

LASER GYROSCOPES

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Frequency response of a gas ring laser with a variable-sign frequency bias in the case of frequency nonreciprocity comparable with the bias amplitude

V.N. Gorshkov, M.E. Grushin, E.G. Lariontsev, I.I. Saveliev, N.I. Khokhlov

Abstract. **The dependence of the beat frequency of counterpropagating waves on the rotation rate is studied theoretically and experimentally in a gas ring laser (GRL) with a Zeeman effect-based, variable-sign frequency bias. Use is made of magneto-optical biases of two types, namely, a rectangular (meander) bias and a combined bias consisting of fast and slow meanders. At high rotation rates, when the frequency nonreciprocity determined by the rotation rate is close in magnitude to the amplitude of the variable-sign frequency bias, the dynamic locking bands take maximum values and there arise the most extensive deviations of the frequency response from the ideal response. Comparison of experimental and theoretical results for the widths of dynamic bands that appear in this region of the measured rotation rates shows good agreement between theory and experiment. The results suggest that the frequency response of a GRL in this region can be described by one differential equation for the phase difference of counterpropagating waves.**

Keywords: *ring laser, Zeeman effect, laser gyro, magneto-optic bias, dynamic locking bands, frequency response.*

1. Introduction

The investigation of gas ring lasers (GRLs) with a magneto-optical control of frequency nonreciprocity of counterpropagating waves is important for practical applications in laser gyroscopy. The scientific value of these studies is explained by the possibility of obtaining detailed information about nonreciprocal effects arising from the interaction of counterpropagating wave fields with a laser medium [1, 2]. In GRLs, a variable-sign frequency bias is used to move from a dead zone (a static locking band of the counterpropagating wave frequencies) [3, 4]. For lasers with a magneto-optical nonreciprocity control based on Faraday and Zeeman effects [1, 2], the form of the bias is typically chosen close to rectangular.

Previous studies (see, e.g., [5–7]) showed that in GRLs with a variable-sign frequency bias, instead of a single static locking band (frequency locking of counterpropagating waves) there appears a set of dynamic bands. They turn to be the most wide when the frequency nonreciprocity measured in the GRL and determined by the rotation rate is close to the amplitude of the variable-sign frequency bias [2, 4, 5]. Under the same condition there arise the most extensive deviations of the frequency response of a GRL from the ideal response, which have not yet been investigated. Therefore, this paper is devoted to the theoretical and experimental study of these deviations for a GRL with a magneto-optical control of frequency nonreciprocity.

V.N. Gorshkov, M.E. Grushin, I.I. Saveliev, N.I. Khokhlov OJSC ‘M.F. Stel’makh Polyus Research Institute’, ul. Vvedenskogo 3, Bld. 1, 107342 Moscow, Russia; e-mail: vov4ik818kva@mail.ru, mihail.grushin1968@gmail.com;
E.G. Lariontsev D.V. Skobel’tsyn Institute of Nuclear Physics, M.V. Lomonosov Moscow State University, Vorob’evy gory 1, Bld. 2, 119234 Moscow, Russia; e-mail: e.lariontsev@yahoo.com

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2. Theoretical analysis

Using the results of previous studies [3, 8, 9] and eliminating the competition of the counterpropagating waves and ensuring the conditions that provide stable bidirectional lasing in the GRL, the frequency response and dynamic locking band can be approximately investigated on the basis of the differential equation for the phase difference Φ of counterpropagating waves:

$$\dot{\Phi} = \Omega + \Omega_{\rm B}(t) + \Omega_{\rm L}\sin\Phi, \qquad (1)$$

where $\Omega = K\dot{\vartheta}$ is the frequency nonreciprocity of a resonator, which is proportional to the angular velocity of the GRL rotation $\dot{\vartheta}$; K is the scale factor; and $\Omega_{\rm B}(t)$ is the variable-sign frequency bias.

