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Intramode wave packet in a thin left-handed film with the spectrum near the frequency of the zero group velocity

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Abstract. **We consider the frequency dependence of propagation constants (phase dispersion) and of the spatial distribution of the electromagnetic field (shape dispersion) of guided optical TE modes in a thin left-handed film. It is shown that the spatiotemporal transformation of narrow-band intramode wave packets with the spectrum adjacent to the frequency of the zero group velocity is caused by the dispersion of both types. The propagation velocity of the power carried by such wave packets is significantly lower than the group velocity of light in a bulk left-handed metamaterial.**

Keywords: *left-handed metamaterials, planar optical waveguides, wave packets.*

1. Introduction

The study of novel waveguide structures comprising left-handed metamaterials with simultaneously negative permittivity and permeability has revealed new regularities of propagation of guided modes in them, qualitatively different from those of mode propagation in waveguides based of usual right-handed materials with positive permittivity and permeability [1–21]. These structures extend the functional capabilities of metamaterial optics [22–27].

In dielectric waveguides, comprising left-handed metamaterials, the propagation is possible not only for the fast waveguide modes with the phase velocity greater than the velocity of plane electromagnetic waves in a bulk medium, but also for the slow waveguide modes with the phase velocity smaller than that in a bulk medium [1–12]. In these waveguides, the direction of the power transfer for some guided modes in the regions of left-handed materials is opposite to the direction of the power transfer for the same mode in the regions made of right-handed materials. Under certain conditions the total power, transferred by a guided mode through a certain cross section of the waveguide, turns into zero and, hence, its group velocity vanishes as well, i.e., the mode does not leave the location of its initial excitation [1].

The dispersion dependences of propagation constants for the guided modes of waveguides, comprising left-handed metamaterials, on different parameters are generally nonmonotonic [2, 4, 5, 7, 9, 11–17, 23], in contrast to the analogous dependences in common waveguides based on right-handed media (see, e.g., [27]). The point at the continuous branch of the dispersion curve, at which the group velocity of the mode is zero, divides this branch into two parts, one of them corresponding to unidirectional phase and group velocities and the other to oppositely directed ones.

The specific features of dispersion in planar and cylindrical waveguides based on left-handed metamaterials considered in Refs [2, 7, 8, 12, 14, 21] can lead to the appearance of new regularities in the spatiotemporal transformation of wave packets, propagating in such waveguide structures. In the present paper the propagation of a narrow-band intramode wave packet with the frequency spectrum adjacent to the frequency at which the group velocity of the TE mode turns into zero is considered in the case of a planar three-layer waveguide structure with the central film made of a left-handed metamaterial.

2. Dispersion of propagation constants for the guided modes

In Refs [3, 21] it was shown that the propagation constants β of the guided TE modes in thin films with the thickness h, including those based on left-handed media (Fig. 1), are real-valued roots of the dispersion equation

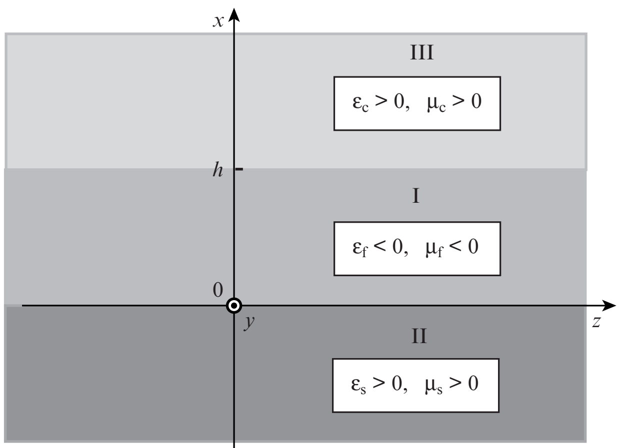

Figure 1. Planar waveguide structure based on the thin film (I) of a left-handed metamaterial and right-handed substrate (II) and covering medium (III) in the chosen coordinate system.

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$$\tanh \chi_f h + \frac{\chi_s \mu_f \chi_f^{-1} \mu_s^{-1} + \chi_c \mu_f \chi_f^{-1} \mu_c^{-1}}{1 + \chi_s \chi_c \mu_f^2 \chi_f^{-2} \mu_s^{-1} \mu_c^{-1}} = f(\beta, \omega) = 0, \qquad (1)$$

where ω is the frequency of the light field; $\chi_* = [\beta^2 - \varepsilon_* \mu_* (\omega/c)^2]^{1/2}$; μ_* and ε_* are the relative permeability and permittivity, respectively; c is the velocity of light in free space; and the symbol '$*$' in the subscript is used instead of the characters 'c' (covering medium), 'f' (film) and 's' (substrate). After the substitution $\mu_* \leftrightarrow \varepsilon_*$ in Eqn (1), the resulting relation will determine the propagation constants β for the TM modes.

