

LIGHT SCATTERING

DOI: 10.1070/QEL16202

Conditions for circular polarisation retention in highly scattering media

E.E. Gorodnichev, A.I. Kuzovlev, D.B. Rogozkin

Abstract. **We analyse conditions for circular polarisation retention in multiple scattering of light by disordered media. The effect of circular polarisation memory is shown to appear when the depolarisation cross section is much smaller than the transport cross section of elastic scattering. This situation occurs in media composed of large (compared to the wavelength of light) weakly refracting scatterers and of resonant Mie particles. Absorption in the medium is shown to result in a significant increase in the depolarisation length.**

Keywords: *multiple scattering of polarised light, Mie particle, depolarisation, circular polarisation.*

1. Introduction

Interest in the study of the depolarisation effect in highly scattering media with different-size inhomogeneities is caused by its use in diffusion spectroscopy of colloidal solutions [1, 2] and, especially, in the diagnostics of biological tissues [3–6]. Among the most notable observed effects we should mention a slow decrease in the degree (attenuation) of circular polarisation in multiple scattering by media with large weakly refracting particles [7, 8]. In contrast to linear polarisation that always attenuates at distances not exceeding the transport mean free path of elastic scattering [8], circular polarisation is 'retained' even after isotropisation of the light flux over directions. An especially strongly pronounced effect of circular polarisation retention has been discovered recently in the scattering of light by dielectric particles of spherical shape in the region of the first two Mie resonances [9, 10].

The differences in the attenuation of circular and linear polarisations of the waves are due to two depolarisation mechanisms, namely, 'dynamic' and 'geometric' [8]. Dynamic depolarisation arises from differences in amplitudes of cross-polarised scattered waves. Depolarisation by the 'geometric' mechanism is caused by the Rytov effect, which results in a turn of the polarisation plane of linearly polarised light along with a change in the direction of a light ray [11]. In the case of multiple scattering of light, the orientation of polarisation planes of different rays becomes chaotic, and depolarisation of light occurs with isotropisation of the beam over directions. In scattering of circularly polarised light, depolarisation is due to the dynamic mechanism, while in scattering of linearly polarised light, depolarisation is due to both mechanisms. Therefore, the linear polarisation decay length obviously cannot exceed the transport mean free path of elastic scattering, at which the light flux becomes isotropic over directions [8, 12, 13].

In media with large weakly refractive scatterers, single scattering occurs mainly through small angles where the amplitudes $A_{||}$ and $A_{\perp}$ of cross-polarised scattered waves differ weakly. Therefore, dynamic depolarisation occurs slowly and becomes noticeable at distances greater than the transport mean free path of elastic scattering [8, 12, 13].

Light scattering in dielectric particles with a relatively high refractive index near Mie resonances is determined in the first approximation by electric and magnetic dipole contributions [14–18]. This approximation ensures the fulfilment of the so-called first Kerker condition [19], when 'exactly backward' scattering is absent and the equality $A_{||} = A_{\perp}$ is achieved at all scattering angles. The dynamic mechanism of depolarisation in this case is 'turned off', and circular polarisation is completely retained in scattering by a single particle and by a disordered ensemble of particles [9]. We have shown in Ref. [10] that the results of paper [9] can be considered only as a first approximation. The quadrupole contribution and contribution of higher-order multipoles into amplitudes $A_{||}$ and $A_{\perp}$ make it impossible to exactly fulfil the first Kerker condition and hence to completely retain circular polarisation of scattered light.

In this paper, we analyse conditions under which the effect of a slow decay of circular polarization should be observed in a multiply scattering medium. It is shown that the rate of depolarisation of light, first, depends on the ratio $\sigma_{\rm tr}/\sigma_{\rm dep}$, where

$$\sigma_{\rm dep} = \frac{1}{2}\int {\rm d}n' \left| A_{||}(\boldsymbol{nn}') - A_{\perp}(\boldsymbol{nn}') \right|^2 \qquad (1)$$

