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TERAHERTZ RADIATION
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Terahertz radiation in laser-induced charge separation in the irradiated plasma target

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Abstract. **Irradiation of a thin target by an intense laser pulse makes it possible to produce both bulk and surface terahertz (THz) pulses as a result of their generation by a bunch of fast electrons leaving the target and/or by a moving front of the spatial charge separation of expanding plasma. These mechanisms responsible for generating bulk and surface THz radiation are refined and compared. The dependence of the parameters of the generated THz radiation on the laser pulse energy, focal spot size and duration is found.**

Keywords: *terahertz radiation, surface waves, relativistic short laser pulse.*

1. Introduction

To date, one of the fastest-growing areas of study of high-intensity laser radiation – matter interaction is related to the problem of the development of a high-power source of terahertz (THz) radiation for a wide range of practical applications [1]. Sources of THz pulses, which have a number of unique properties that are due to the nondestructive impact of this type of radiation, can be used in medicine and biotechnology, diagnostics (inspection) and information technology. However, currently used compact THz radiation sources have significant limitations on the power and efficiency of laser pulse energy conversion into energy of THz radiation, which restricts their practical use. At the same time, strong secondary electromagnetic fields in a small bulk are already routinely produced when high-power femto/picosecond laser pulses irradiate solid targets (e.g., thin metal foils). In this way, we should expect much more powerful (than ever) sources of THz radiation.

A typical mechanism for generating THz radiation is the coherent transition radiation produced by an electron bunch passing through a metal foil. This mechanism is the basis for a high-power THz source using an ultra-relativistic electron beam with a charge of a few nC from a large linear accelerator [2]. Such a charge can be produced by relativistic electrons generated by a high-power short laser pulse in the direction of its propagation under irradiation of a foil [3]. Crossing the back surface of the foil, electrons leaving the target can generate coherent transition radiation in the forward direction and surface radiation, both of them lying in the THz range. Perhaps this is due to the observed THz radiation in experiments on irradiation of solid targets by high-intensity laser radiation [4 – 8]. It should be noted that THz radiation can be produced at much lower intensities of laser radiation [9 – 11].

The electron bunch from the target includes a small part of all laser-accelerated (heated) electrons: only those whose energy is sufficient to overcome the confining electrostatic potential. In a typical case, this is a bunch of moving relativistic particles. The greater part of hot electrons remains in the target, producing a strong electrostatic field at the rear surface, which leads to target ionisation and subsequent plasma expansion. With this expansion, the front of the expanding plasma has a moving uncompensated electric dipole [12], which is basically equivalent to a charge flying away from the target and, hence, can also generate transition radiation [5, 6]. This radiation mechanism, unlike the first one, is associated with the slow movement of the charge. However, the magnitude of a recoil charge is large compared with a charge of a relativistic bunch, and its role in radiation should be considered.

Speaking of these two radiation mechanisms, it should be noted that in both cases both bulk and surface THz pulses can be generated. Therefore, in this paper we refine and compare the mechanisms of generation of bulk and surface radiation produced by a relativistic electron bunch leaving the target and by a nonquasi-neutral expansion of the plasma. At the same time, using a model relationship of corresponding electrical currents with the parameters of heating laser radiation, we solve the problem of the characterisation of generated THz radiation as a function of laser pulse energy, focal spot size and duration.

2. Generation of electromagnetic fields under irradiation of a flat target by a short laser pulse

To study the generation of electromagnetic fields given by the currents arising from irradiation of a target by short laser pulses, we will use the system of Maxwell's equations in a cylindrical coordinate system. Let us apply a Fourier transformation in time

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$$f(z,\boldsymbol{r}_\perp,\omega) = \int_{-\infty}^{\infty} \mathrm{d}t f(z,\boldsymbol{r}_\perp,t)\exp(\mathrm{i}\omega t).$$

Assuming third-party sources and generated fields to be axially symmetric, we will use the expansion in the transverse radius vector $\boldsymbol{r}_\perp$ in the target plane

$$f(z,\boldsymbol{k}_\perp,\omega) = 2\pi\int_0^{\infty} \boldsymbol{r}_\perp \mathrm{d}\boldsymbol{r}_\perp J_0(k_\perp r_\perp) f(z,\boldsymbol{r}_\perp,\omega)$$

and accordingly the expansion

$$f(z,\boldsymbol{r}_\perp,\omega) = \frac{1}{2\pi}\int_0^{\infty} \boldsymbol{k}_\perp \mathrm{d}\boldsymbol{k}_\perp J_0(k_\perp r_\perp) f(z,\boldsymbol{k}_\perp,\omega),$$

where J_0 is the zero-order Bessel function. Maxwell's equations in this case take the form

$$\mathrm{i}\frac{\omega}{c}B_\varphi(z,\boldsymbol{k}_\perp,\omega) = \frac{\partial E_r(z,\boldsymbol{k}_\perp,\omega)}{\partial z} - \mathrm{i}\boldsymbol{k}_\perp E_z(z,\boldsymbol{k}_\perp,\omega),$$

$$-\frac{\partial B_\varphi(z,\boldsymbol{k}_\perp,\omega)}{\partial z} = -\mathrm{i}\frac{\omega}{c}\varepsilon(\omega,z)E_r(z,\boldsymbol{k}_\perp,\omega) + \frac{4\pi}{c}j_r(z,\boldsymbol{k}_\perp,\omega), \quad (1)$$

$$\mathrm{i}k_\perp B_\varphi(z,\boldsymbol{k}_\perp,\omega) = -\mathrm{i}\frac{\omega}{c}\varepsilon(\omega,z)E_z(z,\boldsymbol{k}_\perp,\omega) + \frac{4\pi}{c}j_z(z,\boldsymbol{k}_\perp,\omega).$$

Here, $\varepsilon(\omega,z)$ is the permittivity of the medium. Expressing axial $[E_z(z,\boldsymbol{k}_\perp,\omega)]$ and radial $[E_r(z,\boldsymbol{k}_\perp,\omega)]$ components of the electric field through the azimuthal component of the magnetic field $B_\varphi(z,\boldsymbol{k}_\perp,\omega)$, for the latter we obtain the equation

$$\varepsilon\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial B_\varphi}{\partial z}\right) - k^2 B_\varphi = \frac{4\pi\varepsilon}{c}\left[\mathrm{rot}\left(\frac{\boldsymbol{j}}{\varepsilon}\right)\right]_\varphi$$

$$= -\frac{4\pi\varepsilon}{c}\left[\frac{\partial}{\partial z}\left(\frac{j_r}{\varepsilon}\right) - \mathrm{i}k_\perp\frac{j_z}{\varepsilon}\right] \equiv Q, \quad (2)$$

where $k^2 = k_\perp^2 - (\omega^2/c^2)\varepsilon$.

