

DOI: 10.1070/QEL16131

Intrinsic spontaneous emission-induced fluctuations of the output optical beam power and phase in a diode amplifier

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Abstract. **Output optical beam intensity and phase fluctuations are analysed in a classical approach to describing the propagation and amplification of spontaneous emission in the active region of a laser diode with a gain saturated by input monochromatic light. We find their spectral densities and dispersion and the correlation coefficient of the two-dimensional probability distribution function of the fluctuations.**

Keywords: *diode amplifier, optical beam intensity and phase fluctuations, amplified spontaneous emission.*

1. Introduction

As shown earlier [1, 2], a diode amplifier operating as a monochromatic signal power amplifier is capable of serving as a building block of a highly efficient, high-power laser system with coherent summation of beams. In a previous report [3], we calculated the amplitude of the spectral density of spontaneous emission, which is a source of 'natural' noise in such systems.

As a result of intrinsic spontaneous emission in an output beam of an amplifier, its total output power and phase randomly fluctuate because of the beating of the amplified input signal amplitude with random amplitudes of spontaneous emission components in the spectral vicinity of the input monochromatic light frequency ω_0. It might appear that the problem of these fluctuations can be obviated through the spectral filtering of the output beam of the amplifier with a sufficiently narrow-band optical filter tuned to the frequency ω_0. This is, however, not so. First, the input signal may be not ideally monochromatic and have some spectral width, caused e.g. by amplitude and phase modulation. Second, systems with coherent summation of beams have the advantageous possibility of 'fast' control over the spatial position of the output beam, and it is quite desirable to use it. To this end, 'fast' mutual phase tuning in each channel is necessary, which also requires a certain transmission band Ω_0. Finally, the operation of such a system even under steady-state conditions requires continuous phase self-tuning in the channels in order to compensate for the technical phase shift caused by changes in external conditions. The faster the technical phase shift and the larger the number of channels in a system, the broader the transmission band Ω_0 should be.

In the light of the above, the purpose of this work was to find the spectral density of intrinsic spontaneous emission-induced fluctuations in the output optical beam power and phase of an amplifier in a frequency band Ω_0 near the optical frequency ω_0. The data obtained here allow us to find parameters of the fluctuation probability distribution function and the correlation coefficient of the fluctuations.

Since in our case fluctuations originate from spontaneous emission and an expression for its spectral amplitude was obtained in our previous work [3], this work is essentially a continuation of that study. In connection with this, we use not only the physical model for an amplifier presented in Ref. [3] but also the same designations, expressions and quantities. Recall that numerical calculations refer to a typical single-transverse-mode ridge-waveguide diode amplifier operating at a wavelength of 850 nm.

2. Calculation of the spectral densities of output beam power and phase fluctuations

Figure 1 shows a block diagram of a coherent beam summation system that was used to analyse fluctuations. According to previous results [3], the expression for the electric field amplitude of an output emission wave can be written in the form (Eqn (13) in Ref. [3])

$$\mathcal{E}(L,t) = \frac{1}{2}\{[E_0(L)\exp(\mathrm{i}k_0L - \mathrm{i}\omega_0 t) + E_{\mathrm{sp}}(L,t)] + \mathrm{c.c.}, \quad (1)$$

where $E_0(L)$ is a determinate function corresponding to the amplified monochromatic wave; $E_{\mathrm{sp}}(L,t) = \exp(\mathrm{i}k_0L - \mathrm{i}\omega_0 t) \times\tilde{E}_{\mathrm{sp}}(L,t)$ is the random function corresponding to the amplitude of amplified spontaneous emission that passed through an optical filter with a transmission band from $\omega_0 + \Omega_0$ to $\omega_0 - \Omega_0$ (determined by expressions (13) and (14) in Ref. [3]). The slowly varying envelope of this amplitude is represented by the function $\tilde{E}_{\mathrm{sp}}(L,t)$.