is the depolarisation cross section introduced in Ref. [8]; $\sigma_{\rm tr}$ is the transport cross section of elastic scattering; and $\boldsymbol{n}$ and $\boldsymbol{n}'$ are the unit vectors in the direction of light propagation before and after scattering, respectively. As follows from calculations by Mie formulas presented in this paper, the dependence of $\sigma_{\rm tr}/\sigma_{\rm dep}$ on the parameter k_0a (k_0 is the wave number, and a is the particle radius) strongly changes with increasing refractive index of the particles. For weakly refracting particles, the ratio $\sigma_{\rm tr}/\sigma_{\rm dep}$ increases with k_0a and reaches a limiting value at $k_0a \gg 1$. For particles with a large refractive index n, the dependence of $\sigma_{\rm tr}/\sigma_{\rm dep}$ on k_0a has a sharp peak at the first Kerker point k_0an = 2.74. The maximum value of $\sigma_{\rm tr}/\sigma_{\rm dep}$ increases with increasing refractive index. The depolarisation length of circularly polarised light $l_{\rm circ}$ is calculated by a

E.E. Gorodnichev, A.I. Kuzovlev, D.B. Rogozkin National Research Nuclear University 'MEPhI', Kashirskoe sh. 31, 115409 Moscow, Russia; e-mail: gorodn@theor.mephi.ru

Received 26 August 2016
Kvantovaya Elektronika **46** (10) 947–952 (2016)
Translated by I.A. Ulitkin

method based on the solution of the vector radiative transfer equation in the asymptotic regime [13] and in the spatial diffusion approximation. It is found that for a disordered system of resonant Mie particle, the effect of circular polarisation retention is expressed most strongly, i.e. the depolarisation length $l_{\rm circ}$ in the vicinity of the first Kerker point can be more than an order of magnitude greater than the transport mean free path $l_{\rm tr}$. It is also shown that absorption of light in a medium can lead to a significant increase in the depolarisation length $l_{\rm circ}$.

2. General relations

Multiple scattering of polarised light in a disordered medium is described by the vector radiative transfer equation for the Stokes parameters $\hat{S} = (I, Q, U, V)$ [20, 21]

$$\Big[\mu\frac{\partial}{\partial z} + n_0(\sigma+\sigma_{\rm a})\Big]\hat{S}(z,\mu)$$
$$= n_0\int {\rm d}\boldsymbol{n}'\,\hat{L}(\pi-\beta)\hat{d}(\boldsymbol{nn}')\hat{L}(-\beta')\hat{S}(z,\mu'), \qquad (2)$$

where σ and $\sigma_{\rm a}$ are the cross sections of elastic scattering and absorption; n_0 is the number of scatterers per unit volume; $\mu = \boldsymbol{nn}_{\rm int}$; $\mu' = \boldsymbol{n}'\boldsymbol{n}_{\rm int}$; $\boldsymbol{n}_{\rm int}$ is the unit vector along the internal normal to the front surface of a sample; and the z axis is perpendicular to the layer surface. Matrices $\hat{L}(\pi-\beta)$ and $\hat{L}(-\beta')$ describe rotations in the space of directions $\boldsymbol{n}$. Expressions for the matrices $\hat{L}$ and angles β and β' are presented in Ref. [20]. The matrix $\hat{d}(\boldsymbol{nn}')$ in the scattering plane, appearing in (2), is given by [20, 21]

$$\hat{d}(\boldsymbol{nn}') = \begin{pmatrix} a_1 & b_1 & 0 & 0 \\ b_1 & a_1 & 0 & 0 \\ 0 & 0 & a_2 & b_2 \\ 0 & 0 & -b_2 & a_2 \end{pmatrix}. \qquad (3)$$

The elements a_1, a_2, b_1 and b_2 are expressed in terms of the amplitudes of the cross-polarised singly scattered waves,

$$a_1 = \frac{1}{2}\big(|A_\parallel|^2 + |A_\perp|^2\big),\ \ a_2 = {\rm Re}A_\parallel A_\perp^*,$$
$$b_1 = \frac{1}{2}\big(|A_\parallel|^2 - |A_\perp|^2\big),\ \ b_2 = {\rm Im}A_\parallel A_\perp^*, \qquad (4)$$

and are calculated according to the Mie formulas [20]. The value of a_1 has the meaning of a differential single-scattering cross section.