It is well known (see, e.g., [27–30]) that the values of real propagation constants in planar waveguides made of common right-handed materials in which both the permittivity and the permeability are positive ($\varepsilon_{c,s} > 0$ and $\mu_{c,s} > 0$) are bounded by the condition $\max(k_c;\ k_s) \leqslant \beta \leqslant k_f$. Here $k_* = \sqrt{\varepsilon_* \mu_*}\,\omega/c$ are the wavenumbers for bulk materials of the waveguide layers. In this case the coefficient χ_f takes imaginary values. The phase velocity of the guided (fast) modes of such waveguides is greater than the phase velocity of plane waves in a bulk medium made of the same material as the film.

If the central film of the waveguide is made of a left-handed metamaterial (see Fig. 1), then the coefficient χ_f can take real values, too [1–11]. In this case, the slow modes can also propagate through the waveguide, for which the phase velocity is smaller than the phase velocity of plane electromagnetic waves in bulk media with material parameters, corresponding to those of the waveguide.

Following Refs [1, 2, 4, 5, 7, 8, 12, 14, 21, 26], we assume that the dispersion dependences of the permittivity (ε_f) and permeability (μ_f) of the bulk medium made of the left-handed metamaterial on the frequency ω of the electromagnetic filed are described by the relations

$$\varepsilon_f = 1 - \omega_p^2/\omega^2, \quad \mu_f = 1 - F\omega^2(\omega^2 - \omega_m^2)^{-1}, \qquad (2)$$

where ω_p is the plasma frequency; ω_m is the magnetic resonance frequency; and F is the filling factor of the metamaterial ($0 < F < 1$).

From the analysis of relations (2) it follows that under the condition $\omega_m < \omega_p < \omega_m/\sqrt{1-F}$ the interval of frequencies ω, in which both the permittivity and the permeability of such media are simultaneously negative, lies within the limits $\omega_m < \omega < \omega_p$. In the case $\omega_p > \omega_m/\sqrt{1-F}$, this interval has the limits from ω_m to $\omega_m/\sqrt{1-F}$ and is determined only by the region of negative permeability. The dispersion of the material parameters in a common right-handed material within the above intervals of the optical-range wavelength is negligibly small as compared to the dispersion of left-handed material and will not be taken into account below.

Figure 2 presents the dispersion dependences of the propagation constants $\beta(\omega)$ of the guided TE modes in the left-handed film having the thickness $h = 330$ nm in the case of air covering the medium ($\varepsilon_c = 1$ and $\mu_c = 1$) and nonmagnetic substrate ($\mu_s = 1$) with relative permittivity $\varepsilon_s = 2$, calculated at the parameter values $\omega_p = 3.46\times10^{15}$ rad s^{-1}, $\omega_m = 1.63\times10^{15}$ rad s^{-1} and $F = 0.5$, which agree by the order of magnitude with the data of Refs [20, 31]. In this case the frequency range in which the conditions $\varepsilon_f \leqslant 0$ and $\mu_f \leqslant 0$ simultaneously hold lies within the limits from 1.63×10^{15} rad s^{-1} (the corresponding wavelength of light being $\lambda = 1.16$ μm) to 2.31×10^{15} rad s^{-1} ($\lambda = 0.82$ μm). Note that the indices used here to specify the guided modes of the considered left-handed film are the same as for the guided modes of common right-

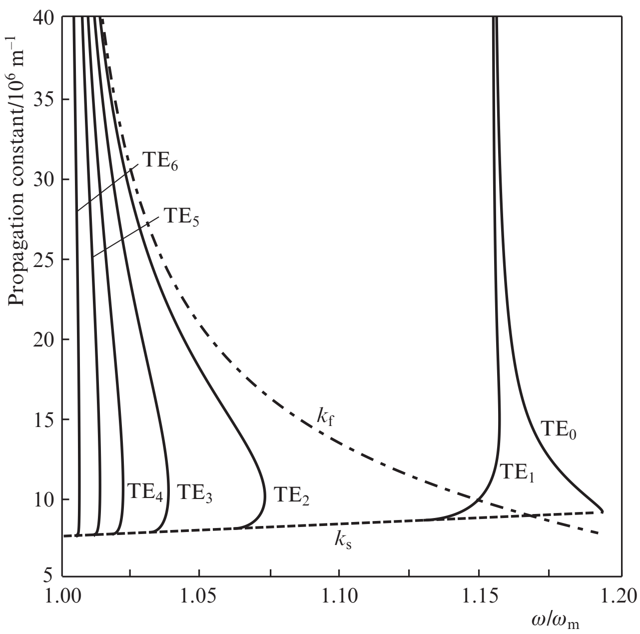

Figure 2. Dispersion dependences of the propagation constants for TE modes in the left-handed film having the thickness $h = 330$ nm in the case of air covering the medium and nonmagnetic substrate with the relative permittivity $\varepsilon_s = 2$; k_f and k_s are the dispersion dependences of the wavenumbers for bulk materials of the film and the substrate, respectively.

handed films, where the mode subscript corresponds to the number of zeros in the transverse distribution of the y component of the electric field strength of the mode [29].

