

# Threshold characteristics of a semiconductor quantum-well laser: inclusion of global electroneutrality in the structure

Z.N. Sokolova, N.A. Pikhtin, I.S. Tarasov, L.V. Asryan

**Abstract.** Threshold characteristics of a semiconductor quantum-well (QW) laser are calculated using the global electroneutrality condition, which includes charge carriers both in the active and waveguide regions and thus presents an equality of the total charge of electrons to the total charge of holes in these two regions. It is shown that at the lasing threshold, the densities of electrons in the QW and the waveguide region are not equal to the densities of holes in these regions, i.e., the local electroneutrality condition is violated in each of the regions. Depending on the velocities of the carrier capture from the waveguide region into the QW, the electron density can be either higher or lower than the hole density (both in the QW and in the waveguide region). The charge of the carriers of each sign in the waveguide region is shown to be greater than that in the QW.

**Keywords:** semiconductor lasers, quantum wells, electroneutrality condition.

## 1. Introduction

In modern semiconductor laser structures, stimulated emission is generated in a low-dimensional active region surrounded by a wideband bulk waveguide region (optical confinement layer, OCL) [1, 2]. In such structures, electrons and holes are first injected from emitters into the OCL, and then captured by the active region [3]. For a high output power to be obtained, the threshold current of the laser structure should be low, and therefore, threshold densities of charge carriers in the active region and the OCL should be low, too.

In calculating threshold characteristics of heterostructure lasers, it is generally assumed that densities of electrons and holes in the active region are equal [4], i.e. there is a local electroneutrality. In reality, however, due to differences between electron and hole parameters (primarily between the parameters that control the capture in a nanosized active region and the position of sub-bands or size quantization levels therein), the carrier densities in the active region may be different, which means violation of the local electroneutrality condition.

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Received 15 July 2016  
Kvantovaya Elektronika 46 (9) 777–781 (2016)  
Translated by I.A. Uliitkin

In our paper [5] we calculated threshold characteristics of a Fabry–Perot cavity planar injection laser at different ratios between the densities of electrons and holes in a quantum well (QW).

In this paper, threshold characteristics of a semiconductor QW laser are calculated using a global electroneutrality condition, which includes charge carriers both in the active region and the OCL. This condition presents an equality of the total charge of electrons to the total charge of holes in these two regions. It is shown that at the lasing threshold, the densities of electrons in the QW and in the OCL are not equal to the densities of holes in them. Depending on the velocity of carrier capture from the OCL into the QW, the electron density can be either higher or lower than the hole density (both in the QW and OCL).

## 2. Theoretical model

To calculate the characteristics of the laser, we used five steady-state rate equations [6]:

for free electrons in the bulk OCL of thickness  $b$  [ $b(\partial n^{\text{OCL}}/\partial t) = 0$ ]

$$\begin{aligned} \frac{j}{e} + N_{\text{QW}} \frac{n^{\text{QW}}}{\tau_{\text{nesc}}} - N_{\text{QW}} v_{\text{ncapt}0} (1 - f_n) n^{\text{OCL}} \\ - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}} = 0; \end{aligned} \quad (1)$$

for free holes in the OCL [ $b(\partial p^{\text{OCL}}/\partial t) = 0$ ]

$$\begin{aligned} \frac{j}{e} + N_{\text{QW}} \frac{p^{\text{QW}}}{\tau_{\text{pesc}}} - N_{\text{QW}} v_{\text{pcapt}0} (1 - f_p) p^{\text{OCL}} \\ - b B_{3\text{D}} n^{\text{OCL}} p^{\text{OCL}} = 0; \end{aligned} \quad (2)$$

for electrons confined in the QW ( $\partial n^{\text{QW}}/\partial t = 0$ )

$$\begin{aligned} v_{\text{ncapt}0} (1 - f_n) n^{\text{OCL}} - \frac{n^{\text{QW}}}{\tau_{\text{nesc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} \\ - v_{\text{gr}} g^{\text{max}} (f_n + f_p - 1) \frac{N}{S} = 0; \end{aligned} \quad (3)$$

for holes confined in the QW ( $\partial p^{\text{QW}}/\partial t = 0$ )