In the case of normal incidence of a circularly polarised wave on the surface of a medium, the vector equation (2) is reduced to two independent systems of equations that couple pairwise the first and second,third and fourth Stokes parameters, respectively (see, e.g., [13, 22]):

$$\Big[\mu\frac{\partial}{\partial z} + n_0(\sigma+\sigma_{\rm a})\Big]\begin{pmatrix} I(z,\mu) \\ Q(z,\mu)\end{pmatrix}$$
$$= n_0\int{\rm d}\boldsymbol{n}'\begin{pmatrix} a_1 & b_1\cos\beta \\ b_1\cos\beta' & a_1\cos\beta\cos\beta' - a_2\sin\beta\sin\beta'\end{pmatrix}\begin{pmatrix} I(z,\mu') \\ Q(z,\mu')\end{pmatrix}, \qquad (5)$$

$$\Big[\mu\frac{\partial}{\partial z} + n_0(\sigma+\sigma_{\rm a})\Big]\begin{pmatrix} U(z,\mu) \\ V(z,\mu)\end{pmatrix}$$
$$= n_0\int{\rm d}\boldsymbol{n}'\begin{pmatrix} a_2\cos\beta\cos\beta' - a_1\sin\beta\sin\beta' & b_2\cos\beta \\ -b_2\cos\beta' & a_2\end{pmatrix}\begin{pmatrix} U(z,\mu') \\ V(z,\mu')\end{pmatrix}. \qquad (6)$$

For a unit incident flux, boundary conditions for (5) and (6) have the form

$$I(z=0,\mu) = V(z=0,\mu) = \frac{1}{2\pi}\delta(1-\mu). \qquad (7)$$

The solution of system (5), (6) allows one to determine the polarisation state of multiply scattered light at any values of the medium thickness L.

It is known [23] that with increasing thickness L, the solution of equations (5), (6) tends to the asymptotic form when the angular and spatial dependences of the Stokes parameters are separated out. The angular dependence does not change with L. The intensity $I(L, \mu)$ in a nonabsorbing medium is proportional to $1/L$. In the presence of absorption, the intensity decreases exponentially: $I \propto \exp(-\varepsilon_I L)$. As for the Stokes parameter V characterising circular polarisation of light, in the asymptotic regime it always decreases according to the law $I \propto \exp(-\varepsilon_V L)$.

In an experiment, the dependence of the degree of circular polarisation of scattered light $P_{\rm C} = V/I$ on the thickness L is conveniently characterised by a circular polarisation decay length $l_{\rm circ}$ [3–5, 7]. In a nonabsorbing medium, we have $l_{\rm circ} = \varepsilon_V^{-1}$, and in the presence of absorption we have

$$l_{\rm circ} = (\varepsilon_V - \varepsilon_I)^{-1}. \qquad (8)$$

To determine the depolarisation coefficients ε_I and ε_V, there is no need to solve system (5) and (6) for all values of z. It is sufficient to consider only the asymptotic regime. In this case, system (5), (6) is reduced to a system of characteristic equations (see Appendix A in Ref. [13]). The values of ε_I and ε_V are found as the smallest roots of this system [13].

3. Basic mode approximation

If nondiagonal elements b_1 and b_2 are small compared with diagonal elements a_1 and a_2, separate equations for I and V can be retained in system (5) and (6). These equations follow from equations (5) and (6), if we neglect the contribution of terms quadratic with respect to small values of b_1 and b_2 [12, 13], and have the form

$$\Big[\mu\frac{\partial}{\partial z} + n_0(\sigma+\sigma_{\rm a})\Big]I(z,\mu) = n_0\int{\rm d}\boldsymbol{n}'a_1(\boldsymbol{nn}')I(z,\mu'), \qquad (9)$$

$$\Big[\mu\frac{\partial}{\partial z} + n_0(\sigma+\sigma_{\rm a})\Big]V(z,\mu) = n_0\int{\rm d}\boldsymbol{n}'a_2(\boldsymbol{nn}')V(z,\mu'). \qquad (10)$$

Equations (9) and (10) correspond to the basic mode approximation [12, 13] in the vector radiative transfer equation.