The obtained dependences $\beta(\omega)$ for TE modes possess two singular points. The first of them is due to the resonance of the bulk material permeability $\mu_f(\omega)$ at the frequency ω_m. The second singularity is located at point $\lambda = 1$ μm, where in the considered case $\mu_f = -1$. Near this point the dispersion curves of the slow guided modes asymptotically approach the dispersion curves of the surface electromagnetic s-type waves [32, 33], for which the phase velocities turn into zero at equal permeabilities ($\mu_s^2 = \mu_f^2 = \mu_c^2$). Under the assumed conditions, this equality is valid at the wavelength of 1 μm.

From Fig. 2 it follows that the distinctive feature of the considered left-handed film is the existence of three frequency ranges, such that only one guided mode, TE$_0$, TE$_1$ or TE$_2$, can propagate in each of them. This single-mode regime in common planar thin-film waveguides made of right-handed materials is implemented only for the lowest-order mode [27–30].

For the modes TE$_0$ and TE$_1$ the dispersion dependences $\beta(\omega)$ lie not only in the region of fast modes ($k_s < \beta < k_f$), which is characteristic for usual films of right-handed materials, but also in the region of slow modes [($\beta > \max(k_f;\ k_s)$], which is absent in usual films [27–30]. For the mode TE$_1$ the dependence $\beta(\omega)$ is divided into two branches by the point, where the derivative is infinite, $d\beta/d\omega = \infty$, and the group velocity turns into zero, $v_g = d\omega/d\beta = 0$. The phase and the group velocity of the mode, corresponding to the lower branch of the dispersion dependence, have similar directions, while those corresponding to the upper branch have the opposite directions. The dispersion dependences $\beta(\omega)$ of the rest modes lie in the region of fast modes. However, in contrast to the common film waveguides made of right-handed materials, in which the dispersion dependences of modes are monotonic, the dependences $\beta(\omega)$ for the fast guided modes of the waveguides with the left-handed film are divided into two branches by the point, corresponding to the case $v_g = 0$ (similarly to the dependences for the TE_1 modes). In this case, with the growth of the mode number of the left-handed film the corresponding dispersion curves practically merge as they approach the region of singular permeability μ_f.

The appearance of two branches in the dispersion dependences of the propagation constants of guided modes in the left-handed film, corresponding to opposite directions of the mode group velocity, is due to the fact that the direction of power transfer by the light field in the film is opposite to that in the right-handed covering medium and substrate (see Refs [1–4, 8, 14]). In the general case, depending on the frequency ω of the mode light field, the direction of the total power transfer can coincide with the direction of the power transfer either in the covering medium and the substrate, or in the film. In the particular case of a certain frequency ω_0, the power, carried by the mode in the film, appears to be equal to that carried in the covering medium and the substrate, so that both the total transferred power and the corresponding group velocity turn into zero. In common right-handed films this effect is absent [27–30].

3. Intramode dispersion near the frequency of the zero group velocity

Let us derive the analytical relations describing the dispersion properties of the guided modes for the left-handed film in the vicinity of the frequency ω_0, at which the group velocity of the mode turns into zero ($v_g = d\omega/d\beta = 0$). The frequency ω_0 and the corresponding propagation constant β_0 can be determined as the roots of the dispersion equation (1), implicitly defining the function $\omega(\beta)$, and the equation $d\omega/d\beta = -(\partial f/\partial\beta)/(\partial f/\partial\omega) = 0$ [21]. Generally, the frequency ω_0 belongs not only to the domain of definition of the dispersion function $\beta(\omega)$ of the mode, for which at this point $v_g = 0$, but also to the domain of definition of $\beta(\omega)$ for other modes (Fig. 2). The dispersion properties of these modes near the frequency ω_0 can be described within the frameworks of the traditional dispersion theory, based on the expansion of the function $\beta(\omega)$ into a truncated series in integer powers of the frequency increment $\omega - \omega_0$ [29, 34, 35], and are not considered below.

From Fig. 2 it follows that for the considered case at the frequency ω near the point ω_0 ($\omega < \omega_0$) the existence of two guided modes is possible. One of them corresponds to the lower branch of the dependence $\beta(\omega)$, its phase and group velocities being unidirectional. The other mode corresponds to the upper branch of the dependence $\beta(\omega)$ and for it the directions of the phase and group velocity are opposite. The dependences $\beta(\omega)$ near the point ω_0 cannot be approximated by a truncated series expansion in integer powers of the frequency increment $\Delta\omega = \omega_0 - \omega$. In the considered case, they can be presented as a series in powers of the square root of the frequency increment ($\Delta\omega^{1/2}$). The dispersion dependences of the propagation constant β^+ (the mode with similar directions of the phase and group velocity) and the constant β^- (the mode with oppositely directed velocities) in the first approximation with respect to the quantity $\Delta\omega^{1/2}$ can be described by the expressions:

$$\beta^{\pm} = \beta_0 \mp \sqrt{\Delta\omega/a} = \beta_0(1 \mp \delta\beta), \tag{3}$$

where $\beta_0 = \beta(\omega_0)$; $a = -0.5(d^2\omega/d\beta^2)_{\omega=\omega_0}$; and $\delta\beta = \beta_0^{-1}\sqrt{\Delta\omega/a}$.

