0^*(z')}. \qquad (14b)$$

The dimensionless complex function $K(L,z',\Omega)$ is defined in the Appendix section. The difference between this function and unity characterises the amplitude of inversion oscillations at the frequency Ω and, accordingly, the nonlinear interaction between waves. For $K \to 1$, inversion oscillations disappear and waves of different frequencies propagate without interaction and are amplified with a mean (static) saturated gain.

Note that, as distinct from that in Ref. [3], the final expression (13) for the output emission wave amplitude is written in integral form. The difference frequency Ω varies continuously, assuming both positive and negative values, which corresponds to a continuous spontaneous emission spectrum concentrated around the frequency ω_0. It can be seen from (14) that the spectral amplitude of the output spontaneous emission at the frequency $\omega_0+\Omega$ is contributed by spontaneous emission sources not only at the frequency $\omega_0+\Omega$ but also at $\omega_0-\Omega$. This is a fundamental result of nonlinear interaction between fields through the strong field at the frequency ω_0. All of the quantities in (13) and (14) can be found using data on the amplifier design and well-known parameters of the active layer material.

Expressions (13) and (14) for the complex random function $E_{\rm sp}(L,t)$ (spontaneous emission wave amplitude) allow the expression for its spectral density $X(\omega)$ to be written in the form

$$\langle E_{\rm sp}(L,\omega)E_{\rm sp}^*(L,\omega')\rangle = X(\omega)\delta(\omega-\omega'), \qquad (15)$$

$$X(\omega_0+\Omega) = |E_0(L)|^2\int_0^L [|F_1(L,z',\Omega)|^2\rho_0(z',\omega_0+\Omega)$$

$$+|F_2(L,z',\Omega)|^2\rho_0(z',\omega_0-\Omega)]\,{\rm d}z'. \qquad (16)$$

Accordingly, for the spectral distribution of the output power of spontaneous emission at the frequency $\omega_0+\Omega$ in the frequency range $\delta\omega$ we obtain

$$P(\omega_0+\Omega)\delta\omega = S\frac{cn}{8\pi}X(\omega_0+\Omega)\delta\omega$$

$$=\beta S\frac{\hbar\omega}{\tau}\frac{\delta\omega}{\Delta\omega}\int_0^L\frac{|E_0(L)|^2}{|E_0(z')|^2}N(z')[|F_1(L,z',\Omega)|^2H(\omega_0+\Omega)$$

$$+|F_2(L,z',\Omega)|^2H(\omega_0-\Omega)]\,{\rm d}z'. \qquad (17)$$

Equations (15)–(17) take into account relations (7) and (8) for $a_0(z',\omega)$ and $\rho_0(\omega)$. Thus, Eqn (17) completely determines the spectral distribution of the output power of spontaneous emission near the frequency ω_0.

4. Calculated spontaneous emission spectrum of the amplifier near the frequency ω_0 of the input monochromatic wave

In our calculations, we used amplifier parameters typical of single-transverse-mode ridge-waveguide laser diodes, which are well-known from the literature and are similar to those used in experimental work in Ref. [4]. The parameters are given in Table 1.

Table 1.

Parameter	Notation	Value
Spontaneous lifetime	τ/ns	1.0
Stimulated transition cross section	$\sigma/10^{-15}$ cm^2	1.9
Optical confinement factor	Γ (%)	2.0
Thickness of the active region	$d_{\rm a}$/nm	9.2
Ridge width	W_N/μm	5.0
Amplifier length	L/mm	4
Background loss coefficient of the waveguide	α/cm^{-1}	1–10
Transparency carrier concentration	$N_{\rm tr}/10^{18}$ cm^{-3}	2.0
Transparency current	$J_{\rm tr}$/mA	59
Saturation intensity	$I_{\rm s}/10^5$ W cm^{-2}	1.23
Effective cross section of the optical beam	$S/10^{-8}$ cm^2	2.3
Wavelength of monochromatic light	λ_0/nm	850
Amplitude–phase coupling factor of the amplifier	R	3
Refractive index of the active region	$n_{\rm a}$	3.6
Effective refractive index	n	3.4
Dipole moment anisotropy factor	γ	1.6

Figure 2 illustrates the evolution of the spontaneous emission spectrum at a constant monochromatic input signal amplitude as the pump current of the amplifier increases from values near the transparency threshold to those corresponding to an output power of ~1 W. As seen from Fig. 2, the spectral density of spontaneous emission increases with pump current, which is an obvious consequence of the increase in carrier concentration. The significant spectral distortion of the spectral density near the frequency ω_0, a less obvious circumstance, is a consequence of nonlinear interaction between the monochromatic wave and spontaneous emission. The interaction has an asymmetric character: the spectral density on the red side of the spectrum considerably exceeds that on the blue side. This feature was reported previously, e.g. by Shtaif et al. [12] and O'Duill et al. [13], and the mechanism of such interaction was analysed by Bogatov et al. [11].