The difference between the differential scattering cross sections a_1 and a_2 in (9) and (10) leads to an exponential decay of the basic mode of circular polarisation V even in the absence of absorption. The quantity

$$\sigma_{\rm dep} = \int {\rm d}\boldsymbol{n}'[a_1(\boldsymbol{nn}') - a_2(\boldsymbol{nn}')] \qquad (11)$$

plays the role of an additional absorption cross section in equation (10). In its physical meaning $\sigma_{\rm dep}$ is a depolarisation cross section of circularly polarised light. Formula (1) is obtained from (11) if we substitute expression (4) into (11).

The depolarisation cross section (11) introduced in Ref. [8] is the main factor determining the rate of a decrease in the degree of circular polarisation of light in the process of multiple scattering. As shown in Ref. [8] (see also [12, 13]), if the depolarisation cross section $\sigma_{\rm dep}$ is small compared to the transport cross section

$$\sigma_{\rm tr} = \int {\rm d}\boldsymbol{n}'(1 - \boldsymbol{nn}')a_1(\boldsymbol{nn}'), \qquad (12)$$

the effect of circular polarisation retention should be observed in a multiply scattering medium, i.e. light is depolarised after a transition to the spatial diffusion regime.

Thus, a necessary condition for circular polarisation retention is the proximity of the diagonal elements a_1 and a_2 of the scattering matrix (3).

Since in forward scattering of light the equality $a_1 = a_2$ always holds true [20], scattering by large (greater than the wavelength) weakly refractive particles is one of the cases when the depolarisation cross section is small. Single scattering in a medium consisting of such particles occurs mainly through small angles. The most typical examples of media with large particles can be water droplets in the air and aqueous suspension of latex particles, which are most frequently used in experiments [3–7] (dependences of the elements a_1, a_2, b_1 and b_2 on the angle γ between the vectors $\boldsymbol{n}$ and $\boldsymbol{n}'$ for this medium are shown in Fig. 1a).

Another example, when the depolarisation cross section is small, is scattering of light by resonant Mie particles with a large refractive index [9, 14–18]. As shown in Ref. [15], scattering in this case is determined with good accuracy by the electric and magnetic dipole contributions, and the first Kerker condition is satisfied between the first two Mie resonances when $a_1 \approx a_2$ at all angles. Angular dependences of the elements a_1, a_2, b_1 and b_2 at the first Kerker point are shown in Fig. 1b. In the approximation, when only the electric and magnetic dipole contributions are taken into account, the equalities $A_\parallel = A_\perp$ hold true, and therefore, $a_1 = a_2$, $b_1 = b_2 = 0$. The inclusion of the quadrupole contribution and multipole contributions of higher order into the scattered field violates these equalities and leads, in particular, to the difference between the elements a_1 and a_2 near the backward direction (Fig. 1b).

Dependences of the ratio $\sigma_{\rm tr}/\sigma_{\rm dep}$ on the parameter k_0an is shown in Fig. 2. According to calculations, for weakly refracting particles, this ratio increases with their size. The peak at the first Kerker point is present, but its amplitude is less than the value of $\sigma_{\rm tr}/\sigma_{\rm dep}$ at large k_0an. With increasing refractive index the ratio $\sigma_{\rm tr}/\sigma_{\rm dep}$ at the first Kerker point sharply increases, and at large k_0an it decreases and proves to be on the order of unity. Therefore, circular polarisation retention is observed at large radii a and at the Kerker point for weakly refracting and highly refractive particles, respectively.