Equation (2) implies that the generation of the magnetic field is determined by the source $\mathrm{rot}[\boldsymbol{j}/\varepsilon(z)]$. If the currents $\boldsymbol{j}$ excited by plasma are potential, for example in the generation of vortex fields due to thermal electromotive force or ponderomotive exposure of the target to the laser pulse, such a source has the form $\mathrm{rot}[\boldsymbol{j}/\varepsilon(z)] = \nabla\varepsilon^{-1}(z)\times\boldsymbol{j}$ and affects only the surface region, where there is a strong inhomogeneity of the electron density. This paper deals with the generation of bulk radiation and surface electromagnetic waves (SEWs) by currents arising from the departure of a nonquasi-neutral hot plasma bunch from the surface under the action of a short intense laser pulse.

The solution to equation (2) for the magnetic field inside the target ($z < 0$) has the form

$$B_\varphi^{\mathrm{p}}(z,\omega,\boldsymbol{r}) = \Big[B_\varphi(z=-0,\omega,\boldsymbol{k}_\perp)$$

$$+\frac{1}{2k}\int_{-\infty}^{0}\mathrm{d}z\exp(kz)\,Q(z,\omega,\boldsymbol{k}_\perp)\Big]\exp(kz)$$

$$-\frac{1}{2k}\Big[\exp(-kz)\int_{-\infty}^{z}\mathrm{d}z\exp(kz)\,Q(z,\omega,\boldsymbol{k}_\perp)$$

$$+\exp(kz)\int_z^0\mathrm{d}z\exp(-kz)\,Q(z,\omega,\boldsymbol{k}_\perp)\Big], \quad (3)$$

where $B_\varphi(z=0,\omega,\boldsymbol{k}_\perp) \equiv B_\varphi^0(\omega,\boldsymbol{k}_\perp)$ is the magnetic field intensity at the target boundary. For the magnetic field in vacuum $B_\varphi^{\mathrm{v}}(z,\omega,\boldsymbol{r})$, from (2) we obtain the expression

$$B_\varphi^{\mathrm{v}}(z,\omega,\boldsymbol{r}) = \Big[B_\varphi(z=+0,\omega,\boldsymbol{k}_\perp)$$

$$+\frac{1}{2k_0}\int_0^{\infty}\mathrm{d}z\exp(-k_0 z)\,Q(z,\omega,\boldsymbol{k}_\perp)\Big]\exp(-k_0 z)$$

$$-\frac{1}{2k_0}\Big[\exp(k_0 z)\int_z^{\infty}\mathrm{d}z\exp(-k_0 z)\,Q(z,\omega,\boldsymbol{k}_\perp)$$

$$+\exp(-k_0 z)\int_0^z\mathrm{d}z\exp(k_0 z)\,Q(z,\omega,\boldsymbol{k}_\perp)\Big], \quad (4)$$

where $k_0^2 = k_\perp^2 - \omega^2/c^2$.

Integrating equation (2) in the vicinity of $z = 0$, we find the magnetic field at the target boundary:

$$B_\varphi^0(\omega,\boldsymbol{k}_\perp) = -\frac{1}{D(\omega,\boldsymbol{k}_\perp)}\Big[\int_0^{\infty}\mathrm{d}z\exp(-k_0 z)\,Q(z,\omega,\boldsymbol{k}_\perp)$$

$$+\int_{-\infty}^{0}\mathrm{d}z\exp(kz)\,Q(z,\omega,\boldsymbol{k}_\perp) + \int_{-0}^{+0}\mathrm{d}z\frac{Q(z,\omega,\boldsymbol{k}_\perp)}{\varepsilon(z,\omega)}\Big]$$

$$= \frac{4\pi}{cD(\omega,\boldsymbol{k}_\perp)}\Big\{\int_0^{\infty}\mathrm{d}z\exp(-k_0 z)[k_0 j_r(z,\omega,\boldsymbol{k}_\perp) - \mathrm{i}k_\perp j_z]$$

$$+\frac{1}{\varepsilon}\int_{-\infty}^{0}\mathrm{d}z\exp(kz)[kj_r(z,\omega,\boldsymbol{k}_\perp) + \mathrm{i}k_\perp j_z]\Big\}, \quad (5)$$

where

$$D(\omega,\boldsymbol{k}_\perp) = k/\varepsilon + k_0. \quad (6)$$

The relationship $D(\omega,\boldsymbol{k}_\perp) = 0$ is the dispersion equation for the SEWs. We investigate the generation of electromagnetic radiation and SEWs by the current component j_z, normal to the target surface. This, for example, takes place on the back side of the target. Then, the expression for the magnetic field in vacuum in view of (4) and (5) takes the form

$$B_\varphi^{\mathrm{v}}(z,\omega,\boldsymbol{r}) = -\mathrm{i}\int_0^{\infty}\frac{k_\perp^2\mathrm{d}\boldsymbol{k}_\perp}{ck_0}J_0(k_\perp r_\perp)$$

$$\times\Big\{\Big[\frac{2k_0}{D(\omega,\boldsymbol{k}_\perp)} - 1\Big]\int_0^{\infty}\mathrm{d}z'\exp(-k_0 z')j_z(z',\omega,\boldsymbol{k}_\perp)\exp(-k_0 z)$$

$$+\int_0^{\infty}\mathrm{d}z'\exp[k_0(z-z')]j_z(z',\omega,\boldsymbol{k}_\perp)$$

$$+\int_0^{z}\mathrm{d}z'\exp[k_0(z'-z)]j_z(z',\omega,\boldsymbol{k}_\perp) + \frac{2k_0}{\varepsilon D(\omega,\boldsymbol{k}_\perp)}$$

$$\times\int_{-\infty}^{0}\mathrm{d}z'\exp(kz')j_z(z',\omega,\boldsymbol{k}_\perp)\exp(-k_0 z)\Big\}. \quad (7)$$

This expression describes the electromagnetic field generated by the current or the field of the moving charge.

3. Expressions for the currents

Consider the currents generated on the metal surface of the back side of a flat target (target–vacuum interface) by short intense laser pulses irradiating its front surface. They are generated in the spot size of the order of the laser focal spot size (typically from several to ten micrometres), wherein the substance is rapidly ionised, transforming into plasma. The laser pulse energy is mainly converted into the energy of electrons generated in the direction of the laser beam propagation. A part of these electrons with the highest energy leaves the target, while the majority of the hot electrons, flying out to a distance of the order of the Debye radius, forms a charge separation field, resulting in plasma expansion into vacuum at the front of which a thin (Debye) charge separation layer is produced.