$$v_{\text{pcapt}0} (1 - f_p) p^{\text{OCL}} - \frac{p^{\text{QW}}}{\tau_{\text{pesc}}} - B_{2\text{D}} n^{\text{QW}} p^{\text{QW}} -$$

$$-v_{\text{gr}}g^{\text{max}}(f_n + f_p - 1)\frac{N}{S} = 0; \quad (4)$$

and for photons in the lasing mode ( $\partial N/\partial t = 0$ )

$$v_{\text{gr}}N_{\text{QW}}g^{\text{max}}(f_n + f_p - 1)N - v_{\text{gr}}(\beta + \alpha_{\text{int}})N = 0. \quad (5)$$

Here,  $n^{\text{OCL}}$  and  $p^{\text{OCL}}$  are the densities of free electrons and holes in the OCL;  $n^{\text{QW}}$  and  $p^{\text{QW}}$  are the two-dimensional densities of electrons and holes confined in the QW;  $N$  is the number of stimulated-emission photons; and  $f_n$  and  $f_p$  are the filling factors (occupancies) of the states corresponding to the lower edge of the electron quantum-confinement sub-band and to the upper edge of the hole quantum-confinement sub-band in the QW.

The occupancies  $f_n$  and  $f_p$  are expressed in terms of two-dimensional densities of electrons and holes in the QW,  $n^{\text{QW}}$  and  $p^{\text{QW}}$ , as follows [7, 8]:

$$f_n = 1 - \exp\left(-\frac{n^{\text{QW}}}{N_c^{\text{2D}}}\right), \quad f_p = 1 - \exp\left(-\frac{p^{\text{QW}}}{N_v^{\text{2D}}}\right), \quad (6)$$

where  $N_{c,v}^{\text{2D}} = m_{c,h}^{\text{QW}}T/(\pi\hbar^2)$  are the two-dimensional effective densities of states in the conduction and valence bands in the QW;  $m_{c,h}^{\text{QW}}$  are the effective masses of electrons and holes in the QW; and  $T$  is the temperature in energy units.

Equations (1)–(5) also include the following parameters:  $j$  is the injection current density;  $e$  is the electron charge;  $N_{\text{QW}}$  is the number of identical (having the same width and the same composition) QWs;  $\tau_{\text{nesc}}$  and  $\tau_{\text{pesc}}$  are the thermal escape times of electrons and holes from the QW into the OCL;  $v_{\text{ncapt}0}$  and  $v_{\text{pcapt}0}$  are the capture velocities of electrons and holes into an empty (at  $f_n = 0$  and  $f_p = 0$ ) single QW ( $\text{cm s}^{-1}$ );  $B_{3\text{D}}$  and  $B_{2\text{D}}$  are the coefficients of spontaneous radiative recombination in the three-dimensional (OCL) region and in the two-dimensional (QW) region ( $\text{cm}^3 \text{s}^{-1}$  and  $\text{cm}^2 \text{s}^{-1}$ , respectively, see expressions for  $B_{3\text{D}}$  and  $B_{2\text{D}}$  in [9] and [10]);  $v_{\text{gr}}$  is the group velocity of light;  $g^{\text{max}}$  is the maximum gain in each QW;  $S = WL$ ;  $W$  is the width of the stripe contact;  $L$  is the length of Fabry–Perot cavity;  $\beta = (1/L)\ln(1/R)$  are the losses associated with the emission output from the cavity;  $R$  is the reflectance of the mirrors; and  $\alpha_{\text{int}}$  is the internal optical loss coefficient in the laser structure.

In accordance with [6, 11], the thermal escape times of electrons and holes from the QW into the OCL have the form

$$\tau_{\text{nesc}} = \frac{1}{v_{\text{ncapt}0}(1-f_n)} \frac{N_c^{\text{2D}}}{n_1}, \quad (7)$$

$$\tau_{\text{pesc}} = \frac{1}{v_{\text{pcapt}0}(1-f_p)} \frac{N_v^{\text{2D}}}{p_1},$$

where

$$n_1 = N_c^{\text{3D}} \exp\left(-\frac{\Delta E_c - \varepsilon_n^{\text{QW}}}{T}\right); \quad (8)$$

$$p_1 = N_v^{\text{3D}} \exp\left(-\frac{\Delta E_v - \varepsilon_p^{\text{QW}}}{T}\right);$$