The optical power at the amplifier output is given by

$$P(t) = S\frac{cn}{4\pi}\overline{\mathcal{E}^2(L,t)} = P_0 + \delta P(t), \quad (2)$$

where

$$P_0 = S\frac{cn}{8\pi}|E_0(L)|^2 \quad (3)$$

is the output power of an amplified monochromatic wave;

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Received 12 May 2016
Kvantovaya Elektronika **46** (8) 699 – 702 (2016)
Translated by O.M. Tsarev

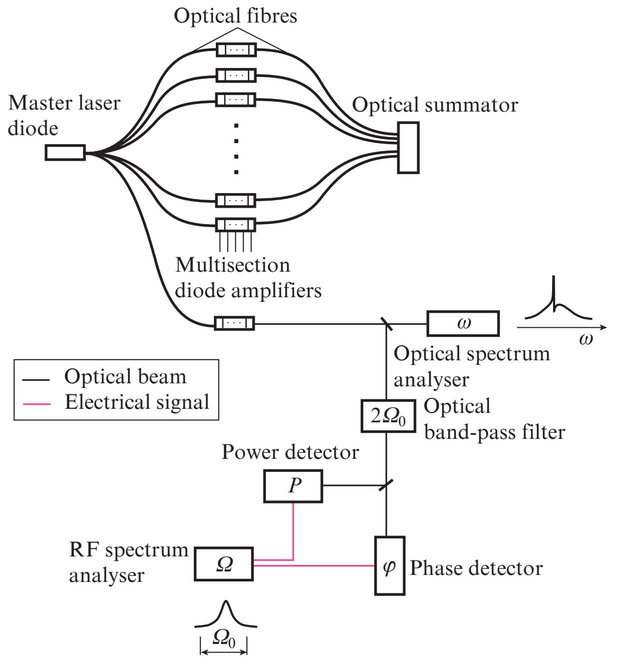

Figure 1. Block diagram of a coherent beam summation system that was used to analyse fluctuations.

$$\delta P(t) = S\frac{cn}{8\pi}[E_0^*(L)\tilde{E}_{\rm sp}(L,t) + {\rm c.c.}]$$

$$= \frac{1}{2}\int_{-\Omega_0}^{\Omega_0} p(\Omega)\exp(-{\rm i}\Omega t){\rm d}\Omega \quad (4)$$

is a random function corresponding to dynamic fluctuations in the output power of the optical beam; and

$$p(\Omega) = 2P_0[a(\omega_0+\Omega) + a^*(\omega_0-\Omega)]. \quad (5)$$

The Fourier amplitude $a(\omega)$ is determined by Eqn (14) in Ref. [3].

Therefore, the dispersion of output power fluctuations is

$$\sigma_P^2 = \langle \delta P^2(t)\rangle = \int_0^{\Omega_0} G(\Omega){\rm d}\Omega. \quad (6)$$

Here, $G(\Omega)$ is the spectral density of output power fluctuations (at positive frequencies), which meets the equation

$$\langle p(\Omega)p^*(\Omega')\rangle = 2G(\Omega)\delta(\Omega-\Omega'). \quad (7)$$

Using Eqn (5), we obtain

$$G(\Omega) = 2P_0 S\beta\frac{\hbar\omega}{\Delta\omega\tau}\int_0^L \frac{u(L)}{u(z')}N(z')$$

$$\times|K(L,z',\Omega)|^2[H(\omega_0+\Omega) + H(\omega_0-\Omega)]{\rm d}z'. \quad (8)$$

Representing the wave amplitude of the output beam in the form (1), we can also find the random function $\delta\varphi(t)$, which describes fluctuations in the phase of the wave:

$$\delta\varphi(t) = \frac{1}{2}\int_{-\Omega_0}^{\Omega_0}\varphi(\Omega)\exp(-{\rm i}\Omega t){\rm d}\Omega,$$

$$\delta\varphi(\Omega) = \frac{1}{\rm i}[a(\omega_0+\Omega) - a^*(\omega_0-\Omega)]. \quad (9)$$

Here and in what follows, $\delta\varphi(t)$ is taken to mean a random deviation of the output beam phase from its average, $\varphi_0 = \varphi(L)$, determined by relation (4) in Ref. [1]: $\delta\varphi(t) = \varphi(t) - \varphi_0$.