Let us examine the current generated by a bunch of the fastest electrons, which have overcome the potential barrier of the near-wall plasma. The bunch of such electrons with concentration n_f, moving from the target surface along the z axis at the velocity V_f, can be presented in the form of a cylinder of length L uniform along z. Assuming that the concentration distribution in the radial direction is Gaussian, proportional to $\exp(-r^2/R^2)$, where the characteristic size R is determined by the focal spot size of the laser pulse, we write the expression for the current in the form

$$j_z(z,\boldsymbol{r}_\perp t) = en_\mathrm{f} V_\mathrm{f}\Theta(t)\Theta(z - V_\mathrm{f}t + L)$$

$$\times\Theta(V_\mathrm{f}t - z)\exp(-r_\perp^2/R^2), \tag{8}$$

where e is the electron charge, and $\Theta(x)$ is the Heaviside function. The value of the current in question is determined by the energy (velocity) of fast electrons and their concentration n_f, which can be related to the characteristics of the incident laser pulse. To do this, we should evaluate the efficiency of the target heating and find the value of the confining potential. To find the temperature T of heated electrons we will use its dependence on the laser intensity I_0 in the form specified by the ponderomotive potential [13]:

$$T = mc^2(\sqrt{1+\eta a_0^2/2} - 1).$$

Here,

$$a_0^2 = \frac{2I_0}{n_\mathrm{c}mc^3} = 0.85\sqrt{\frac{I_0}{10^{18}\lambda^2}};$$

m is the mass of the electrons; n_c is the critical concentration of electrons; λ is the wavelength of the laser radiation in micrometres; η is the coefficient of laser radiation absorption; and I_0 is taken in W cm^{-2}. The electron density can be found from the condition of conservation of energy flow. Thus, equating $n\langle\mathcal{E}\rangle v_\mathrm{h}$ to ηI_0, where $v_\mathrm{h} = ca_0\sqrt{\eta}/\sqrt{2+\eta a_0^2}$ is the mean velocity of electrons calculated by their average energy $\langle\mathcal{E}\rangle$, which is defined as the effective temperature T, we obtain the expression for the concentration of heated electrons:

$$n = a_0 n_\mathrm{c}\sqrt{\frac{\eta}{2}}\frac{\sqrt{1+\eta a_0^2/2}}{\sqrt{1+\eta a_0^2/2} - 1}. \tag{9}$$

Equation (9) will be used below to assess the efficiency of the laser pulse energy conversion into the secondary radiation energy.

Knowing the characteristics of the laser-heated electrons, we can find the characteristic value of the confining potential at the target boundary, $e\Phi_\mathrm{min} = -2T\ln[R/(r_\mathrm{D}\sqrt{2})]$, where $r_\mathrm{D} = \sqrt{T/(4\pi e^2 n)}$ is the Debye radius of hot electrons, and estimate the concentration of fast electrons, $n_\mathrm{f} = n\exp(e\Phi_\mathrm{min}/T)$, capable of overcoming this potential, assuming the distribution of hot electrons to be Maxwellian–Boltzmannian. Typically, the concentration n_f is a fraction of a percentage of the total concentration of the laser-heated electrons n.

It should be noted that the problem of transition radiation of a uniformly moving electron bunch was solved in different formulations [14], including the one with laser-heated electrons [15, 16]; however, the issue of the dependence of the efficiency of the electromagnetic radiation and SEW generation on the laser parameters was not discussed in detail.

The temperature of hot electrons and their density determine the effective current arising at the front of the plasma expanding in vacuum due to quasi-neutrality violations [12]. It can be shown that this current is significant only at the Debye scale at the expanding plasma front, where there is a charge separation*. The propagation velocity of the plasma front V initially increases linearly with time,

$$V = 2c_\mathrm{s}\ln(\tau + \sqrt{1+\tau^2}), \tag{10}$$

and then with logarithmic accuracy reaches a constant value of several ion sound velocities $c_\mathrm{s} = \sqrt{T/M}$. Here, $\tau = \omega_\mathrm{pl}t/\sqrt{2\mathrm{e}} \equiv t/\tau_0$; $\mathrm{e} = \exp(1)$; $\omega_\mathrm{pl} = c_\mathrm{s}/r_\mathrm{D}$; and M is the mass of the plasma ions. In this case, the electron concentration at the front is reduced according to the law [12]

$$n_\mathrm{e} = \frac{n}{2.718(1+\tau^2)}.$$

Accordingly, the current expansion will be modelled by the expression:

$$j_z(z,\boldsymbol{r}_\perp,t) = enc_\mathrm{s}r_\mathrm{D}\Theta(t)\exp(-\alpha t/\tau_0)[\delta(z - c_\mathrm{s}t\tau)\tau\Theta(\tau_0 - t)$$

$$+\delta(z - 2c_\mathrm{s}t + c_\mathrm{s}\tau_0)\Theta(t-\tau_0)]\exp(-r_\perp^2/R^2). \tag{11}$$

The distribution of the current in the lateral direction is also assumed here to be Gaussian. The first term in (11) describes a rapidly moving, up to a time τ_0, electron bunch, and the second term corresponds to the uniform motion in the next, $t > \tau_0$, instants of time. The constant α determines the rate of a decrease in the electron concentration due to a reduced charge separation in the plasma expansion. In this model, the characteristic times of the discharge prove to be on the order of τ_0 ($\alpha \approx 1$). Note that the same times of the current disappearance are typical for adiabatic cooling of electrons after the laser pulse termination [17].