$N_{c,v}^{\text{3D}} = 2[m_{c,v}^{\text{OCL}}T/(2\pi\hbar^2)]^{3/2}$  are the three-dimensional effective densities of states in the conduction and valence bands in

the OCL;  $m_{c,v}^{\text{OCL}}$  are the effective masses of electrons and holes in the OCL;  $\Delta E_{c,v}$  are the offsets of the edges of the conduction and valence bands at a heterojunction between a QW and OCL; and  $\varepsilon_n^{\text{QW}}$  and  $\varepsilon_p^{\text{QW}}$  are, respectively, the energies of the lower and upper edges of the electron and hole quantum-confinement sub-bands in QWs.

The capture velocities of electrons and holes ( $v_{\text{ncapt}0}$  and  $v_{\text{pcapt}0}$ ) from the OCL into an empty QW are the characteristics of a QW, depend on its width and depth, i.e. on the composition of the QW material and its surrounding layers, and can vary greatly in different laser structures. In papers [12, 13], we found the electron capture velocities into a single QW in real laser structures. To do this, we used experimental laser characteristics and theoretical calculations.

The total capture velocities  $v_{\text{ncapt}}$  and  $v_{\text{pcapt}}$  are defined as follows [6]:

$$v_{\text{ncapt}} = v_{\text{ncapt}0}(1-f_n), \quad v_{\text{pcapt}} = v_{\text{pcapt}0}(1-f_p). \quad (9)$$

The global electroneutrality condition for the OCL and the QWs is written in the form:

$$e(N_{\text{QW}}n^{\text{QW}} + bn^{\text{OCL}}) = e(N_{\text{QW}}p^{\text{QW}} + bp^{\text{OCL}}). \quad (10)$$

### 3. Discussion of threshold characteristics of a laser

The set of equations (1)–(5), supplemented with condition (10), was solved at the lasing threshold when the number of coherent photons in the cavity can be assumed equal to zero ( $N = 0$ ). The set was solved numerically for a InGaAs/GaAs/AlGaAs laser structure comprising a single strained QW  $\text{In}_{0.28}\text{Ga}_{0.72}\text{As}$  of thickness  $80 \text{ \AA}$ . The material of the waveguide region (OCL) having a width  $b = 1.7 \text{ \mu m}$  was GaAs; the emitter material was  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ . The lasing wavelength was equal to  $1.044 \text{ \mu m}$ . The length of the Fabry–Perot cavity was  $L = 1.5 \text{ mm}$ , the width of the stripe contact was  $W = 100 \text{ \mu m}$ , the reflectance of the mirrors was  $R = 0.32$ , the mirror loss was  $\beta = 7.6 \text{ cm}^{-1}$ , the temperature was  $T = 300 \text{ K}$ , and the internal optical loss was  $\alpha_{\text{int}} = 1 \text{ cm}^{-1}$  (in this paper  $\alpha_{\text{int}}$  was considered to be independent of the injection current). The maximum modal gain in the QW was  $g^{\text{max}} = 49.1 \text{ cm}^{-1}$ . The capture velocities of electrons and holes into an empty QW,  $v_{\text{ncapt}0}$  and  $v_{\text{pcapt}0}$ , were varied from  $10^5$  to  $10^6 \text{ cm s}^{-1}$ .