For the phase dispersion, we obtain

$$\sigma_\varphi^2 = \langle\delta\varphi(t)^2\rangle = \int_0^{\Omega_0} Q(\Omega){\rm d}\Omega, \quad (10)$$

where the spectral density of phase fluctuations, $Q(\Omega)$, meets a relation analogous to (7):

$$\langle\delta\varphi(\Omega)\delta\varphi^*(\Omega')\rangle = 2Q(\Omega)\delta(\Omega-\Omega'). \quad (11)$$

Using (9) and Eqns (14) from Ref. [3], we obtain

$$Q(\Omega) = S\beta\frac{\hbar\omega}{2P_0\Delta\omega\tau}\int_0^L\left|\frac{E_0(L)}{E_0(z')}\right|^2 N(z')$$

$$\times\{H(\omega_0+\Omega)\,|1+{\rm i}R[1-K(L,z',\Omega)]|^2$$

$$+H(\omega_0-\Omega)\,|1-{\rm i}R[1-K(L,z',\Omega)]|^2\}\,{\rm d}z'. \quad (12)$$

Since power and phase fluctuations emerge from emission of the same source, we can introduce their mutual correlation coefficient $\mu(\tau)$:

$$\mu(\tau) = \frac{\langle\delta P(t)\delta\varphi(t+\tau)\rangle}{\sigma_P\sigma_\varphi}. \quad (13)$$

Using the spectral decompositions of $\delta P(t)$ and $\varphi(t+\tau)$ in the form of Eqns (5) and (9), we obtain

$$\mu(\tau) = \int_0^{\Omega_0}{\rm Re}[U(\Omega)\exp({\rm i}\Omega\tau)]{\rm d}\Omega, \quad (14)$$

where the complex function has the following form:

$$U(\Omega) = \frac{1}{\sigma_P\sigma_\varphi}\frac{S\beta\hbar\omega}{\Delta\omega\tau}\int_0^L\left|\frac{E_0(L)}{E_0(z')}\right|^2 N(z')$$

$$\times\{H(\omega_0+\Omega)[RK(L,z',\Omega) - R|K(L,z',\Omega)|^2 + {\rm i}K(L,z',\Omega)]$$

$$+H(\omega_0-\Omega)[RK(L,z',\Omega) - R|K(L,z',\Omega)|^2$$

$$-{\rm i}K(L,z',\Omega)]\}\,dz'. \quad (15)$$

The quantities σ_P, σ_φ and $\mu(\tau)$ found above are parameters of a two-dimensional normal distribution function, $M[P(t),\varphi(t+\tau)]$, of the output power $P(t)$ and phase $\varphi(t+\tau)$. The probability that the output power $P(t)$ lies in the interval ${\rm d}P$ at time t and that the phase φ lies in the interval ${\rm d}\varphi$ at time $t+\tau$ is $M[P(t),\varphi(t+\tau)]$, where we have according to Ref. [4]

$$M[P(t),\varphi(t+\tau)] = \exp\Big\{-\frac{1}{2[1-\mu(\tau)]}$$

$$\times\Big[\frac{[P(t)-P_0]^2}{2\sigma_P^2} + \frac{[\varphi(t+\tau)-\varphi_0]^2}{2\sigma_\varphi^2} -$$

$$- \frac{2\mu(\tau)[P(t)-P_0][\varphi(t+\tau)-\varphi_0]}{\sigma_P\sigma_\varphi}\Bigg]\Bigg\}\left[2\pi\sigma_P\sigma_\varphi\sqrt{1-\mu(\tau)}\right]^{-1}. \quad (16)$$