4. Generation of bulk radiation in vacuum

Consider the excitation of bulk electromagnetic waves in the wave zone (away from the source), propagating from the surface of the target with $|\varepsilon| \gg 1$. Given that for the bulk waves $k_0 = -\mathrm{i}\sqrt{\omega^2/c^2 - k_\perp^2}$ and $0 < k_\perp < \omega/c$, we rewrite relation (7), taking into account the asymptotic expansion of the Bessel functions for $k_\perp r \gg 1$, in the form

*Generally speaking, there is also a current within the plasma, which is associated with the rarefaction wave. However, because its radiation is negligible, this current is no of interest for the problem under study.

$$B_\varphi^{\mathrm{w}}(z,\omega,\boldsymbol{r}) = \int_0^{\omega/c} \frac{k_\perp^2 \mathrm{d}\boldsymbol{k}_\perp}{2c\,|k_0|}\sqrt{\frac{2}{\pi k_\perp r_\perp}}$$

$$\times \exp\Big[\mathrm{i}\Big(k_\perp r_\perp + |k_0|\,z - \frac{\pi}{4}\Big)\Big] K(|k_0|,\boldsymbol{k}_\perp), \quad (12)$$

where

$$K(|k_0|,\boldsymbol{k}_\perp) = \int_0^\infty \mathrm{d}z' \exp(\mathrm{i}|k_0|z') j_z(z',\omega,\boldsymbol{k}_\perp)$$

$$+ \exp(2\mathrm{i}|k_0|z) \int_z^\infty \mathrm{d}z' \exp(-\mathrm{i}|k_0|z') j_z(z',\omega,\boldsymbol{k}_\perp)$$

$$+ \int_0^z \mathrm{d}z' \exp(-\mathrm{i}|k_0|z) j_z(z',\omega,\boldsymbol{k}_\perp). \quad (13)$$

Expression (12) retains only the term in the asymptotic expansion of the Bessel function, corresponding to a wave leaving the target surface for vacuum. In calculating $K(|k_0|,\boldsymbol{k}_\perp)$, we take into account only terms that do not depend explicitly on z. Using the saddle point method [18], we obtain

$$B_\varphi^{\mathrm{w}}(z,\omega,\boldsymbol{r}) = \frac{\omega r_\perp}{c^2 r}\frac{\exp(\mathrm{i}\omega r/c)}{r} K\Big(\frac{\omega z}{cr}, \frac{\omega r_\perp}{cr}\Big). \quad (14)$$

Knowing the magnetic field (14), we determine the energy emitted into the solid angle $\mathrm{d}o = 2\pi \sin\theta \mathrm{d}\theta$ and frequency interval $\mathrm{d}\omega$:

$$\mathrm{d}W^{\mathrm{w}}(\omega,\theta) = \frac{cr^2}{4\pi^2}|B_\varphi^{\mathrm{w}}|^2 \mathrm{d}\omega \mathrm{d}o. \quad (15)$$

It is known that radiation is most effective if the bunch particles radiate coherently, i.e. concentrated on a scale much smaller than the emitted wavelength. If the bunch is made up of particles with a charge of the same sign, then the field amplitudes are summed and the density of the radiated energy is much greater than the sum of the radiation energy densities of the individual electrons [19]. Thus, for a uniformly moving electron bunch with a distributed density, crossing the target surface [for current (8)], the expression for the magnetic radiation field has the form

$$B_\varphi^{\mathrm{w}}(z,\omega,\boldsymbol{r}) = B_\varphi^{\mathrm{w}0}(z,\omega,\boldsymbol{r}) F(\omega,\theta),$$

where

$$B_\varphi^{\mathrm{w}0}(z,\omega,\boldsymbol{r}) = \frac{2\mathrm{i}q_{\mathrm{f}} V_{\mathrm{f}} \sin\theta}{rc^2[1-(V_{\mathrm{f}}^2/c^2)\cos^2\theta]}\exp\Big(\frac{\mathrm{i}\omega r}{c}\Big). \quad (16)$$

The form factor $F(\omega,\theta)$ determines the frequency-angular dependence of the radiation field and represents a space–time Fourier transform of the current density $j_z(z,\omega,\boldsymbol{k}_\perp)$ (8), normalised to the product of the total bunch charge $q_{\mathrm{f}} = en_{\mathrm{f}}L\pi R^2$ and its velocity V_{f}:

$$F = \frac{2V_{\mathrm{f}}}{\omega L}\sin\Big(\frac{\omega L}{2V_{\mathrm{f}}}\Big)\exp\Big(\mathrm{i}\frac{\omega L}{2V_{\mathrm{f}}}\Big)\exp\Big(-\frac{R^2\omega^2\sin^2\theta}{4c^2}\Big). \quad (17)$$

For low-frequency waves, $L\omega/c \ll 1$ and $R\omega/c \ll 1$ and the form factor $F \approx 1$. The expression for the spectral density of bulk transition radiation energy, which is generated by the bunch, crossing the target surface, can be written in the form [14]

$$\frac{\mathrm{d}W^{\mathrm{w}0}(\omega,\theta)}{\mathrm{d}\omega \mathrm{d}o} = \frac{\mathrm{d}W^{\mathrm{w}0}}{\mathrm{d}\omega \mathrm{d}o}|F(\omega,\theta)|^2, \quad (18)$$

where $W^{\mathrm{w}0}(\omega,\theta)$ is spectral radiation transition energy density of the charge $q_{\mathrm{f}} = en_{\mathrm{f}}L\pi R^2$ [14]; and

$$\frac{\mathrm{d}W^{\mathrm{w}0}}{\mathrm{d}\omega \mathrm{d}o} = \frac{q_{\mathrm{f}}^2 V_{\mathrm{f}}^2 \sin^2\theta}{\pi^2 c^3 [1-(V_{\mathrm{f}}^2/c^2)\cos^2\theta]^2}. \quad (19)$$

Let us analyse the obtained expression in the relativistic limit, when the kinetic energy of electrons $\mathcal{E}_0$ exceeds the energy at rest: $\mathcal{E}_0 > mc^2$ (Fig. 1). In this case, the expression for the total energy radiated in the frequency range $0-\omega$ can be written as

$$W^{\mathrm{w}}(\omega) = \frac{2q_{\mathrm{f}}^2}{\pi L}\int_0^{\frac{\omega L}{2c}} \mathrm{d}y \frac{\sin^2 y}{y^2}$$

$$\times \int_0^{\frac{\pi}{2}} \frac{\mathcal{E}_0^4 \sin^3 x \mathrm{d}x}{(\mathcal{E}_0^2\sin^2 x + m^2c^4\cos^2 x)^2}\exp\Big(-\frac{2R^2y^2}{L^2}\sin^2 x\Big). \quad (20)$$

In the low-frequency limit ($\omega \ll 2c/L$) we obtain a standard expression for the transition radiation energy of an ultrarelativistic charge, increasing linearly with frequency:

$$W^{\mathrm{w}}(\omega) = \frac{q_{\mathrm{f}}^2\omega}{\pi c}\Big[2\ln\Big(\frac{2\mathcal{E}_0}{mc^2}\Big) - 1\Big]. \quad (21)$$