Using the results of Refs [27–30], the dependence of the y component of the electric filed strength vector E_* of the light field of the guided TE mode, propagating in the plane of the film along the z axis, on the coordinate x orthogonal to the film surface (see Fig. 1) can be presented (to a certain factor) in the form

$$E_c = (-1)^{\eta}(1 - \mu_f^2\chi_c^2\chi_f^{-2})^{-1/2}\exp[\chi_c(h - x)],\ x > h, \tag{4}$$

$$E_f = (1 - \mu_f^2\chi_s^2\chi_f^{-2})^{-1/2} \times [\cosh(\chi_f x) + \mu_f\chi_s\chi_f^{-1}\sinh(\chi_f x)],\ 0 \leqslant x \leqslant h, \tag{5}$$

$$E_s = (1 - \mu_f^2\chi_s^2\chi_f^{-2})^{-1/2}\exp(\chi_s x),\ x < 0, \tag{6}$$

where η is the mode index. Formulae (4)–(6) and the well-known relations, expressing the x and z components of the magnetic field strength of the TE mode (H_{x*} and H_{z*}) in terms of the y component of the electric field strength E_* [27–30], allow the general description of the dispersion dependence of the spatial field distribution (shape dispersion) for both the fast and the slow modes of the left-handed film.

In the first approximation with respect to the parameter $\delta\beta = \beta_0^{-1}\sqrt{\Delta\omega/a}$, the relation describing the shape dispersion of y and x components of the TE mode field with similar ('+') and opposite ('−') directions of the phase and group velocities can be derived in the form

$$E_*^{\pm} = (1 \mp \xi_*\beta_0^2\delta\beta)E_{*0},\ H_*^{\pm} = [1 \mp (1 + \xi_*)\beta_0^2\delta\beta]H_{*0}, \tag{7}$$

where

$$\xi_c(x) = \frac{h - x}{\chi_{c0}} + \frac{\mu_{f0}^2(\chi_{f0}^2 - \chi_{c0}^2)}{\chi_{f0}^2(\chi_{f0}^2 - \mu_{f0}^2\chi_{c0}^2)},\ x > h; \tag{8}$$

$$\xi_f(x) = \mu_{f0}\frac{\chi_{f0}^2 - \chi_{s0}^2}{\chi_{s0}\chi_{f0}^3}\frac{\sinh(\chi_{f0}x)}{\cosh(\chi_{f0}x) + \mu_{f0}\chi_{s0}\chi_{f0}^{-1}\sinh(\chi_{f0}x)} + \frac{x}{\chi_{f0}^2}\frac{d}{dx}\ln[\cosh(\chi_{f0}x) + \mu_{f0}\chi_{s0}\chi_{f0}^{-1}\sinh(\chi_{f0}x)] + \frac{\mu_{f0}^2(\chi_{f0}^2 - \chi_{s0}^2)}{\chi_{f0}^2(\chi_{f0}^2 - \mu_{f0}^2\chi_{s0}^2)},\ 0 \leqslant x \leqslant h; \tag{9}$$

$$\xi_s(x) = \frac{x}{\chi_{s0}} + \frac{\mu_{f0}^2(\chi_{f0}^2 - \chi_{s0}^2)}{\chi_{f0}^2(\chi_{f0}^2 - \mu_{f0}^2\chi_{s0}^2)},\ x < 0. \tag{10}$$

The subscript '0' means that the quantities are to be calculated at the frequency ω_0.

Expressions (3)–(10) allow the theoretical analysis of propagation of narrow-band intramode wave packets in the thin left-handed film with the frequency spectrum near ω_0.

4. Power carried by the intramode wave packet with the spectrum near ω_0

Consider the propagation of a transform-limited packet of guided TE modes in the thin left-handed film. Let us assume that the propagation constants for the modes of this packet belong to one dispersion curve, and their frequencies lie in the narrow band adjacent to the frequency ω_0 from below ($\Delta\Omega = \omega_0 - \omega_{\min} \ll \omega_0$, see Fig. 2). Generally, two different pairs of intramode packets correspond to such interval, in which the phase velocities are oppositely directed. In the modes of one packets from each pair (with the propagation constants β^+) the directions of phase and group velocities coincide, as in usual right-handed films [27–30], and in the modes of the other packet from each pair (with the propagation constants β^-) these velocities are oppositely directed, which is impossible in right-handed films.