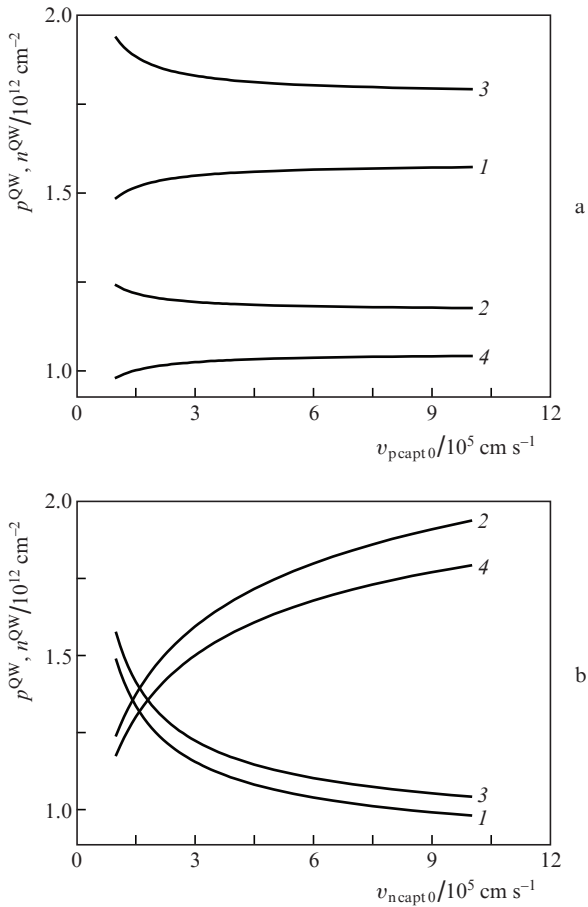
Using equation (5) with (6) taken into account, the electron density  $n^{\text{QW}}$  in the QW can be expressed through the hole density  $p^{\text{QW}}$  in the QW as follows [5]:

$$n^{\text{QW}} = N_c^{\text{2D}} \ln \left[ 1 - \frac{\beta + \alpha_{\text{int}}}{N_{\text{QW}}g^{\text{max}}} - \exp\left(-\frac{p^{\text{QW}}}{N_v^{\text{2D}}}\right) \right]^{-1}. \quad (11)$$

Using expression (11) in equations (3) and (4), the densities of electrons and holes in the OCL,  $n^{\text{OCL}}$  and  $p^{\text{OCL}}$ , can also be expressed through  $p^{\text{QW}}$ . And then using expression (11) for  $n^{\text{QW}}$  and thus obtained expressions for  $n^{\text{OCL}}$  and  $p^{\text{OCL}}$  in (10), we obtain an equation for one unknown ( $p^{\text{QW}}$ ). Determining  $p^{\text{QW}}$  from it, we find  $n^{\text{QW}}$ ,  $n^{\text{OCL}}$  and  $p^{\text{OCL}}$ . The threshold current density is calculated according to the formula

$$j_{\text{th}} = eN_{\text{QW}}B_{2\text{D}}n^{\text{QW}}p^{\text{QW}} + ebB_{3\text{D}}n^{\text{OCL}}p^{\text{OCL}}. \quad (12)$$

Figure 1a shows the dependences of the threshold densities of electrons and holes in the QW on the capture velocity of holes into an empty QW at electron capture velocities  $v_{\text{ncapt}0} = 10^5$  and  $10^6$  cm s<sup>-1</sup>. One can see that when  $v_{\text{ncapt}0}$  is changed by an order of magnitude (from  $10^5$  to  $10^6$  cm s<sup>-1</sup>),  $p^{\text{QW}}$  and  $n^{\text{QW}}$  weakly depend on  $v_{\text{pcapt}0}$ ; at a low capture velocity of electrons ( $v_{\text{ncapt}0} = 10^5$  cm s<sup>-1</sup>), the threshold density of holes in the active region  $p^{\text{QW}}$  [curve (1)] is higher than the threshold density of electrons  $n^{\text{QW}}$  [curve (2)]; at a high capture velocity of electrons ( $v_{\text{ncapt}0} = 10^6$  cm s<sup>-1</sup>), the ratio between  $p^{\text{QW}}$  and  $n^{\text{QW}}$  is reversed, i.e. the density of electrons [curve (4)] in the active region is about twice the density of holes [curve (3)].



**Figure 1.** Dependences of the densities of (1, 3) holes and (2, 4) electrons in the QW at the lasing threshold on (a) the hole capture velocity at  $v_{\text{ncapt}0} = (1, 2) 10^5$  and (3, 4)  $10^6$  cm s<sup>-1</sup> and (b) the electron capture velocity at  $v_{\text{pcapt}0} = (1, 2) 10^5$  and (3, 4)  $10^6$  cm s<sup>-1</sup>.