As shown in [8, 9], the coefficient $\Omega_{\rm L}$ in equation (1) depends both on the amplitude of the frequency bias and on the measured frequency nonreciprocity Ω. This can be explained as follows. The multiplier $\Omega_{\rm L}$ accounts for the effect of complex coupling coefficients of counterpropagating waves through the backscatter $m_{1,2}$ on the rate of change in the phase difference between the waves, Φ. The coefficients $m_{1,2}$, included in the initial system of equations for the amplitudes and phases of the fields of counterpropagating waves, affect Φ both directly and indirectly, through the modulation of wave intensities $I_{1,2}$. When the additional phase modulation associated with the modulation of intensities $I_{1,2}$ is taken into account, there appears a dependence of the multiplier $\Omega_{\rm L}$ on the amplitude of the frequency bias and Ω. In the absence of the frequency bias, the multiplier $\Omega_{\rm L}$ is equal to the half-width of the static locking band.

In this paper, we study the frequency response of a GRL with a bias of two types: with a rectangular variable-sign bias and with a combined bias consisting of fast and slow meanders.

2.1. Rectangular frequency bias

The expression for the rectangular frequency bias can be written in the form:

$$\Omega_{\rm B}(t) = \begin{cases} \Omega_{\rm p} \text{ at } 0 < t < T_{\rm p}/2, \\ -\Omega_{\rm p} \text{ at } T_{\rm p}/2 < t < T_{\rm p}, \end{cases} \quad (2)$$

where $\Omega_{\rm p}$ and $T_{\rm p}$ are the bias amplitude and period.

A GRL with a periodic variable-sign bias exhibits parametric locking of the beat frequencies $\dot{\Phi}$, leading to the emergence of dynamic locking bands. Within these bands the beat frequencies are constant and determined by the formula

$$\dot{\Phi} = 2\pi n/T_{\rm p}, \quad (3)$$

where $n = 0, 1, 2, 3, ...$ is the sequence number of the band.

The dynamic band widths Γ_n in the case of a rectangular bias are given by the approximate expressions (see, e.g., [5]):

$$\Gamma_n = 2\Omega_{\rm L}\frac{\gamma}{\gamma - n}\frac{\sin[(\pi/2)(\gamma + n)]}{(\pi/2)(\gamma + n)}, \quad (4)$$

where $\gamma = T_{\rm p}\sqrt{\Omega_{\rm p}^2 - \Omega_{\rm L}^2}/2\pi$.

At $n = 0$ expression (4) yields the formula for the zero locking band width, obtained in [3]. Equation (4) is approximate, and the greatest deviations from the exact values take place at band widths Γ_n close to the maximum (at $|\gamma - n| \approx 1$), when the measured frequency nonreciprocity $\Omega = K\dot{\vartheta}$ is close to the amplitude of the frequency bias $\Omega_{\rm p}$.

In this paper, the frequency response of the GRL (dependence of the beat frequency of the counterpropagating waves $f_{\rm b}$ on the measured frequency nonreciprocity $\Omega/2\pi$) was calculated by solving numerically the differential equation (1) by the Euler method.

In the presence of a frequency bias, the beat frequency contains a variable-sign component modulated at a frequency $1/T_{\rm p}$. To eliminate these pulsations, we have calculated the average beat frequency. At the given values of the parameters entering into (1), the numerical solution of this equation makes it possible to calculate the change in the phase difference Φ of the counterpropagating waves for a time equal to an integer $n_{\rm p}$ of the bias periods $T_{\rm p}$. The average value of the beat frequency is

$$f_{\rm b} = \langle\dot{\Phi}\rangle/2\pi = \int_{t_0}^{t_0 + n_{\rm p}T_{\rm p}} \dot{\Phi}(t)\,{\rm d}t/(2\pi n_{\rm p}T_{\rm p}). \quad (5)$$