In the first-order approximation with respect to the parameter $\delta\beta = \beta_0^{-1}\sqrt{\Delta\omega/a}$, the spatiotemporal dependences of the y components of the electric field strength vectors ($E_\nu^\pm$) and the x components of the magnetic field strength vectors ($H_\nu^\pm$) of the packets of guided TE modes of the first pair ($\nu = 1$, the phase velocity in the forward direction of the z axis) and the second one ($\nu = 2$, the phase velocity in the backward direction of the z axis) in a certain cross section of the waveguide, corresponding to the coordinate z, can be presented in the form:

$$E_{*\nu}^\pm = E_{*0}(x)\left[\psi_\nu^\pm(z,t) \mp \beta_0 \Delta\psi_\nu^\pm(z,t)\xi_*(x)\right]$$
$$\times \exp\{\mathrm{i}[\omega_0 t + (-1)^\nu \beta_0 z]\} + \text{c. c.}, \quad (11)$$

$$H_{*\nu}^\pm = (-1)^{\nu+1} H_{*0}(x)\{\psi_\nu^\pm(z,t) \mp \beta_0 \Delta\psi_\nu^\pm(z,t)[1 + \xi_*(x)]\}$$
$$\times \exp\{\mathrm{i}[\omega_0 t + (-1)^\nu \beta_0 z]\} + \text{c. c.}, \quad (12)$$

where

$$\psi_\nu^\pm(z,t) = \int_0^{\Delta\Omega} C_\nu^\pm \exp\{-\mathrm{i}[\Delta\omega t \pm (-1)^\nu \sqrt{\Delta\omega/a}z]\}\,\mathrm{d}\Delta\omega; \quad (13)$$

$$\Delta\psi_\nu^\pm(z,t) = \int_0^{\Delta\Omega} C_\nu^\pm \sqrt{\Delta\omega/a}$$
$$\times \exp\{-\mathrm{i}[\Delta\omega t \pm (-1)^\nu \sqrt{\Delta\omega/a}z]\}\,\mathrm{d}\Delta\omega; \quad (14)$$

and $C_\nu^\pm \equiv C_\nu^\pm(\Delta\omega)$ is the spectral density of modes, depending on the frequency detuning.

The fact that the group velocity of the mode at the frequency ω_0 ($v_\mathrm{g} = 0$) equals zero is a consequence of turning into zero of the power carried by the waveguide mode in the left-handed film ($S_0 = \int_{-\infty}^{\infty} E_0 H_0^* \mathrm{d}x = 0$, the superscript '*' means the complex conjugation) [28]. Hence, the power carried by the considered intramode wave packets can be written in the form

$$S_\nu^\pm(z,t) = \pm(-1)^{\nu+1}\frac{4\beta_0^2}{\mu_0\omega_0}$$
$$\times \int_{-\infty}^{\infty} \frac{\xi(x)}{\mu(x)}[E_0^\pm(x)]^2 \mathrm{d}x\, \mathrm{Re}\{\psi_\nu^\pm(z,t)[\Delta\psi_\nu^\pm(z,t)]^*\}, \quad (15)$$

where $\mu(x)$ and $\xi(x)$ are the dependences of the permeability and the parameter ξ on the coordinate x [see Fig. 1 and Eqns (8)–(10)].

The terms in Eqns (11), (12) and (15), proportional to the functions $\psi_\nu^\pm$, describe the influence of phase dispersion on the spatiotemporal transformation of wave packets in the process of their propagation along the film. The terms, proportional to the functions $\Delta\psi_\nu^\pm$, describe the mutual effect of both the phase and the shape dispersion on this transformation. Due to the condition $S_0 = \int_{-\infty}^{\infty} E_0 H_0^* \mathrm{d}x = 0$ valid in the considered case, the power transferred by the intramode wave packets has no components proportional to the squared modules of the functions $\psi_\nu^\pm$ that describe the phase dispersion effect solely. If the spectra of the intramode wave packets propagating in the left-handed film were far from the frequency of the zero group velocity of the mode, then the main contribution to the power carried by them would be provided by the components, subject to the effect of phase dispersion only and proportional to the squared modules of the functions $\psi_\nu^\pm$ [35]. Note, that in a right-handed film the power carried by the mode does not turn into zero at any of the frequencies, and the appropriate component of the transferred power of the mode is always nonzero in such a film and gives the main contribution to the power [27–30]. In the considered case of the left-handed film, the major contribution to the power carried by the intramode wave packet having the spectrum near the frequency ω_0 is given by the component that describes the effect of both the phase and the shape dispersion.