The increase in spontaneous emission intensity on the red side of the spectrum with respect to ω_0, as well as the decrease on the blue side at considerable monochromatic wave intensi-

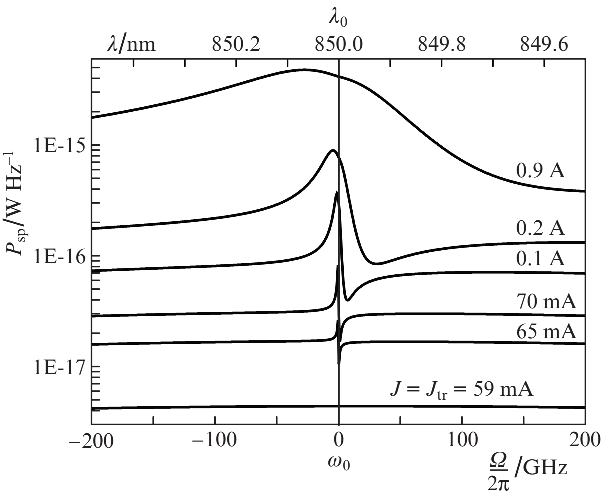

Figure 2. Effect of the pump current of the amplifier on the amplified spontaneous emission spectrum around the frequency of monochromatic light, ω_0. The input power of the monochromatic light is 0.28 mW ($0.1P_s$); $R = 3$; $\alpha = 1$ cm^{-1}. The minimum current (59 mA) corresponds to the transparency current J_{tr}, and the maximum current (0.9 A), to an output power of 1 W.

ties, are only due to an effect that was interpreted previously [11] as stimulated scattering by dynamic inversion (electron concentration) oscillations. Indeed, the additional nonlinear gain due to the strong field is the result of its scattering by dynamic refractive index oscillations. If the refractive index does not vary, i.e. is independent of carrier concentration N, the effect disappears. It is quantified by the amplitude–phase coupling factor R.

Figure 3 shows calculated amplified spontaneous emission spectra at different R values. It is seen that, at $R = 0$ (the refractive index is independent of electron concentration), the spectral density of spontaneous emission near the frequency of the strong wave decreases even in comparison with the case where there is no nonlinear interaction ($K = 1$). This means that, at $R = 0$, where inversion oscillations produce no refractive index oscillations, gain oscillations induce only an additional absorption for weak waves in the spectral range of the strong wave. As seen from the $R = 0$ curve, the induced absorption is spectrally symmetric with respect to the frequency of the strong wave.

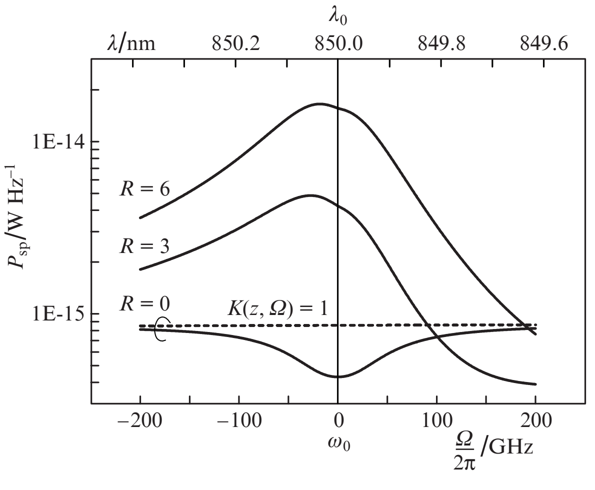

Figure 3. Effect of the amplitude–phase coupling factor R on the amplified spontaneous emission spectrum around the monochromatic light frequency. The dashed line represents the spectrum in the absence of mode interaction.

The shape of the amplified spontaneous emission spectrum is sensitive not only to the pump current (output power) of the amplifier (Fig. 2) and R (Fig. 3), but also to the loss α (Fig. 4). It can be seen that, at a low level of losses ($\alpha \leqslant 3$ cm^{-1}), the spectrum is asymmetric. As α increases to $\sim$10 cm^{-1}, the spectrum becomes more symmetric. High loss values (at a constant output power) correspond to a higher spectral density of spontaneous emission, which is due to the higher pump current needed for maintaining a constant output power.

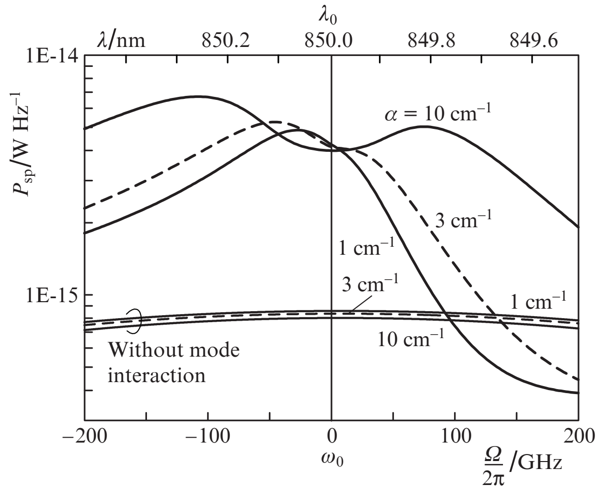

Figure 4. Effect of the loss coefficient of the amplifier, α, on the amplified spontaneous emission spectrum around the monochromatic light frequency. The input power of the monochromatic light is 0.28 mW ($0.1P_s$); $R = 3$.