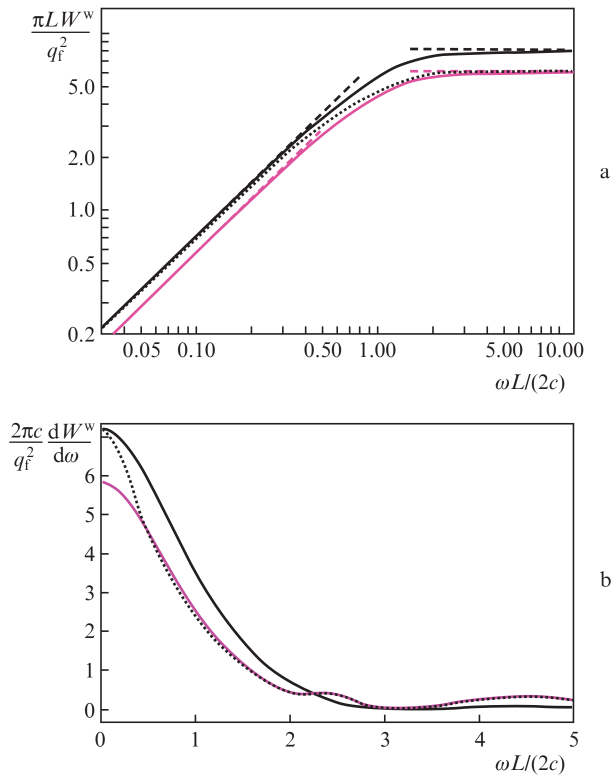

Figure 1. (a) Energy of bulk radiation generated by the electron bunch and (b) emission spectra as functions of the frequency for $\mathcal{E}_0 = 30mc^2$, $R/L = 1$ (solid black curve), $\mathcal{E}_0 = 15mc^2$, $R/L = 1$ (solid gray curve) and $\mathcal{E}_0 = 30mc^2$, $R/L = 2$ (dashed curve). The dashed curves correspond to the limiting cases $\omega \ll 2c/L$ and $\omega \gg 2c/L$.

In the opposite limit ($\omega \gg 2c/L$), we have a frequency-independent expression determined by the beam parameters, which gives the total energy of transition radiation (for all frequencies)

$$W^{\mathrm{w}} = \frac{2q_{\mathrm{f}}^2}{\pi L}\int_0^{\infty}\mathrm{d}y\frac{\sin^2 y}{y^2} \times\int_0^{\infty}\frac{x\mathrm{d}x}{(1+x)^2}\exp\left(-\frac{2R^2m^2c^4y^2}{\mathcal{E}_0^2L^2}x\right). \tag{22}$$

Note that at $\mathcal{E}_0 L/(mc^2R) \gg 1$ the double integral is well approximated by the expression:

$$W^{\mathrm{w}} = \frac{q_{\mathrm{f}}^2}{\pi L}\left[3\ln\left(\frac{\mathcal{E}_0 L}{\sqrt{2}\,mc^2R}\right) - 1\right]. \tag{23}$$

In this limit, the maximum frequency (up to which and above which the frequency spectrum of radiation is almost constant and begins to decrease, respectively) is determined by the longitudinal size of the beam, i.e., by the laser pulse duration: $\omega_{\mathrm{m}} \approx c/L$. Note the weak dependence of the radiation energy on the focal spot size of the laser pulse (electron beam transverse cross section).

The expansion of the plasma into vacuum is determined by the speed of sound, which is much less than the speed of light: $c_{\mathrm{s}} \ll c$. We calculate the magnetic field of bulk radiation for current (11) in a nonrelativistic approximation:

$$B_{\varphi}^{\mathrm{w}}(z,\omega,\boldsymbol{r}) = \frac{2\mathrm{i}qc_{\mathrm{s}}\sin\theta}{rc^2}\exp\left(\mathrm{i}\frac{\omega r}{c}\right)G(\omega,\theta),$$

where

$$G(\omega,\theta) = \frac{\mathrm{i}\omega\tau_0[1-\exp(-\alpha-\mathrm{i}\omega\tau_0)]}{(\alpha+\mathrm{i}\omega\tau_0)^2}\exp\left(-\frac{\omega^2R^2\sin^2\theta}{4c^2}\right) \tag{24}$$

is the form factor, and the total charge q is determined by the Debye radius of hot electron: $q = enr_{\mathrm{D}}\pi R^2$. Accordingly, the radiated energy

$$\frac{\mathrm{d}W^{\mathrm{w}}}{\mathrm{d}\omega\mathrm{d}o} = \frac{q^2c_{\mathrm{s}}^2}{\pi^2c^3}|G(\omega,\theta)|^2\sin^2\theta. \tag{25}$$

In the limit $\alpha = 0$ (motion without changing the bunch density), form factor (24) coincides with the result for form factor (17) with $L/V_{\mathrm{f}} = \tau_0$ obtained for transition radiation of a cylinder,

$$G(\omega,\theta) = \frac{2\sin(\omega\tau_0/2)}{\omega\tau_0}\exp\left(-\frac{\omega^2R^2\sin^2\theta}{4c^2} - \frac{\mathrm{i}\omega\tau_0}{2}\right),$$

although radiation is produced in the case of a uniform charge acceleration up to time τ_0, and not at the moment it crosses the surface as in the case of a uniformly moving charge. In the general case, the expression for the total power radiated in the frequency range $0-\omega$ has the form

$$W^{\mathrm{w}}(\omega) = \frac{q^2c_{\mathrm{s}}^2\tau_0^2}{\pi R^3}\int_0^{\omega\tau_0}\mathrm{d}x\frac{1+\exp(-2\alpha)-2\exp(-\alpha)\cos x}{x(\alpha^2+x^2)^2} \times\left[\sqrt{2\pi}\left(1+\frac{x^2R^2}{c^2\tau_0^2}\right)\exp\left(-\frac{x^2R^2}{2c^2\tau_0^2}\right)\mathrm{Erf}\left(\frac{xR}{c\tau_0\sqrt{2}}\right) - \frac{2xR}{c\tau_0}\right]. \tag{26}$$

In the low-frequency region ($\omega\tau_0 \ll 1$), the energy behaviour (26) is determined by the value of α:

$$W^{\mathrm{w}}(\omega) = \frac{2q^2c_{\mathrm{s}}^2}{3\pi c^3\tau_0}\left\{\left[\frac{\arctan(\omega\tau_0/\alpha)}{\alpha} - \frac{\omega\tau_0}{\alpha^2+\omega^2\tau_0^2}\right]\times\{[1-\exp(-\alpha)]^2 - 3\alpha^2\exp(-\alpha)\} + \frac{2\omega^3\tau_0^3\exp(-\alpha)}{\alpha^2+\omega^2\tau_0^2}\right\}.$$