5. Spatiotemporal transformation of intramode wave packets and the velocity of their propagation

Consider the features of spatiotemporal transformation of the transferred power, general for all intramode wave packets in a thin left-handed film. For this purpose we assume their spectral densities to be similar, i.e., $C_\nu^\pm(\Delta\omega) = C(\Delta\omega)$. In this case from Eqn (15) it follows that the dependences of the power flows on the coordinate z and the time t satisfies the relation $S_2^+(z,t) = S_1^-(z,t) \equiv -S(-z,t)$ and $S_2^-(z,t) = S_1^+(z,t) \equiv S(z,t)$. Below we restrict ourselves to the analysis of a wave packet with the transferred power $S(z,t)$, the spectral density of modes being constant and equal to unity ($C(\Delta\omega) = 1$).

Under the assumed conditions the functions $\psi(z,t)$ and $\Delta\psi^*(z,t)$, for which the real part of the product describes the spatiotemporal transformation of the power $S(z,t)$, carried by the wave packet with the propagation constants β^+ [see Eqn (15)] can be expressed in the form:

$$\psi = \frac{\mathrm{i}}{t}\left[\mathrm{Ex}(z,t) - \frac{z}{2\sqrt{at}}\mathrm{Erf}_\Delta(z,t)\right], \quad (16)$$

$$\Delta\psi^* = \frac{-\mathrm{i}}{t\sqrt{at}}\left[\mathrm{Ex}^*(z,t)\left(\sqrt{\Delta\Omega t} + \frac{z}{2\sqrt{at}}\right)\right.$$
$$\left. - \frac{z}{2\sqrt{at}} - \left(\frac{\mathrm{i}}{2} + \frac{z^2}{4at}\right)\mathrm{Erf}_\Delta^*(z,t)\right], \quad (17)$$

where

$$\mathrm{Ex}(z,t) = \exp\left[\mathrm{i}\left(\sqrt{\frac{\Delta\Omega}{a}}z - \Delta\Omega t\right)\right] - 1; \quad (18)$$

$$\mathrm{Erf}_\Delta(z,t) = \frac{1+\mathrm{i}}{\sqrt{2}}\sqrt{\pi}\exp\left(\frac{\mathrm{i}z^2}{4at}\right)\times$$

$$\times\left\{\operatorname{erf}\left(\frac{1+\mathrm{i}}{\sqrt{2}}\frac{z}{2\sqrt{at}}\right)-\operatorname{erf}\left[\frac{1+\mathrm{i}}{\sqrt{2}}\left(\frac{z}{2\sqrt{at}}-\sqrt{\Delta\Omega t}\right)\right]\right\}; \quad (19)$$

and erf (ζ) is the error integral [36].

From Fig. 2 it follows that for the left-handed film considered above the small vicinity of the frequency $\omega_0 \approx 1.75\times 10^{15}$ rad s^{-1} ($\beta_0 \approx 1.02\times10^7$ m^{-1}) that belongs to the domain of definition of the dispersion dependence of the propagation constant of the guided TE_2 mode with the parameter $\alpha \approx$ 1.66 s cm^{-2} does not belong to the domains of definition of the dispersion dependences for other modes. For this mode the temporal dependence of power $s(t) = S(z,t)/\max[S(0,t)]$, normalised to the maximum of the flow at $z = 0$, transferred by the wave packet with the spectral width $\Delta\Omega = 100$ GHz in different cross sections $z =$ const is presented in Fig. 3. The multiple-lobe shape of this dependence, caused by the rectangular spectrum of the initial wave packet, is preserved in the course of its propagation. The effect of dispersion leads to the distortions that manifest themselves in equalising the maxima of the lobes in the dependence $s(t)$ in the process of the packet temporal broadening, its total energy being conserved in the absence of absorption. Visible dispersion-caused distortions of the packet manifest themselves even at small lengths z (tenths of a millimetre). The numerical analysis has shown that half the first lobe amplitude in the dependences $s(t)$ exceeds the maximum of the second lobe amplitude for $z < 0.8$ mm.

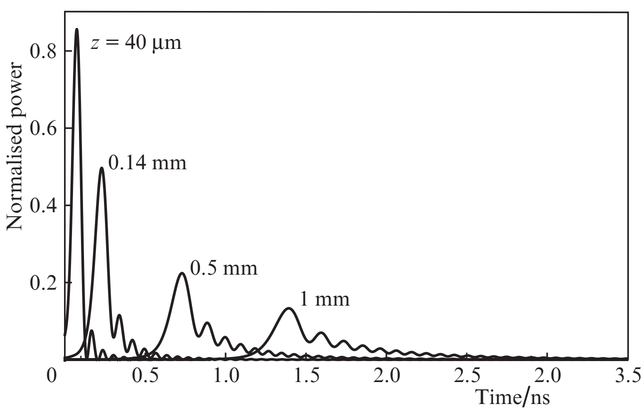

Figure 3. Time dependence of the power transferred by the wave packet of the TE_2 mode, normalised to the maximal value, in different cross sections z = const for the left-handed film and the initial rectangular spectrum of the packet with the width $\Delta\Omega$ = 100 GHz, adjacent from below to the frequency$\omega_0 \approx 1.75\times10^{15}$ rad s^{-1}.