In the calculations presented below, the average value of $f_{\rm b}$ was calculated at $n_{\rm p} = 20$. Figure 1 demonstrates the normalised frequency response $f_{\rm b}T_{\rm p}$ of the GRL with a variable-sign rectangular frequency bias, calculated at $\Omega_{\rm L}/2\pi$ = 300 Hz, the bias amplitude of $\Omega_{\rm p}/2\pi$ = 56888 Hz and the bias period of $T_{\rm p}$ = 0.004 s [curve (*1*)]. This response is shown in a narrow range of Ω values, close to the bias amplitude. As can be seen from Figure 1, the frequency response exhibits broad dynamic locking bands, which follow one another at intervals of 250 Hz (with a frequency of the magnetic field switching, $1/T_{\rm p}$). For the widest dynamic bands, n = 227, 228 and 229.

2.2. Combined frequency bias

In solving numerically equation (1) we also investigated the frequency response of the GRL in the case of the combined frequency bias. The Zeeman effect-based, combined, variable-sign frequency bias consists of main and additional components. The main component is a variable-sign rectangular bias with an amplitude $\Omega_{\rm p}/2\pi$ = 56 888 Hz and a period $T_{\rm p}$ = 0.004 s. The additional variable-sign bias (a slow meander) has a much smaller amplitude ($\Omega_{\rm p1}/2\pi$ = 112 Hz) and a substantially larger switching period ($T_{\rm p1}$ = 4 s).

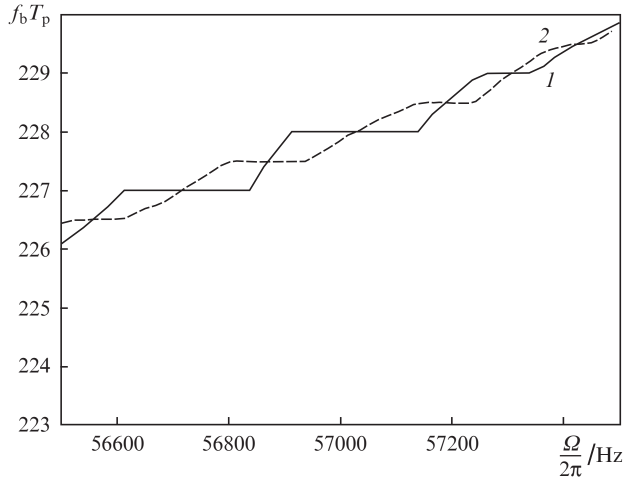

Figure 1. Normalised frequency response (dependence of $f_{\rm b}T_{\rm p}$ on the measured frequency nonreciprocity $\Omega/2\pi$) for (*1*) rectangular and (*2*) combined frequency biases.

In the case of a combined frequency bias, calculations are very cumbersome because of the computational burden. In this paper we propose an approach to repeatedly reduce the computation time. Because the period of a slow meander is considerably larger (by 1000 times) than the period of the main component, we will approximately assume that the frequency response of the Zeeman laser gyroscope (ZLG) consists of two parts. The first part ($f_{\rm b1}$) corresponds to one half-period of the slow meander, i.e., the frequency bias consists of a fast meander and a constant frequency bias equal to the amplitude of the slow meander (112 Hz). The second part ($f_{\rm b2}$) corresponds to the other half-period of the slow meander, and in this case, the frequency bias consists of a fast meander and a constant frequency bias (–112 Hz).

Using the calculated values of $f_{\rm b1}$ and $f_{\rm b2}$, the average value of the beat frequency $f_{\rm b}$ in the GRL with a combined frequency bias will have the form:

$$f_{\rm b} = (f_{\rm b1} + f_{\rm b2})/2. \quad (6)$$

Figure 1 shows the normalised frequency response [calculated using formula (6)] of the GRL with a combined frequency bias [curve (*2*)]. Comparing curves (*1*) and (*2*), corresponding to a rectangular bias and a combined bias, note the following differences between them. In the case of a rectangular bias, the normalised beat frequency, in accordance with the theory [see Eqn (3)], takes on integer values within the dynamic locking band (for a band with the index n, this value is equal to n). In the case of a combined bias, the values of the normalised frequency $f_{\rm b}T_{\rm p}$ within the respective locking bands are half-integral, and the dynamic locking bands turn out more narrow.