When $\alpha \ll \omega\tau_0 \ll 1$, the energy increases linearly with frequency:

$$W^{\mathrm{w}}(\omega) = \frac{4q^2c_{\mathrm{s}}^2\omega}{3\pi c^3}(1-\alpha),$$

whereas at $\omega\tau_0 \ll \alpha$ it is a cubic function of frequency:

$$W^{\mathrm{w}}(\omega) = \frac{4q^2c_{\mathrm{s}}^2\omega^3\tau_0^2}{9\pi c^3\alpha^4}[1-\exp(-\alpha)]^2.$$

In the opposite limiting case ($\omega\tau_0 \gg 1$), the total energy is determined by the electron current parameters. The maximum achievable total energy corresponds to $\alpha = 0$:

$$W^{\mathrm{w}} = \sqrt{\frac{\pi}{2}}\frac{q^2c_{\mathrm{s}}^2}{4c^2R} \quad \text{at} \quad R \gg c\tau_0;$$

$$W^{\mathrm{w}} = \frac{2q^2c_{\mathrm{s}}^2}{3c^3\tau_0} \quad \text{at} \quad R \ll c\tau_0.$$

Thus, the time variation of the charge density in the expanding plasma leads to disruption of coherent radiation, which manifests itself in a significant reduction in the radiation power (Fig. 2). This is particularly important for low frequencies. In the region $\omega \ll 1/\tau_0$, a decrease in the frequency by an order of magnitude results in a reduction of the radiation energy by three order of magnitude.

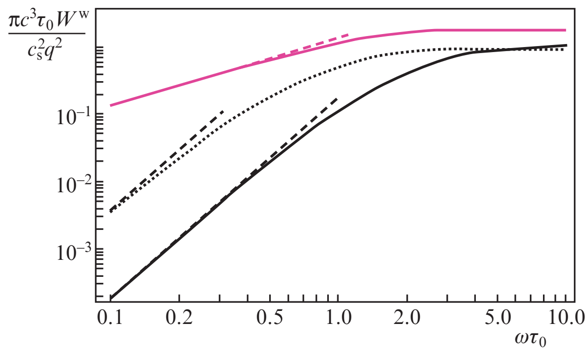

Figure 2. Energy of bulk radiation generated by the plasma expanding into vacuum as a function of the frequency for $\alpha = 1$, $R/(c\tau_0) = 0.1$ (solid black curve), $\alpha = 0$, $R/(c\tau_0) = 1$ (solid gray curve) and $\alpha = 0.3$, $R/(c\tau_0) = 1$ (dashed curve). The dashed curves correspond to the limiting case $\omega\tau_0 \ll 1$.

5. Excitation of SEWs

The magnetic field of the SEWs is given by expression (7) and in the wave zone, taking into account the asymptotic expansion of the Bessel function, has the form

$$B_{\varphi}^{\mathrm{v}}(z,\omega,\boldsymbol{r}) = \frac{2\varepsilon^2 k_0^{\mathrm{s}} k_\perp^{\mathrm{s}}}{c(\varepsilon^2-1)}\sqrt{\frac{2\pi}{k_\perp^{\mathrm{s}} r_\perp}}\exp\left[\mathrm{i}\left(k_\perp^{\mathrm{s}} r_\perp - \frac{\pi}{4}\right) - k_0^{\mathrm{s}} z\right]$$
$$\times\int_0^\infty \mathrm{d}z' \exp(-k_0^{\mathrm{s}} z') j_z(z',\omega,\boldsymbol{k}_\perp^{\mathrm{s}}), \quad (27)$$

where $k_\perp^{\mathrm{s}} = (\omega/c)\sqrt{\varepsilon/(1+\varepsilon)}$; and $k_0^{\mathrm{s}} = \omega/(c\sqrt{-1-\varepsilon})$ [20]. Since the main contribution to the energy of the surface wave is made by the field in vacuum, it is possible to estimate the spectral density of the SEW energy as ($|\varepsilon| \gg 1$)

$$\frac{\mathrm{d}W^{\mathrm{s}}}{\mathrm{d}\omega} = \frac{r_\perp c}{2\pi}\int_0^\infty \mathrm{d}z\, |B_\varphi^v(z)|^2$$
$$= \frac{2\omega\, |k_0^{\mathrm{s}}|^2}{c^2 \mathrm{Re}\, k_0^{\mathrm{s}}}\left|\int_0^\infty \exp(-k_0^{\mathrm{s}} z) j_z(z,\omega,\omega/c)\,\mathrm{d}z\right|^2. \quad (28)$$

Using (28) in the case of current (8), the expression for the emitted SEW energy takes the form

$$\frac{\mathrm{d}W^{\mathrm{s}}}{\mathrm{d}\omega} = \frac{\mathrm{d}W^{\mathrm{s}0}}{\mathrm{d}\omega}|F(\omega,\pi/2)|^2,$$

where $W^{\mathrm{s}0}(\omega)$ is the spectral density of the surface wave energy generated by the intersection of the surface by charge q;

$$\frac{\mathrm{d}W^{\mathrm{s}0}}{\mathrm{d}\omega} = \frac{2q_{\mathrm{f}}^2 V_{\mathrm{f}}^2}{c^3 |\varepsilon|^{1/2} \cos[\arctan(\varepsilon''/\varepsilon')/2]}, \quad (29)$$

and $\varepsilon = \varepsilon' + \mathrm{i}\varepsilon''$.

For the current given by expression (11) we obtain

$$\frac{\mathrm{d}W^{\mathrm{s}}}{\mathrm{d}\omega} = \frac{\mathrm{d}W^{\mathrm{s}0}}{\mathrm{d}\omega}|G(\omega,\pi/2)|^2,$$

where $W^{\mathrm{s}0}(\omega)$ is found from expression (29) when V_{f} is replaced by c_{s} and q_{f} is replaced by q. Here, the frequency dependence of the radiation energy density is determined not only by the form factor [$F(\omega)$ or $G(\omega)$], but also by the dependence on the frequency of the plasma permittivity. This leads to the fact that for metals ($\varepsilon \approx 4\pi\mathrm{i}\sigma/\omega$) the total energy of the surface wave generated by a beam of relativistic electrons at low frequencies ($\omega \ll 2c/L$) increases with frequency faster ($\propto\omega^{3/2}$) than the energy of transition radiation ($\propto\omega$) (Fig. 3):

$$W^{\mathrm{s}}(\omega) = \frac{2\sqrt{2}\, q_{\mathrm{f}}^2 \omega^{3/2}}{3c\sqrt{\pi\sigma}}.$$

Note that in this case, the SEW energy does not depend on the electron beam energy. In the opposite limit ($\omega \gg 2c/L$), the total SEW energy at $L \gg R$ can be presented as

$$W^{\mathrm{s}}(\omega) = \frac{4q_{\mathrm{f}}^2}{L}\sqrt{\frac{c}{\sigma L}}.$$

As in the case of generation of bulk waves, the maximum frequency is determined by the longitudinal size of the fast electron beam (laser pulse duration): $\omega_{\mathrm{m}} \approx c/L$.