Significant temporal broadening of the pulse with the half-maximum time width of the first lobe $2\pi\Delta\Omega^{-1} \approx 6.28$ ps at $z = 0$ and the width of the same lobe ~0.2 ns at $z = 1$ mm is not related to the contribution of material dispersion of the left-handed material. Indeed, the estimation of this contribution, implemented using the relations from Refs [27, 29, 43, 35], yields a negligibly small broadening $\sim 4\times10^{-15}$ s for the considered (small) distances, in spite of strong material dispersion in the film (the ratio of phase velocity to the group one at the frequency $\omega_0 \approx 1.75\times10^{15}$ rad s^{-1} for the bulk left-handed material is equal to ~6.3). In the left-handed film, the pulse broadening is due to the wave dispersion of phase and shape. The contributions of both kinds of dispersion are comparable with each other near the frequency ω_0, in contrast to the contributions at other frequencies, where the contribution of the shape dispersion into the analogous broadening is small as compared to the contribution of the phase dispersion. For guided modes in usual right-handed films the frequency, at which the group velocity turns into zero, does not exist, and, therefore, the influence of the shape dispersion on the propagation of wave packets in them is also negligibly small.

Under the strong distortions caused by the dispersion, the usual notion of the propagation velocity of a narrow-band wave packet as the group velocity (i.e., the velocity of the envelope moving with conserved shape) loses its meaning [37, 38]. In this case, a few approaches exist to define the velocity of wave packets, based on the necessity to use this notion for a particular application [39]. For example, the velocity of energy propagation, equal to the ratio of the power, carried by the wave packet in the direction of propagation, to the energy density [40–42] in the case of planar waveguide structures can be presented in the form

$$v_W(z,t) = \frac{S(z,t)}{W(z,t)}, \quad (20)$$

where $W(z,t)$ is the density of electromagnetic energy of the mode packet, averaged over the cross section of the waveguide structure [28, 42]. In the considered case, the quantity $v_W(z,t)$ carries the information about the instantaneous velocity of the packet energy at each point along the direction of its propagation. The value of this velocity, significant for technical applications, corresponds to the space points of the packet with large transferred power.

The numerical calculations carried out for the considered case have shown that in spite of the strong dispersion distortions of the wave packet, the maximum of the first lobe in the time dependence of the normalised power $s(t)$, carried by the wave packet, is larger than the maxima of other lobes in any cross section of the waveguide at the lengths up to a few metres. The instantaneous velocity v of this maximum at a certain cross section z can be calculated with necessary accuracy using the approximate relation $v \approx \Delta z/\Delta t_{\max}$, where Δz is the separation between two closely spaced cross sections; and $\Delta t_{\max}$ is the difference of times $t_{\max}$ that determine the position of the first maximum in the dependences $s(t)$, calculated at these two cross sections. The mean velocity of the first lobe maximum is determined by the relation $V = z/t_{\max}$.

Figure 4 presents the dependence of the instantaneous [$v(z)$, solid curve] and the mean [$V(z)$, dashed curve] velocity of the maximum of the power, transferred by the wave packet, at different cross sections z for the considered left-handed film. As follows from the figure, the instantaneous velocity grows nonmonotonically from $v_0 = 136\sqrt{a\Delta\Omega}/105 \approx 5.27\times 10^5$ m s^{-1} in the cross section $z \to 0$ to $v_\infty = 2\sqrt{a\Delta\Omega} \approx 8.14\times 10^5$ m s^{-1} in the cross section $z \to \infty$ [43]. The values of v_0 and v_∞ were obtained analytically by performing the appropriate passages to limits in Eqns (15)–(19). The length of the intervals within which the first lobe maximum of the time dependence of the transferred power is accelerated coincide by the order of magnitude with the lengths of the intervals, within which this maximum is decelerated. The total increase in the instantaneous velocity of the transferred power maximum $v(z)$ with the growth of the path length of its propagation z leads to the increase in its mean velocity $V(z)$, too. In this case, the condition $V(z) \leqslant v(z)$ holds, typical for the accelerated (including uniformly accelerated) motion of a material point.

The reason for the growing motion velocity of the transferred power $s(t)$ maximum with increasing z is the increase in the contribution to its formation from the spectral components of the intramode packet with the frequencies $\omega_0 - \Omega \leqslant \omega < \omega_0$, for which the absolute value of the group velocity grows with their frequencies shifted off the frequency ω_0. This contribution increases with the growth of z at the expense of the dispersion broadening of the packet.