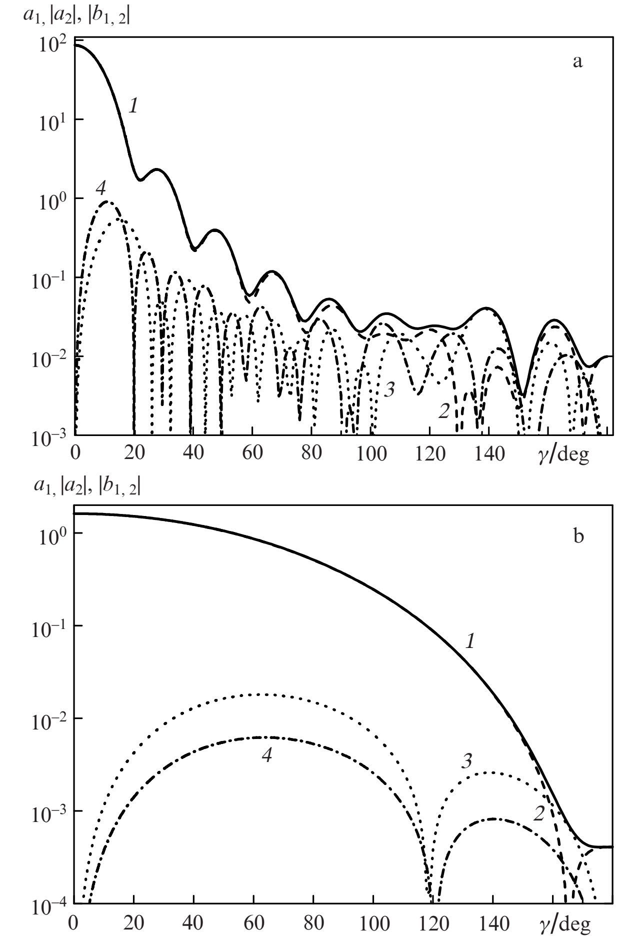

Figure 1. Angular dependences of scattering matrix elements (*1*) a_1, (*2*) $|a_2|$, (*3*) $|b_1|$ and (*4*) $|b_2|$ for (a) latex particles in water (n = 1.2) and (b) Si particles (n = 3.5) at the first Kerker point (k_0a = 0.784).

This dependence of $\sigma_{\rm tr}/\sigma_{\rm dep}$ directly affects the value of the circular polarisation decay length $l_{\rm circ}$. The results of the calculations for the ratio $l_{\rm circ}/l_{\rm tr}$ using the characteristic equations [13], corresponding to system (5), (6), are given in Table 1 and shown Figs 3 and 4. The calculations were car-

Table 1.

Material	Refractive index (λ = 1.6 μm [24])	$k_0a{\rm Re}n$	$\sigma_{\rm dep}/\sigma_{\rm tr}$	$\sigma_{\rm a}/\sigma_{\rm tr}$	$l_{\rm circ}/l_{\rm tr}$
CdTe	2.73+i0.25	2.708	0.31	1.24	10.0 (10.4)
AlAs	2.9	2.743	0.0016	0	14.6 (14.6)
GaP	3.05	2.745	0.0012	0	16.5 (16.5)
InP	3.15	2.744	0.0010	0	17.9 (17.9)
AlSb	3.28	2.745	0.0009	0	19.3 (19.3)
GaAs	3.37	2.743	0.0008	0	20.8 (20.8)
Si	3.47	2.745	0.0007	0	22.0 (22.0)
PbS	4.23+i0.35	2.673	0.35	2.79	28.3 (30.9)
Ge	4.24	2.743	0.0003	0	32.6 (32.6)

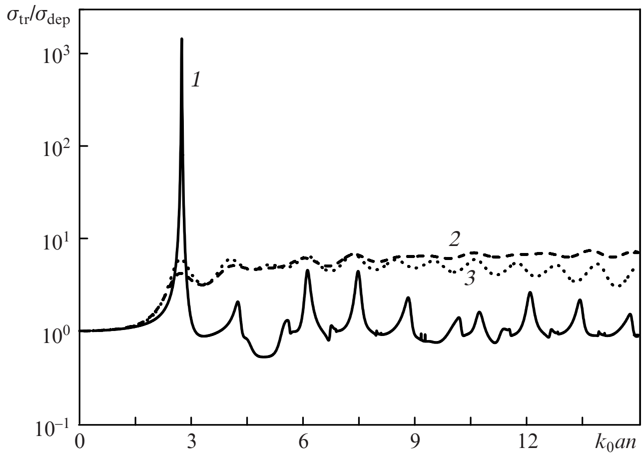

Figure 2. Dependences of the ratio σ_{tr}/σ_{dep} for Si particles [n = 3.5; (*1*)], aqueous suspension of latex particles [n = 1.2; (*2*)] and water droplets in the air [n = 1.33; (*3*)] on the parameter k_0an.