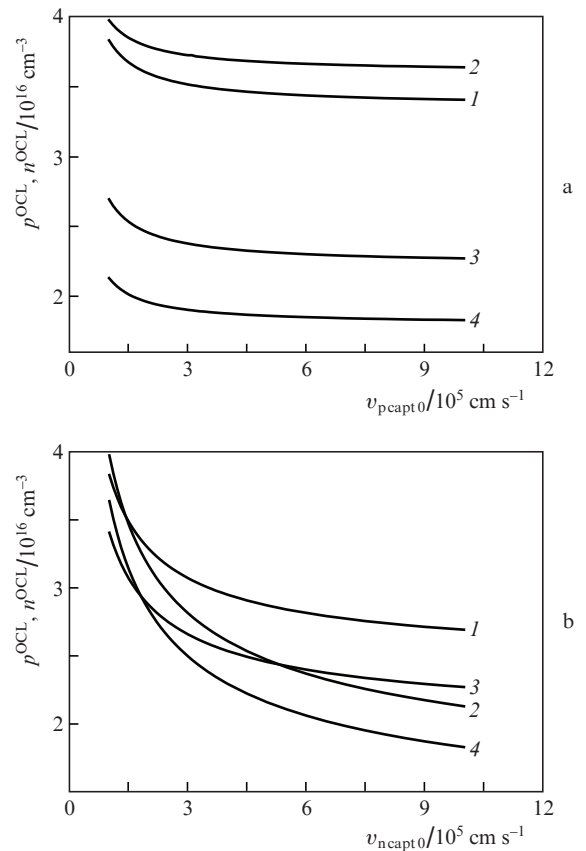
Figure 1b shows the dependences of the densities  $n^{\text{QW}}$  and  $p^{\text{QW}}$  on the capture velocity of electrons into an empty QW at capture velocities of holes  $v_{\text{pcapt}0} = 10^5$  and  $10^6$  cm s<sup>-1</sup>. One can see that  $p^{\text{QW}}$  and  $n^{\text{QW}}$  quite strongly depend on  $v_{\text{ncapt}0}$ ; at small  $v_{\text{ncapt}0}$ , the density of holes in the QW [curve (1)] is greater than that of electrons [curve (2)]; at large  $v_{\text{pcapt}0}$ , the density of electrons in the QW can significantly exceed the density of holes, which coincides with the results shown in Fig. 1a.

The results obtained (Fig. 1) demonstrate that in laser structures the electron and hole densities in the active region at the lasing threshold can differ greatly, which means viola-

tion of the local electroneutrality condition in it [5]. Depending on the capture velocity, the electron density in the QW can be either higher or lower than the hole density. Note that the densities  $n^{\text{QW}}$  and  $p^{\text{QW}}$  are strongly dependent on the electron capture velocity and are weakly dependent on the hole capture velocity.

Our calculations show that at the lasing threshold when changing  $v_{\text{ncapt}0}$  and  $v_{\text{pcapt}0}$  from  $10^5$  to  $10^6$  cm s<sup>-1</sup>, the occupancy of the lower edge of the electron quantum-confinement sub-band in the QW,  $f_n$  [see (6)], varies from 0.843 to 0.952, and the occupancy of the upper edge of the hole quantum-confinement sub-band in the QW,  $f_p$  [see (6)], varies from 0.223 to 0.332. Thus, holes in the QW at the lasing threshold are non-degenerate. In the studied range of the capture velocities, the quasi-Fermi level of holes is located in the forbidden zone. Electrons in the QW at the lasing threshold are strongly degenerate, i.e. the quasi-Fermi level of electrons is located in the quantum-confinement sub-band. Thus, already at the lasing threshold, unlike the hole quantum-confinement sub-band, the electron quantum-confinement sub-band in the QW is strongly filled.