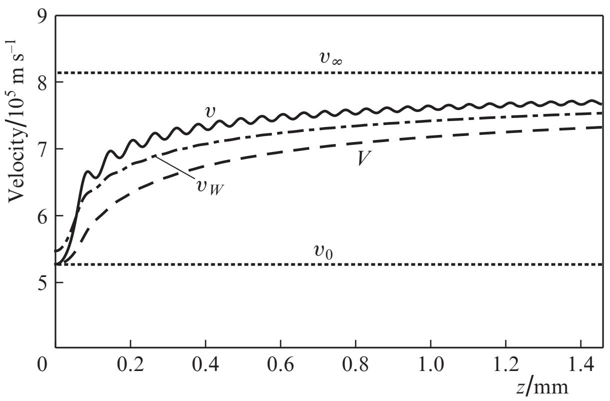

Figure 4. Dependences of the instantaneous (v) and mean (V) velocities of the maximum of the power transferred by the wave packet of the TE_2 mode in the left-handed film on the coordinate z along the direction of its propagation (v_0 and v_∞ are the limiting velocities); v_W is the propagation velocity of the packet energy at point z at the time moments, corresponding to the arrival of the maximum of the ransferred power at this point.

The results of calculating the velocity of the energy propagation in the considered wave packet $v_W(z, t_{max})$ using the general relation (20) at the time moments t_{max}, corresponding to the arrival of the first lobe maximum in the temporal dependence of the transferred power $s(t)$ at the point z, are presented in Fig. 4 by the dashed-dotted line. It is seen that the spatial dependence of the velocity $v_W(z, t_{max})$ is similar in its character to the spatial dependence of the instantaneous velocity $v(z)$ and the dependence of the mean velocity $V(z)$.

Note that in the unlimited (bulk) weakly absorbing left-handed medium the group velocity of the light waves does not turn into zero at any of the frequencies in the range, where both its permittivity and its permeability are simultaneously negative. In Ref. [35] it is shown that inside this range the frequency of zero group velocity dispersion can exist, at which a narrow-band light pulse in a left-handed bulk material is practically free of dispersion-caused distortions. For the medium parameters used above in the numerical calculations, this frequency is approximately equal to 2.15×10^{15} rad s^{-1} and lies far beyond the limits of the considered range. In this case, at the frequency $\omega_0 \approx 1.75\times10^{15}$ rad s^{-1} the group velocity in the bulk medium amounts to $\sim4\times10^7$ m s^{-1}. This is more than by 50 times greater than the velocity of the power transfer by the considered intramode wave packet, equal to $\sim7\times10^5$ m s^{-1} for its spectral width $\Delta\Omega$ = 100 GHz. The reduction of $\Delta\Omega$ leads to the decrease in this velocity in the left-handed film. At $\Delta\Omega$ = 1 GHz the power transfer velocity will be already by 500 times smaller than the group velocity in the bulk material. In the limit case $\Delta\Omega \to 0$ the considered velocity of wave packet propagation in the film also tends to zero, since in this limit case the wave packet in the film degenerates into a single guided mode with zero group velocity at the frequency ω_0.

From Fig. 2 it follows that for the accepted conditions only the guided mode TE_0 has the group velocity, different from zero in the entire frequency range considered. Therefore, the intramode packets with the spectra, adjacent from below to the frequencies of zero group velocities, can be formed by other modes, too, and not only by the TE_2 mode. The propagation character of these packets will be analogous to that of the mode formed by the TE_2 mode analysed above. The deceleration of propagation of intramode wave packages, formed by these modes, relative to the wave packets, propagating in the bulk left-handed medium, will be expressed weaker, since the parameter a for them is considerably smaller than for the TE_2 mode ($a \approx 0.46$ s m^{-2} for the mode TE_1 and $a \approx 0.03$ s m^{-2} for the mode TE_6).

6. Conclusions

Thus, it is shown that the dispersion properties of the guided optical TE modes in a thin left-handed film are qualitatively different from those in common right-handed films. Under the assumed conditions, the first-order mode at any frequency is slow, the second-order mode can be either fast or slow depending on the frequency, and the rest modes are fast. The single-mode regime can be implemented for three modes of the lowest order. Generally, the group velocity of the mode can coincide with the phase velocity, be oppositely directed to it, or turn into zero. Only the slow guided mode TE_0 has the group velocity different from zero in the entire frequency range considered.

In the left-handed film at a single frequency near the frequency ω_0 of a certain guided optical mode in the general case four guided modes can propagate having the propagation constants belonging to one dispersion branch. In two of these modes, the directions of phase and group velocities are similar, as in common right-handed films. Two other modes have the oppositely directed phase and group velocities, which is impossible in right-handed films. The spatiotemporal transformation of the transferred power envelope for the modes of narrow-band wave packets with similar initial spectrum, adjacent from below to the frequency ω_0, satisfy simple symmetric relations and up to a sign can be described by a single function. The considerable temporal broadening of the power flow in such wave packet that manifests itself already at the distances $\sim$1 mm is caused by the waveguide dispersion, to which the dispersion of both the phase and the shape of the mode contribute. The velocity of the power maximum propagation in such wave packet is proportional to the square root of its spectral width and for $\Delta\Omega$ = 100 GHz can be reduced by two orders of magnitude as compared to the group velocity of light in the bulk left-handed material.

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