2.3. Widths of the dynamic locking bands

For the GRL with a rectangular frequency bias, Fig. 2 shows the widths of the dynamic locking bands Γ_n as functions of the band number n, calculated on the basis of the numerical solution of equation (1) and formula (4).

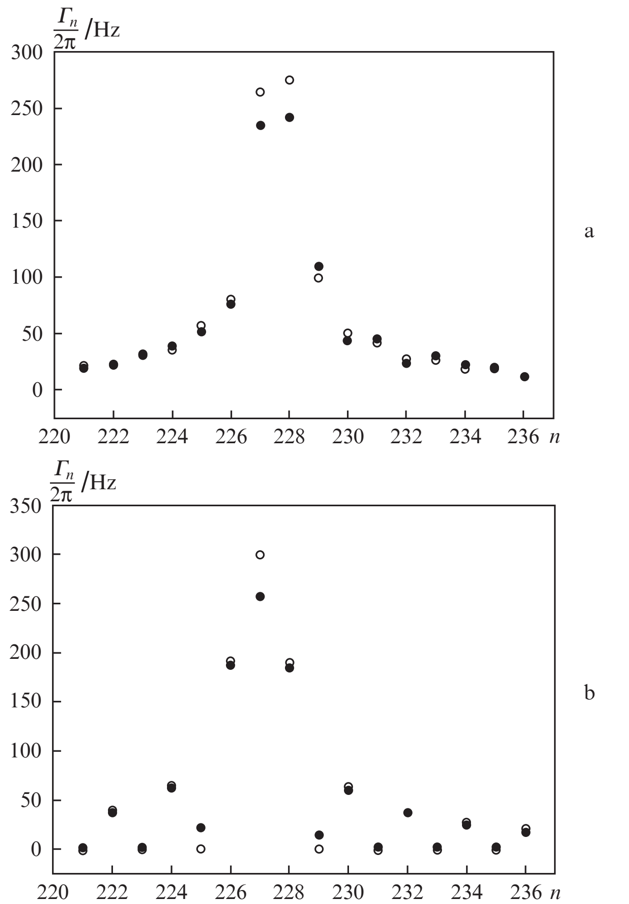

Figure 2. Dependences of the widths Γ_n of dynamic locking bands on the band number n at $\Omega_L/2\pi$ = 300 Hz and $\Omega_p/2\pi$ = (a) 56 888 and (b) 56750 Hz. Filled and empty points show the values of Γ_n, calculated by solving numerically equation (1) and by formula (4), respectively.

One can see that the approximate analytical dependence that determines the widths of the dynamic bands [formula (4)], is in fairly good agreement with the results obtained by solving numerically equation (1). This conclusion is valid even for the widest dynamic bands which are observed at $\Omega = K\dot{\vartheta}$, close to the amplitude of the frequency bias Ω_p.

At different amplitudes of the rectangular variable-sign bias, one can observe a monotonic decrease in the locking band widths Γ_n with a change in the band index n with the distance from the widest band Γ_{max} (Fig. 2a), as well as their nonmonotonic decrease (Fig. 2b).

3. Comparison with the experiment

Experimental studies of the frequency response and dynamic locking band widths were performed using a ZLG [2]. In a ring He–Ne laser with a nonplanar resonator, whose active medium was subjected to the longitudinal magnetic field, lasing occurred in the regime of a single longitudinal mode at the $3s^2 - 2p^4$ transition of neon with a wavelength of 633 nm. Due to circular anisotropy of the nonplanar ring resonator, the counterpropagating waves were circularly polarised.