Figure 2a shows the threshold densities of electrons and holes in the OCL as functions of the capture velocities of holes,  $v_{\text{pcapt}0}$ , for two values of the electron capture velocity  $v_{\text{ncapt}0}$ . One can see a weak dependence of the threshold densities in the OCL on the capture velocity of holes. It is also seen that at small values of  $v_{\text{ncapt}0}$ , the densities  $n^{\text{OCL}}$  and  $p^{\text{OCL}}$  do not significantly differ, the electron density in the OCL



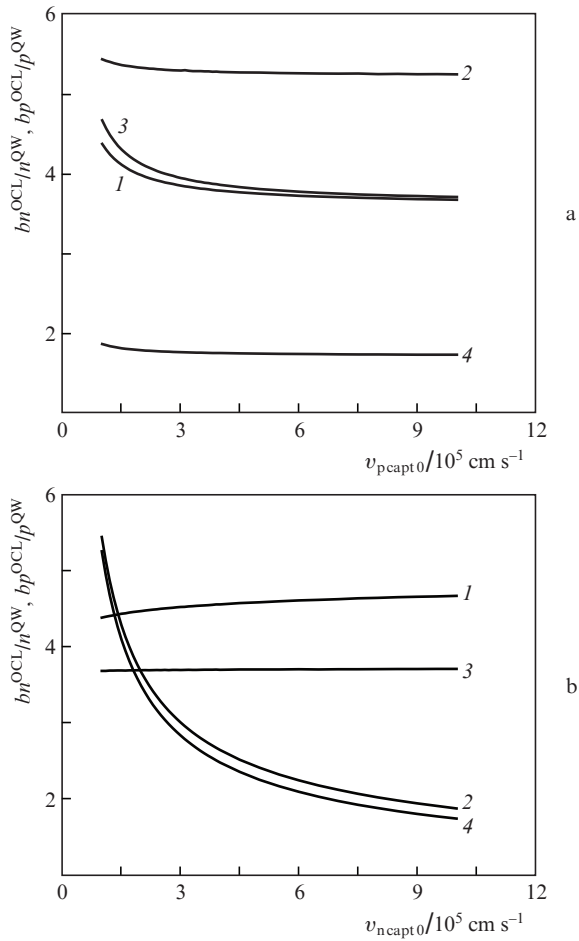
**Figure 2.** Dependences of the densities of (1, 3) holes and (2, 4) electrons in the OCL at the lasing threshold on (a) the hole capture velocity at  $v_{\text{ncapt}0} = (1, 2) 10^5$  and (3, 4)  $10^6$  cm s<sup>-1</sup> and (b) the electron capture velocity at  $v_{\text{pcapt}0} = (1, 2) 10^5$  and (3, 4)  $10^6$  cm s<sup>-1</sup>.

being higher than the hole density. At high capture velocities of electrons ( $v_{\text{ncapt}0} = 10^6 \text{ cm s}^{-1}$ ), the hole density in the OCL exceeds the electron density.

Figure 2b shows the threshold densities of electrons and holes in the OCL as functions of the capture velocities of electrons,  $v_{\text{ncapt}0}$ , for two values of the hole capture velocity  $v_{\text{pcapt}0}$ . One can see that there is a strong dependence of the threshold densities in the OCL on the electron capture velocity  $v_{\text{ncapt}0}$ . At  $v_{\text{ncapt}0} > 2 \times 10^5 \text{ cm s}^{-1}$ , the hole density in the OCL starts to exceed the electron density.

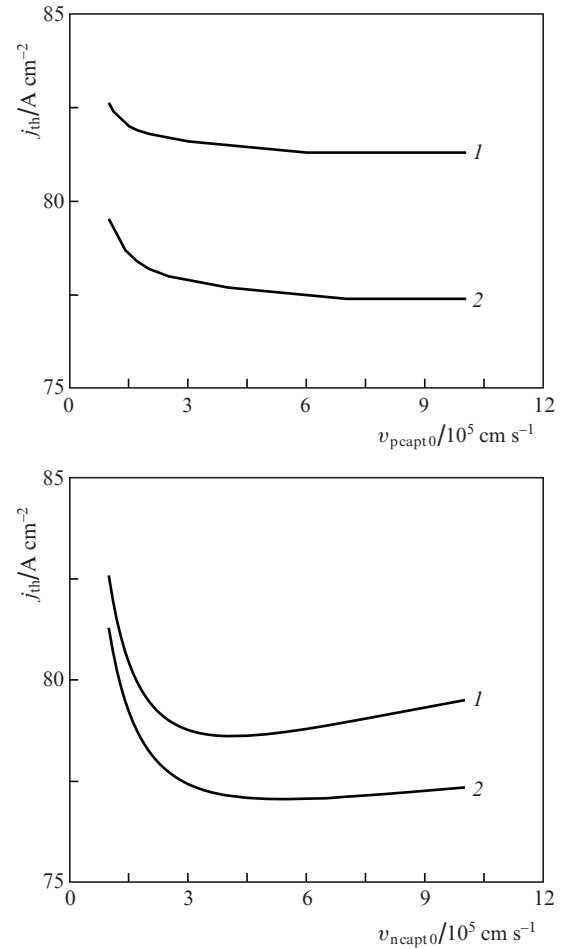
Thus, as follows from Figure 2, with increasing capture velocities of both electrons and holes, the threshold densities of the charge carriers in the OCL decrease.