3. Calculation results

Figure 2 shows spectral densities of output power fluctuations at different pump currents of the amplifier (at different output powers P_0) and a constant input power (P_{in} = 0.28 mW), which amounts to 0.1 of the saturation power ($P_s \simeq$ 2.8 mW). The lowermost curve corresponds to the inversion threshold current J_{tr}. It is seen that, at low currents (low output powers), the spectral density rises monotonically with a very small slope. Further increasing the output power of the amplifier is accompanied by distortion of the curves at low frequencies, where nonlinear interaction between optical waves is most efficient. In our calculations, the dynamic nonlinear interaction between optical harmonics is quantified by the variation of $K(L,z',\Omega)$ from unity. At $K(L,z',\Omega) = 1$, e.g. at high frequencies, where $\Omega\tau$ rises with no limit, there is no dynamic interaction and only steady-state gain saturation occurs. It is for this reason that the interaction between harmonics is most efficient at low frequencies.

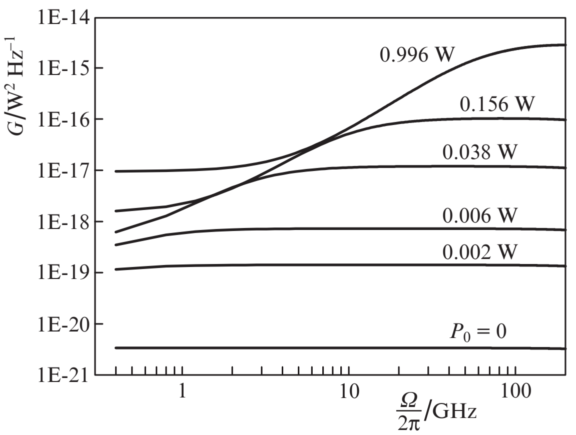

Figure 2. Spectral densities of output power fluctuations, $G(\Omega)$, at different pump currents of the amplifier, corresponding to $P_0 = 0-0.996$ W. For all the curves, P_{in} = 0.28 mW ($0.1P_s$).

Figure 3 illustrates the behaviour of the spectral density of phase fluctuations. As follows from these data, the behaviour is nonmonotonic at both low and high frequencies. It is worth noting that, in the limit of high frequencies, the spectral density of fluctuations drops. The reason for this is that, because of the inversion saturation, the amplitude of spontaneous emission waves grows more slowly or even stops growing, whereas the monochromatic wave amplitude rises linearly. Accordingly, their ratio, which determines phase fluctuations, drops. At low frequencies, phase fluctuations first increase with increasing power, which is a consequence of the transformation of amplitude fluctuations (see Ref. [1]), and then stop growing, and the spectral density drops to some level because the interaction saturates. At the same time, the output power rises, maintaining the level of phase fluctuations almost constant.

Figure 4 shows the rms power fluctuation σ_P as a function of the input signal level at a pump current of 0.9 A, corresponding to an output power of 1 W. The calculation was

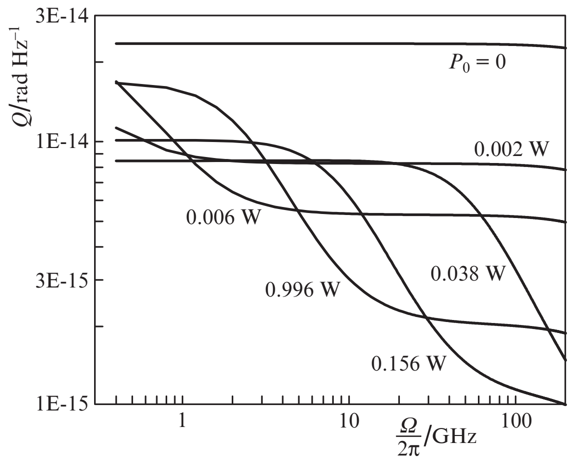

Figure 3. Spectral density of output beam phase fluctuations, $Q(\Omega)$, at the same parameters as in Fig. 2.