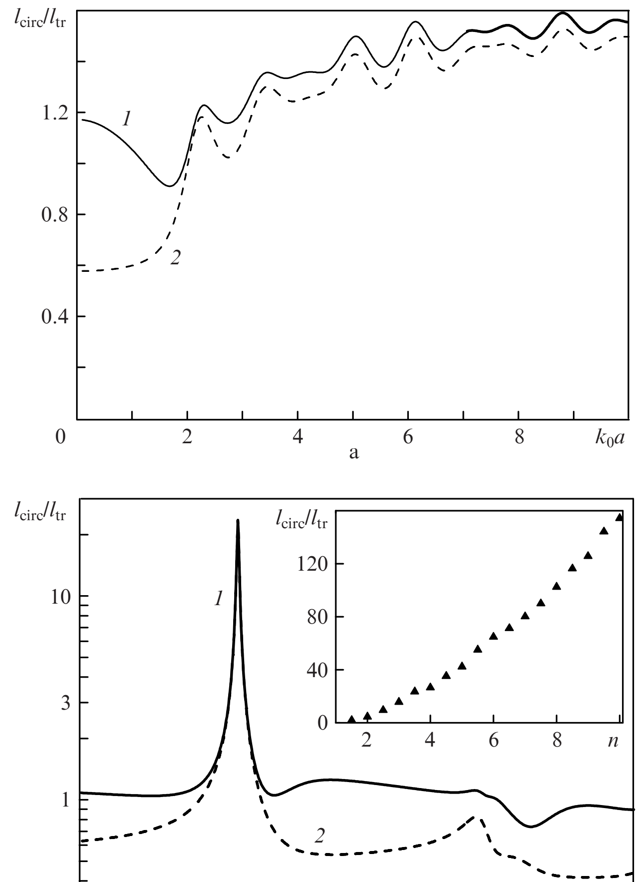

Figure 3. Dependences of the ratio l_{circ}/l_{tr} on the parameter k_0a for (a) an aqueous suspension of latex particles and (b) a disordered ensemble of Si particles, obtained (*1*) with the numerical solution of the characteristic equation [13] and (*2*) by the diffusion equation (16). The inset shows the dependence of l_{circ}/l_{tr} at the first Kerker point on the refractive index of particles.

ried out for commonly used semiconductor materials [24] and an aqueous suspension of latex particles most frequently used in laboratory experiments (see, e.g., [3, 4, 7]). Table 1 also shows the values of $k_0a\mathrm{Re}\,n$, σ_{dep}/σ_{tr}, σ_a/σ_{tr} at the first Kerker point and the results of calculations of l_{circ}/l_{tr} using the characteristic equations corresponding to the approximation of basic modes (9), (10) (see the value in parentheses).

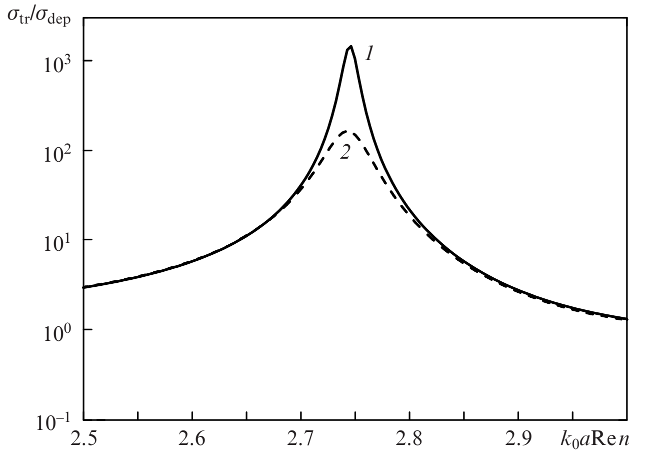

Figure 4. Dependences of the ratio σ_{tr}/σ_{dep} of Si particles on the parameter $k_0a\mathrm{Re}\,n$ for wavelengths λ = 1.6 μm (n = 3.5 [24]; (*1*)) and 0.6 μm (n = 3.94 + i0.03 [24]; (*2*)).