The sharp dependence of the frequency for SEWs generated during plasma expansion into vacuum leads at $\omega < 1/\tau_0$ to a very low efficiency of the excitation of low-frequency surface waves as a result of the action of the mechanism in question (Fig. 3).

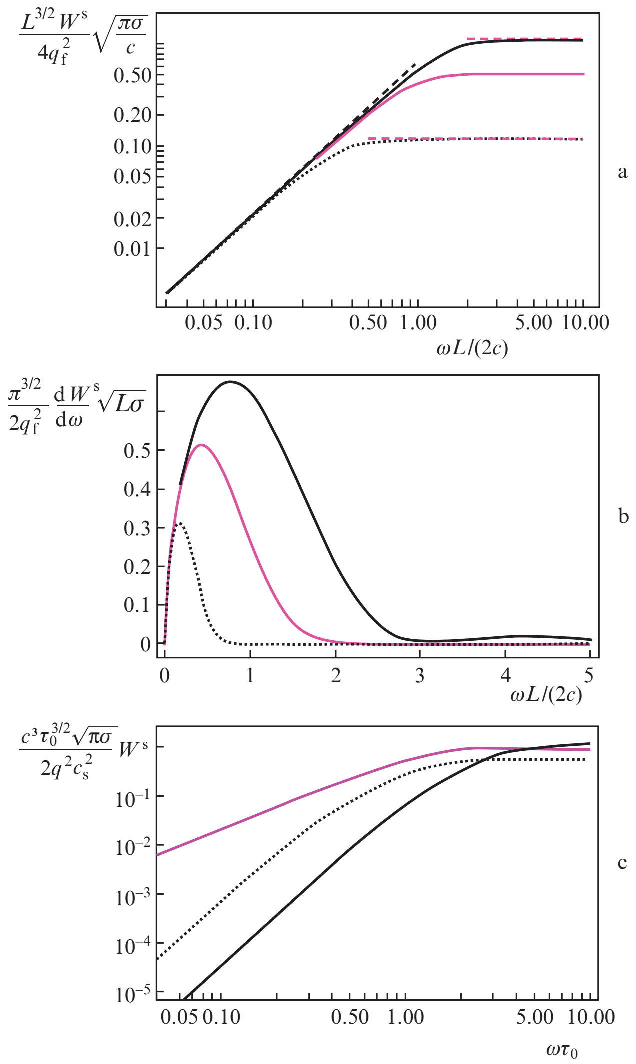

Figure 3. (a) Frequency dependences of the energy of surface waves generated by the electron bunch and (b) emission spectra for R/L = 3 (solid black curve), 1 (solid gray curve) and 3 (dashed curve), as well as (c) frequency dependence of the energy of surface waves generated by the plasma expanding into vacuum for the parameters corresponding to Fig. 2. The dashed curves in Fig. 3a correspond to the limiting cases $\omega \ll 2c/L$ and $\omega \gg 2cL$.

6. Discussion of the results

We first estimate the total energy emitted by a small part of accelerated fast electrons leaving the target. In the relativistic limit, $a_0 \gg 1$, the hot electrons are accelerated in the direction of the laser pulse propagation, pass through the target and fly from its back side. In this case, the electrons with maximum energy, capable of overcoming the potential barrier, leave the target. The number of such fast electrons is small compared with the total number of hot electrons, but they have relativistic velocities, which significantly increases the energy of their radiation. We estimate the total charge q_{f} of fast electrons leaving the target, assuming that the longitudinal beam size L is determined by the laser pulse duration τ_{L} ($L = c\tau_{\mathrm{L}}$): $q_{\mathrm{f}} = ea_0 n_{\mathrm{c}} \lambda^2 c\tau_{\mathrm{L}}\sqrt{\eta/(8\pi^2)}$. Assuming that the characteristic energy of fast electrons $\mathcal{E}_0 \approx -e\Phi_{\min} = 2T\ln[R/(r_{\mathrm{D}}\sqrt{2})]$, the radiation energy is determined by the maximum frequency ω_{m} as follows [see formula (21)]

$$W^{\mathrm{w}} = \frac{m^2c^4a_0^2\eta}{4\pi e^2}\omega_{\mathrm{m}}c\tau_{\mathrm{L}}^2\left\{\ln\left[4a_0\sqrt{\frac{\eta}{2}}\ln\left(\frac{\pi\sqrt{2}R}{\lambda}\right)\right]-\frac{1}{2}\right\}.$$

Or, expressing the energy in joules, we have

$$W^{\mathrm{w}} \approx 2\times10^{-7}\left(\frac{\omega_{\mathrm{m}}}{10^{12}\ \mathrm{s}^{-1}}\right)\left(\frac{\tau_{\mathrm{L}}}{30\ \mathrm{fs}}\right)^2a_0^2(1+\ln a_0).$$

In the estimates used here and below, $\eta = 0.3$. We can estimate the conversion efficiency of the laser pulse energy $W_{\mathrm{L}} = \pi R^2\tau_{\mathrm{L}}I_0$ into the secondary transition radiation energy in the frequency range up to ω_{m}:

$$\frac{W^{\mathrm{w}}}{W_{\mathrm{L}}} = \frac{\eta}{2\pi^3}\omega_{\mathrm{m}}\tau_{\mathrm{L}}\frac{\lambda^2}{R^2}\left\{\ln\left[4a_0\sqrt{\frac{\eta}{2}}\ln\left(\frac{\pi\sqrt{2}R}{\lambda}\right)\right]-\frac{1}{2}\right\}.$$

This value is slightly dependent on the laser radiation intensity and reaches a maximum for tightly focused pulses ($R \approx \lambda$):

$$\frac{W^{\mathrm{w}}}{W_{\mathrm{L}}} \approx 9\times10^{-6}\left(\frac{\omega_{\mathrm{m}}}{10^{12}\ \mathrm{s}^{-1}}\right)\left(\frac{\tau_{\mathrm{L}}}{30\ \mathrm{fs}}\right)^2\left(\frac{4\lambda}{R}\right)^2(1+\ln a_0).$$