performed for boundary frequencies $\Omega_0/2\pi$ of 50 and 200 GHz. Figure 5 shows analogous curves for the rms phase fluctuation σ_φ. As would be expected, the curves are monotonic and give a quantitative idea of the fluctuating quantities. It is seen from comparison of the data in Figs 4 and 5 that, whereas the amplitude–phase coupling factor for power fluctuations (R from 0 to 6) is almost insignificant, phase fluctuations increase markedly with increasing R.

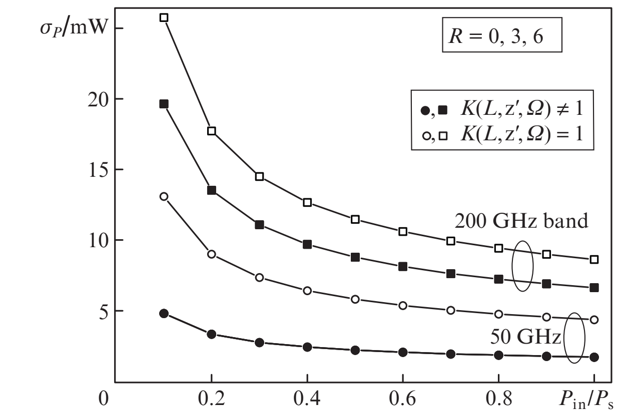

Figure 4. σ_P as a function of the input signal level at a pump current of 0.9 A, corresponding to P_0 = 1 W. The calculation was performed for boundary frequencies $\Omega_0/2\pi$ = 50 and 200 GHz and R = 0, 3 and 6 with $[K(L,z',\Omega) \neq 1]$ and without $[K(L,z',\Omega) \equiv 1]$ interaction between modes.

The correlation between optical beam power and phase fluctuations is characterised by the quantity $\mu(\tau)$, whose calculated values are presented in Fig. 6. It is seen that $\mu(\tau)$ has a maximum at certain positive values of τ, because the phase fluctuations slightly lag behind the output power fluctuations. The reason for this is that inversion (refractive index and phase) oscillations lag behind output power oscillations in the amplifier. The data in Fig. 6 demonstrate that the fluctuations correlate only when the coefficient R differs from zero, and this is an additional argument for R to be referred to as the amplitude–phase coupling factor. It also can be concluded from Fig. 6 that this correlation is only significant at a sufficiently high power, where nonlinear interaction between optical fields tells on the spontaneous emission gain.

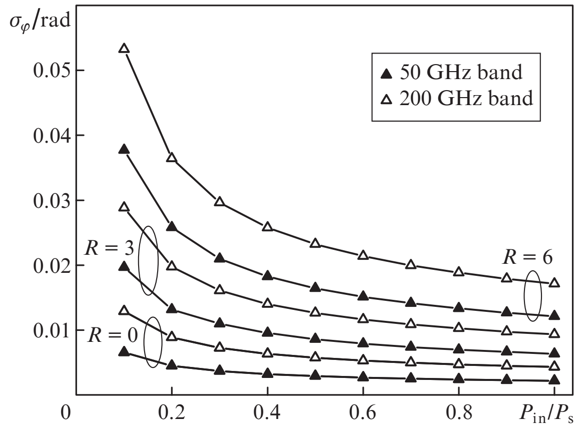

Figure 5. σ_φ as a function of the input signal level at the same parameters as in Fig. 4 and in the presence of interaction between modes $[K(L, z', \Omega) \neq 1]$.