4. Effect of circular polarisation 'memory' in spatial diffusion of light

If $\sigma_{dep} \ll \sigma_{tr}$, then equations (9) and (10) can be solved using the spatial diffusion approximation [8, 12, 13]. This approximation assumes that the angular distributions of the Stokes parameters $I(z, \mu)$ and $V(z, \mu)$ are close to isotropic and these parameters are determined by expressions [8]

$$I(z,\mu) = \frac{1}{4\pi}\left[I(z) - \mu l_{tr}\frac{\partial I(z)}{\partial z}\right],$$
$$V(z,\mu) = \frac{1}{4\pi}\left[V(z) - \mu l_{tr}\frac{\partial V(z)}{\partial z}\right]. \qquad (13)$$

Quantities $I(z)$ and $V(z)$ in (13) obey the diffusion equations

$$\frac{\partial^2 I(z)}{\partial z^2} - \varepsilon_I^2 I(z) = 0, \quad \frac{\partial^2 V(z)}{\partial z^2} - \varepsilon_V^2 V(z) = 0, \qquad (14)$$

where $\varepsilon_I = n_0(3\sigma_{tr}\sigma_a)^{1/2}$; and $\varepsilon_V = n_0[3(\sigma_{tr} + \sigma_{dep} + \sigma_a)\times(\sigma_{dep} + \sigma_a)]^{1/2}$. In a nonabsorbing medium the solutions of these equations have the form

$$I(z) = 3^{3/2}\frac{L - z}{L}, \quad V(z) = 3^{3/2}\frac{\sinh(\varepsilon_V(L - z))}{\sinh(\varepsilon_V L)}, \qquad (15)$$

which implies that the depolarisation length is

$$l_{circ} = \sqrt{\frac{l_{tr}l_{dep}}{3}}, \qquad (16)$$

where $l_{\rm tr} = (n_0\sigma_{\rm tr})^{-1}$ is the transport mean free path of elastic scattering, and $l_{\rm dep} = (n_0\sigma_{\rm dep})^{-1}$ is the mean free path for depolarising collisions. The results of the calculation by formula (16) and the numerical solution of the characteristic equation [13] are shown in Fig. 3.

It should be noted that the spread of the particles in size leads to a reduction of the maximum value of the ratio $l_{\rm circ}/l_{\rm tr}$ near the first Kerker point. For example, at $\Delta a/a = 0.05$ we obtain $l_{\rm circ}/l_{\rm tr} = 4$. However, even this number is much higher than the values obtained previously for large (greater than the wavelength) particles (see Fig. 3a, and also [12, 13]). Thus, at the first Kerker point a minimum depolarising ability of a medium is reached.

5. Effect of absorption

If the light is scattered by absorbing particles, then there appear two effects. On the one hand, absorption results in an increase in $\sigma_{\rm dep}$ and smoothing of the dependence of $\sigma_{\rm dep}$ on k_0a. This is illustrated by the results of calculations given in Table 1 and in Fig. 4. On the other hand, the presence of absorption in a medium leads to a suppression of multiple scattering in the backward hemisphere [12], i.e. the angular intensity distribution of the scattered light in the asymptotic regime is anisotropic (elongated in the forward direction). Since in this region of angles the elements $a_1(\gamma)$ and $a_2(\gamma)$ do not virtually differ, it can give rise to a noticeable increase in the depolarisation length. This effect is also preserved, if the particles themselves do not absorb light and are immersed in an absorbing medium.

The effect of absorption on a decrease in the depolarisation rate can be easily demonstrated in the diffusion approximation when the ratios $\sigma_{\rm dep}/\sigma_{\rm tr}$ and $\sigma_{\rm a}/\sigma_{\rm tr}$ are assumed small. In the presence of absorption, formula (16) with (8) taken into account is generalised as follows [25]:

$$l_{\rm circ} = \frac{1}{\varepsilon_V - \varepsilon_I} = l_{\rm d}\left(\frac{l_{\rm dep}}{l_{\rm a}}\right)\left(1 + \sqrt{1 + \frac{l_{\rm a}}{l_{\rm dep}}}\right)$$

$$= \begin{cases} \sqrt{\dfrac{l_{\rm tr} l_{\rm dep}}{3}}, & l_{\rm dep} < l_{\rm a}, \\ 2l_{\rm d}\dfrac{l_{\rm dep}}{l_{\rm a}}, & l_{\rm dep} > l_{\rm a}, \end{cases} \tag{17}$$