Figure 3 shows the dependence of the ratio of the electron charge in the OCL to that in the QW and the ratio of the hole charge in the OCL to that in the QW on the capture velocities of electrons and holes into the QW. It follows from Fig. 3a that these ratios are virtually independent of the capture velocity of holes. The ratio for the holes is also independent of the electron capture velocity, while the ratio for the electrons decreases by approximately three times with increasing electron capture velocity by an order of magnitude (Fig. 3b). However, the charge of both electrons and holes in the OCL is larger than that in the QW.



**Figure 3.** Ratios of (1, 3) the charge of holes in the OCL to the charge of holes in the QW and (2, 4) the charge of electrons in the OCL to the charge of electrons in the QW vs. (a) the capture velocity of holes at  $v_{\text{ncapt}0} = (1, 2) 10^5$  and  $(3, 4) 10^6 \text{ cm s}^{-1}$  and (b) the electron capture velocity at  $v_{\text{pcapt}0} = (1, 2) 10^5$  and  $(3, 4) 10^6 \text{ cm s}^{-1}$ .

The dependences of the threshold current density on the capture velocity of holes in the QW at electron capture velocities  $v_{\text{ncapt}0} = 10^5$  and  $10^6 \text{ cm s}^{-1}$  are shown in Fig. 4a, and the dependences of the threshold current density on the capture velocity of electrons at hole capture velocities  $v_{\text{pcapt}0} = 10^5$  and  $10^6 \text{ cm s}^{-1}$  – in Fig. 4b. One can see from Fig. 4 that in the laser structure in question, the threshold current density is weakly dependent on the capture velocities of both holes and electrons.



**Figure 4.** Dependences of the threshold current density on (a) the hole capture velocity at  $v_{\text{ncapt}0} = (1, 2) 10^5$  and  $(3, 4) 10^6 \text{ cm s}^{-1}$  and (b) the electron capture velocity at  $v_{\text{pcapt}0} = (1, 2) 10^5$  and  $(3, 4) 10^6 \text{ cm s}^{-1}$ .

## 4. Conclusions

We have used the global electroneutrality condition, i.e. the condition of the equality of the total charge of electrons in the OCL and QW to the total charge of holes in the OCL and QW.

Calculations with the global electroneutrality condition taken into account have shown that at the lasing threshold in laser structures, the densities of electrons and holes in the active region are strongly different (see Fig. 1), which means a violation of the local electroneutrality condition. Depending on the capture velocity, the electron density in the QW can be either higher or lower than the hole density. It is shown that the densities  $n^{\text{QW}}$  and  $p^{\text{QW}}$  strongly depend on the electron capture velocity and weakly – on the hole capture velocity.

The densities of electrons and holes in the OCL at the lasing threshold are also not equal: depending on the capture velocity, the electron density in the OCL can be either higher or lower than the hole density. With increasing capture velocity, the densities of both types of charge carriers in the OCL are reduced; however, the charge of the carriers of each sign in the OCL is greater than that in the QW.

Thus, we have shown that at the lasing threshold, the local electroneutrality condition is violated in laser layers – in the active region and the OCL (under the condition of the global electroneutrality).

**Acknowledgements.** This work was carried out in accordance with the State Programme of the Ioffe Institute. L.V. Asryan thanks the US Army Research Office (Grant No. W911NF-13-1-0445) for support of this work.

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