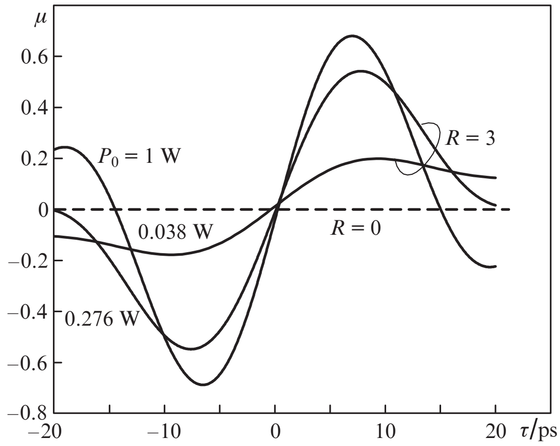

Figure 6. $\mu(\tau)$ curves at different P_0 (pump current) and R values; boundary frequency $\Omega_0/2\pi = 50$ GHz.

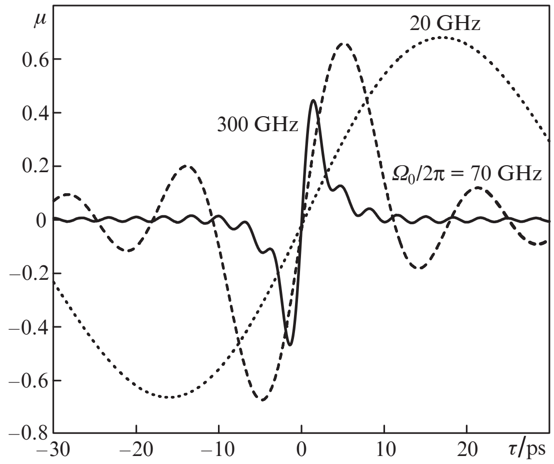

Figure 7. Mutual correlation function $\mu(\tau)$ for the output beam power and phase at different boundary frequencies $\Omega_0/2\pi$, $P_0 = 1$ W and $R = 3$.

Figure 7 illustrates how the transmission band of the optical filter (i.e. the spectral range of the spontaneous emission harmonics involved in beating with a monochromatic wave) influences the $\mu(\tau)$ curve. The oscillations in the curves are a consequence of the cutoff of the spectral density near frequencies of $\pm\Omega_0$.

As a result, according to (16) three parameters – σ_P, σ_φ and $\mu(\tau)$ – completely determine the probability distribution function $M[P(t), \varphi(t+\tau)]$.

Thus, in combination with previous results [3], the theory presented in this work allows one to assess spontaneous emission-related fluctuations in parameters of the output optical beam of a diode amplifier. Note that, in so doing, only those amplifier parameters are used that can be determined from independent measurements or are known with good accuracy in advance.

In combination with the average values of P_0 and $\varphi_0(L)$ presented previously [1, 3], the distribution function $M(P, \varphi)$ found above completely defines the statistical properties of a stationary random process with a normal distribution, including the dynamics of output optical beam intensity and phase fluctuations. This allows optical beam quality and error generation rate in a given laser system to be modelled in a rather broad range of its parameters. Even though all the calculations in this work were performed for one individual amplifier channel, it is obvious that they can readily be generalised for an entire large system. This is because fluctuations in each individual channel are statistically independent of those in the other channels, because they are related to independent spontaneous emission sources. Thus, starting from a required optical beam quality (or an acceptable error generation rate in communication systems), one can optimise the entire laser system, e.g. in terms of transmitted power or the acceptable number of beams for coherent summation.

In conclusion, note that the numerical calculations presented here were performed for a symmetric position of the transmission band of the optical filter with respect to the carrier frequency ω_0. In some particular cases, an asymmetric filter can be used, and the behaviour of the parameter σ_P, σ_φ or $\mu(\tau)$ may then differ somewhat from that described here, but the above theory allows one to readily take this into account, because all the relations involved were written an analytical form.

Acknowledgements. This work was supported by the RF Ministry of Education and Science (state research task No. 0023-2014-0171).

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