3.1. Frequency response

In processing the beat signal of the counterpropagating waves, we excluded the variable-sign part modulated by the frequencies $1/T_p$ and $1/T_{p1}$ and measured only the average beat frequency f_b of the counterpropagating waves, arising due to rotation. The ZLG normalised frequency response (dependence of f_bT_p on the rotation rate $\dot{\vartheta} = \Omega/K$), measured on the ZLG with a combined frequency bias, is shown by points in Fig. 3.

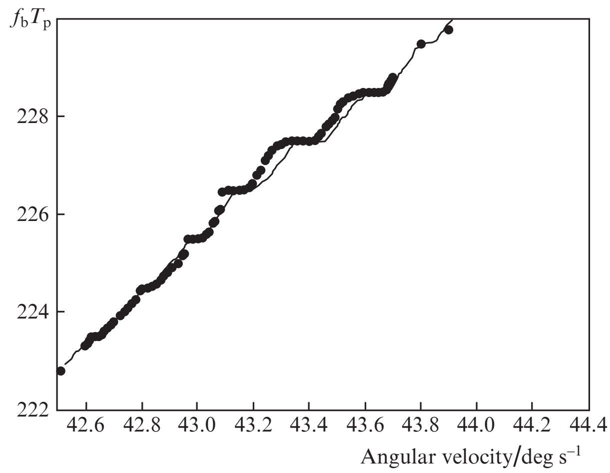

Figure 3. Experimentally measured (points) and calculated [based on equation (1)] (solid curve) normalised frequency responses of the ZLG; $\Omega_p/2\pi$ = 56 888 Hz, T_p = 0.004 s, $\Omega_{p1}/2\pi$ = 112 Hz, T_{p1} = 4.

The plot of the frequency response in Fig. 3 corresponds to the rotation rates at which the measured frequency nonreciprocity $\Omega = K\dot{\vartheta}$ is close to the amplitude of the main components of the bias. The frequency response exhibits broad dynamic locking bands following one another at intervals of 250 Hz (with a frequency of the magnetic field switching, $1/T_p$).

To compare the experiment with the theory, the frequency response was also investigated using the numerical solution of equation (1). The solid curve in Fig. 3 shows the frequency response of the ZLG in the case of a combined bias, the parameters of which are equal to the corresponding values of the combined bias in the experiment. The only unknown parameter in equation (1) is Ω_L. The values of this parameter and its dependence on the measured frequency nonreciprocity Ω can, in principle, be calculated theoretically, considering the system of equations for the amplitudes and phases of counterpropagating waves in a Zeeman ring laser and formulas for the parameter Ω_L obtained in [9]. However, this calculation proved to be very cumbersome and was not performed in this work; the value of the parameter Ω_L in numerical calculations was varied. The best fit of the experimentally measured and calculated dependences (Fig. 3) was obtained at $\Omega_L/2\pi$ = 300 Hz. One can see that these frequency responses are close enough to each other. This concerns the dynamic band widths and their positions on the frequency response.

3.2. Dynamic locking bands

We also performed special experiments to investigate the dependence of the dynamic band widths Γ_n on their number n for the given amplitude of the frequency bias Ω_p. Studies were carried out on a bench imitating the sensor rotation. In these experiments, the frequency nonreciprocity Ω in the ZLG was obtained not by rotation, but by an alternating current supplied to one of two coils of the nonreciprocal device. The rectangular variable-sign bias $\Omega_p(t)$ was produced by the constant current, which was fed into the second coil.