The typical maximum frequency of transition radiation is determined by the longitudinal or transverse sizes of the beam of fast electrons [see formulae (22), (23)]. For relativistic laser pulses, the longitudinal size of the laser pulse, determined by its duration, is greater than its transverse size, which leads to the formation of a hot electron beam with $L > R$. In this case, the maximum frequency is independent of the focal spot diameter and is completely determined by the laser pulse duration: $\omega_{\mathrm{m}} \approx 1/\tau_{\mathrm{L}}$. For short ($\tau_{\mathrm{L}} \approx 30$ fs) tightly focused [$R \approx (2-10)\lambda$] laser pulses, these characteristic frequencies amount to $\sim$30 THz. Thus, up to 30% of energy of total bulk radiation lies in the THz frequency region (up to 10 THz). For example, a laser pulse with an energy of 2 J ($R = 4\lambda$, $\tau_{\mathrm{L}} = 30$ fs, $a_0 = 10$), the energy of bulk radiation in the THz region reaches 0.6 mJ, which corresponds to the $\sim$0.03% conversion. Increasing the pulse duration results in a shift of the transition radiation spectrum to lower frequencies. Thus, at a pulse duration $\tau_{\mathrm{L}} > 100$ fs, virtually the entire spectrum of bulk radiation will be in the THz region.

In the case of plasma expansion into vacuum, the total number of hot electrons participating in the formation of charge separation, is somewhat greater than the number of fast electrons: $q = ea_0n_{\mathrm{c}}\sqrt{\eta/8}\,R^2\lambda$, whereas the characteristic expansion velocity (equal to several ion-sound velocities) is much lower than the velocity of fast electrons:

$$\frac{c_{\mathrm{s}}}{c} = \sqrt{\sqrt{\frac{\eta a_0^2}{2}}\frac{Zm}{M}} \ll 1,$$

where Z is the ion charge. Below we assume that in estimates $Zm/M = m/(2m_{\mathrm{p}}) \approx 0.00027$ (m_{p} is the proton mass). The smallness of the expansion velocity results in a significantly lower (by about three to four orders of magnitude) total energy of the secondary radiation compared to the case of fast electron beam emission. In practice, the radiation energy is much less because of the rapid disappearance of the charge separation field (and currents) on the characteristic scale of the order of the Debye length, which for high-power laser pulses is of the order of the laser radiation wavelength. Furthermore, as shown above, the discharge during absorption ($\alpha = 1$) leads to the 'cutting' of the low-frequency radiation field. It is possible to estimate the total energy for the low-frequency range as

$$W^{\mathrm{w}}(\omega) = 9.6\times10^{-3}\frac{\eta a_0^2\omega_{\mathrm{m}}^3R^4m^2c}{e^2}.$$

Thus, for the above considered laser pulse, the radiation energy in the THz region is only 0.08 μJ, and the conversion rate is

$$\frac{W^{\mathrm{w}}}{W_{\mathrm{L}}} = 1.95\times10^{-3}\eta\frac{\lambda^2R^2\omega_{\mathrm{m}}^3}{c^4\tau_{\mathrm{L}}} \approx 4\times10^{-8}.$$

Therefore, the efficiency of generation of secondary radiation as a result of plasma expansion into vacuum is always much less than the efficiency of generation of radiation due to the emission of fast electron beams. It is worth noting different angular directions of the generated transition radiation. If the fast electron beam causes the appearance of radiation directed along the beam axis (almost along the normal to the target or along the direction of incidence of the laser pulse onto the target), the radiation associated with the plasma expansion into vacuum is directed generally along the target surface.

The efficiency of SEW generation is below the generation efficiency of bulk radiation. This is due both to the high permittivity of metals (plasma) and with its additional dependence on the frequency. The latter leads to a decrease in the SEW energy in the low-frequency region

$$W^{\mathrm{s}} = \frac{m^2c^4a_0^2\eta}{6e^2\sqrt{2\pi\sigma}}\omega_{\mathrm{m}}^{3/2}c\tau_{\mathrm{L}}^2$$

or

$$W^{\mathrm{s}} \approx 1.6\times10^{-8}\left(\frac{\omega_{\mathrm{m}}}{10^{13}\ \mathrm{s}^{-1}}\right)^{3/2}\left(\frac{\tau_{\mathrm{L}}}{30\ \mathrm{fs}}\right)^2a_0^2.$$

In the estimates we used the value $\sigma \approx 10^{17}\ \mathrm{s}^{-1}$, characteristic of metals, and considered only SEWs generated by a relativistic electron beam. For the above example, the energy of the surface wave in the THz frequency range is equal to $\sim$1.7 μJ, which corresponds to a conversion on the order of 10^{-6}. The surface wave arising from the emission of the fast electron beam contains more energy than the bulk wave generated by the plasma expanding into vacuum.

Note also that with distance from the source the intensity of surface waves, due to geometrical factor, decreases more slowly than that of the bulk waves, and therefore, away from the source SEWs can govern the experimentally measured effect. Thus, in measuring THz radiation along the surface of a bounded target, we can expect the main contribution to be made by the radiation of surface waves rather than by radiation produced as a result of expansion of plasma into vacuum. It is possible that at a certain design of the surface (for example, using strips of dielectric materials [21]) and/or using a specific method of focusing of laser-generated surface waves, we can achieve a significant concentration of its energy and pass to solving the problem of a directional THz antenna.

7. Conclusions

Thus, this paper describes the theory of generation of secondary electromagnetic waves by the currents resulting from irradiation of a target by short intense laser pulses. As a source of these waves we consider the currents of fast electrons leaving the target and the currents that arise due to violation of quasi-neutrality of the hot plasma bunch expanding from the target surface. It is shown that the generation efficiency of bulk and surface radiation is much greater during emission of a laser-generated beam of relativistic electrons leaving the target than in the case of plasma expansion into vacuum. The difference can be several orders of magnitude, particularly in the low-frequency (terahertz) region of the spectrum. We have obtained simple expressions relating the energy of secondary electromagnetic radiation with the laser pulse parameters.

The generation efficiency of surface waves by a beam of fast electrons exceeds the generation efficiency of bulk transition radiation in the charge-uncompensated plasma expansion into vacuum. Given the lower divergence of surface waves propagating from the source, we cannot exclude the fact that SEWs are responsible for the observed in experiments [5, 6] THz radiation propagating along the surface of a metal target, which, according to our estimates, prevails over radiation of the dipole field of charge separation during plasma expansion.

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