Figure 5 illustrates the effect of the input power of monochromatic light on the spectral density of amplified spontaneous emission at a constant output power. Increasing the input power leads to a monotonic decrease in spontaneous emission power, as would be expected because of the monotonic decrease in inversion due to the increase in the degree of saturation of the amplifier.

5. Conclusions

We have obtained the analytical expression (17) for the amplified spontaneous emission amplitude at the output of an amplifier saturated by input monochromatic light. Written as a random function, the expression allows one to find the optical spectrum of amplified spontaneous emission, whose shape depends on monochromatic wave intensity. The dependence is the result of nonlinear interaction of the strong wave with spontaneous emission waves through inversion oscillations (carrier concentration oscillations in the case in question). As a result of this interaction, there is a tendency for waves to be amplified on the red side of the spectrum and suppressed on the blue side, in agreement with previous results (see e.g. Refs [11–13]). At a high level of losses, there may be deviations from this tendency, and the spectral density of sponta-

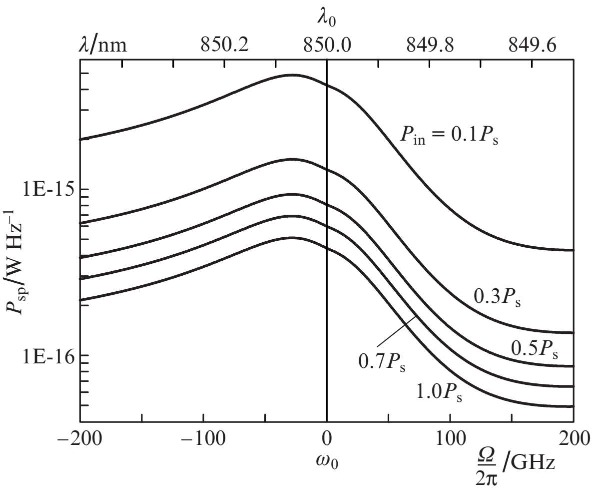

Figure 5. Effect of the power of input monochromatic light on the amplified spontaneous emission spectrum around the monochromatic light frequency at $R = 3$, $\alpha = 1$ cm^{-1} and $P_s = 2.8$ mW. The pump current for each curve corresponds to an output power of 1 W.

neous emission may be observed to rise in a considerable part of the spectrum on both sides with respect to the monochromatic wave frequency ω_0.

In conclusion, note that the final results of this study include only calculated spontaneous emission spectra, but the relation obtained for the amplitude of spontaneous emission waves [Eqn (13)] in principle offers the possibility of calculating amplifier output power and phase fluctuations due to the beating of the sum amplitude of the monochromatic wave and spontaneous emission waves in the close vicinity of the frequency of the monochromatic wave. Fluctuations in characteristics of the output beam of an amplifier will be calculated in our subsequent work.

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Appendix

According to relations (11) and (12) in Ref. [3], the function $K(L, z', \Omega)$ is an implicit function of the z' coordinate [through the dimensionless intensity $u(z')$] and frequency Ω:

$$K(L, z', \Omega) = \exp\left\{\frac{g+1}{g+1-\mathrm{i}\Omega\tau} \times \ln\left[\frac{(1-u(L)/g)(1+u(z')-\mathrm{i}\Omega\tau)}{(1-u(z')/g)(1+u(L)-\mathrm{i}\Omega\tau)}\right]\right\}, \quad \text{(A1)}$$

where

$$g = \frac{\Gamma\sigma\tau}{\alpha e d_a W_N L}(J - J_{tr}) - 1;$$

e is the electron charge; J is the pump current of the laser; and J_{tr} is the transparency current. The function

$$u(z) = \frac{cn}{8\pi I_s}|E_0(z)|^2$$

can be found by solving the transcendental equation

$$\frac{u(z)}{u_0}\left|\frac{g-u_0}{g-u(z)}\right|^{g+1} = \exp(g\alpha z), \quad \text{(A2)}$$

where $u_0 = P_{in}/P_s$ is the dimensionless input beam intensity in the amplifier; P_{in} is the input power of the amplifier; $P_s = I_s S$ is the saturation power of the amplifier; and $I_s = \hbar\omega_0/(\sigma\tau)$ is the saturation intensity. The field of the strong wave, $E_0(z)$, is given by $E_0(z) = |E_0(z)|\exp[\mathrm{i}\varphi(z)]$, where $\varphi(z) = \varphi(0) - (R/2) \times\{\alpha z + \ln[u(z)/u_0]\}$. The carrier concentration is

$$N(z) - N_{tr} = \left(\frac{J}{J_{tr}} - 1\right)\frac{N_{tr}}{1+u(z)},$$

where $N_{tr} = J_{tr}\tau/(d_a L W_N e)$. The other designations are explained in the main text.

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