At a fixed amplitude of the bias there are many dynamic bands with different indices n. In the experiments, we measured only the band widths $\Gamma_n/2\pi$, which occur in the vicinity of the frequency nonreciprocity values Ω, close in magnitude to the amplitude of the frequency bias Ω_p. This region of values is characterised by the widest dynamic bands. The dependence of the width $\Gamma_n/2\pi$ of the dynamic bands on their number n, measured on the simulation bench, is shown in Fig. 4. The number n of the band was determined from the measured (average for the bias period T_p) beat frequency of the counterpropagating waves, $f_b = \Omega/2\pi$, inside this band:

$$n = f_b T_p. \quad (7)$$

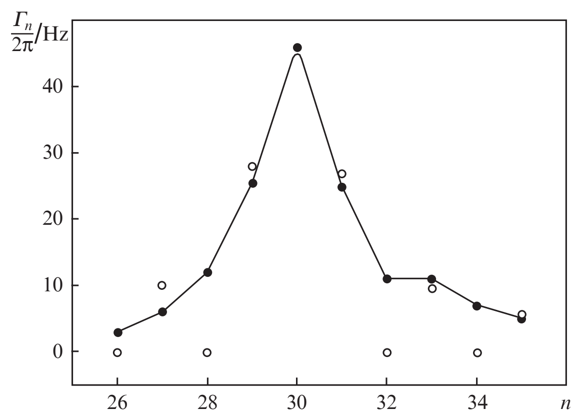

Figure 4. Experimental (●) and calculated using formula (4) (○) dependences of the widths $\Gamma_n/2\pi$ of dynamic locking bands on the band number n in the case of a rectangular variable-sign bias.

For comparison with the experiment, the dynamic locking band widths $\Gamma_n/2\pi$ were calculated using formula (4). In calculations, the amplitude of the frequency bias was assumed equal to the experimentally measured value $\Omega_p/2\pi = 7500$ Hz at the bias period $T_p = 0.004$ s. The coefficient Ω_L was the only parameter that was varied in the calculations. The best fit of the experimental results with the calculated dependence $\Gamma_n/2\pi\ (n)$ was obtained at $\Omega_L/2\pi = 42$ Hz.

Comparison of the theoretical and experimental dependences shows their good quantitative agreement for some dynamic locking bands. However, there is a discrepancy: in numerical calculations, the even numbered bands, $n = 26, 28, 32, 34$, are absent (have a zero width), while the widths of the experimentally observed bands with these numbers are nonzero.

In measurements on the simulation bench, the amplitudes of the frequency biases are approximately seven times less than those in normal operation of the ZLG (see Fig. 3). As mentioned above, the value of the parameter Ω_L depends on the bias amplitude. In this connection, we have obtained different values of the parameter Ω_L: in normal operation, when the amplitude of the variable-sign frequency bias is $\Omega_p/2\pi = 56888$ Hz, we have $\Omega_L/2\pi = 300$ Hz, and in the experiment on the simulation bench this parameter is less by 6.7 times.

4. Conclusions

The study performed in this paper shows that the frequency response of the GRL with a Zeeman effect-based, variable-sign bias can be analysed with good accuracy, using only equation (1) for the phase difference of the counterpropagating waves. In this description, one should take into account the fact that the multiplier Ω_L, which determines the effect of the coupling coefficients $m_{1,2}$ of the counterpropagating waves, depends both on the amplitude of the variable-sign frequency bias Ω_p and on the measured frequency nonreciprocity Ω.

We have found that for a rectangular frequency bias the widths of the dynamic locking bands in the region of high rotation rates, when the measured frequency nonreciprocity is close to the amplitude of the frequency bias, is described well by the approximate formula (4), although some discrepancies are observed at the boundary of the band nonlinearity.

The frequency responses of the GRL have been investigated for two types of a variable-sign magneto-optical bias: for a rectangular bias (meander) and a combined bias consisting of fast and slow meanders. It is found that for the values of the measured frequency nonreciprocity Ω, comparable to the bias amplitude, the dynamic locking bands are narrower than those in the case of a combined bias.

The studies performed in this paper can be used for correcting the frequency response of the GRL in the region of the measured rotation rates that are close to the amplitude of the frequency bias.

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