where $l_{\rm d} = \varepsilon_I^{-1} = \sqrt{l_{\rm tr} l_{\rm a}/3}$ is the diffusion length, at which the intensity I undergoes a transition to the asymptotic propagation regime; and $l_{\rm a}$ is the absorption length. According to (17), in the case of a relatively strong absorption ($l_{\rm a} < l_{\rm dep}$) the depolarisation length $l_{\rm circ}$ exceeds the diffusion length $l_{\rm d}$, i.e. the degree of circular polarisation decreases after a transition to the asymptotic regime. With increasing absorption, the diffusion equation (17) becomes inapplicable. At arbitrary values of the ratios $l_{\rm a}/l_{\rm tr}$ and $l_{\rm dep}/l_{\rm tr}$, the length $l_{\rm circ}$ should be calculated using the solution of the characteristic equations corresponding to systems (5), (6) or, if we neglect the off-diagonal elements b_1 and b_2, to systems (9), (10).

The results of a numerical calculation of $l_{\rm circ}$ for an absorbing medium with the vector radiative transfer equations (5) and (6) are shown in Table 1 and in Fig. 5. For comparison, Fig. 5 shows a dependence of $l_{\rm circ}/l_{\rm tr}$, obtained in the diffusion approximation. Note that even at $l_{\rm a}/l_{\rm tr} < 1$, when the spatial diffusion approximation is not applicable, the diffusion equation (17) makes it possible to evaluate $l_{\rm circ}$, with a small error

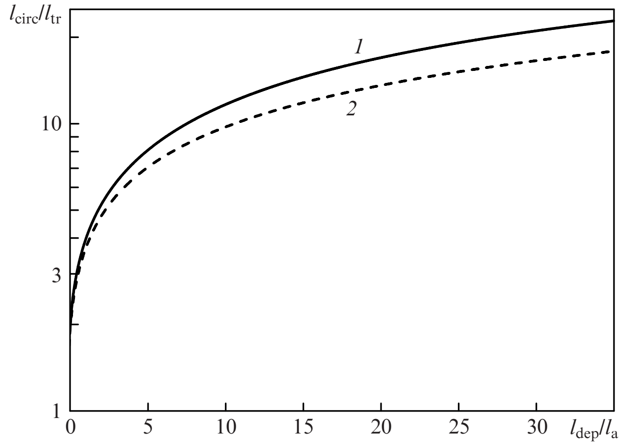

Figure 5. Dependences of the ratio $l_{\rm circ}/l_{\rm tr}$ on the ratio $l_{\rm dep}/l_{\rm a}$ for latex particles immersed in an absorbing medium, obtained (*1*) with the numerical solution of the system of characteristic equations [13] and (*2*) by the diffusion formula (17).

[even at $l_{\rm a}/l_{\rm tr} = 5$ the value of $l_{\rm circ}$, calculated by formula (17), differs from the exact one by 25%].

The applicability of the diffusion approximation for the optical wavelength range ($\lambda = 0.6$ μm) is also confirmed by the calculations. In particular, for AlSb ($n = 4.01 + {\rm i}0.006$) and Si ($n = 3.94 + {\rm i}0.03$) [24], the ratio $l_{\rm circ}/l_{\rm tr}$ in the diffusion approximation is 517.5 and 115.3, respectively, and the solution of the system of characteristic equations yields $l_{\rm circ}/l_{\rm tr} = 526.0$ and 136.8.

Figure 6 shows how $l_{\rm circ}/l_{\rm tr}$ changes at the first Kerker point with increasing imaginary part of the refractive index (i.e. the light absorption coefficient in the material of particles). With a growth of the imaginary part, absorption and depolarisation cross sections increase monotonically. The competition of these factors leads to a nonmonotonic dependence of $l_{\rm circ}/l_{\rm tr}$. The reduction of the ratio $l_{\rm circ}/l_{\rm tr}$ begins almost simultaneously with the growth of the cross-section ratios $\sigma_{\rm dep}/\sigma_{\rm tr}$.

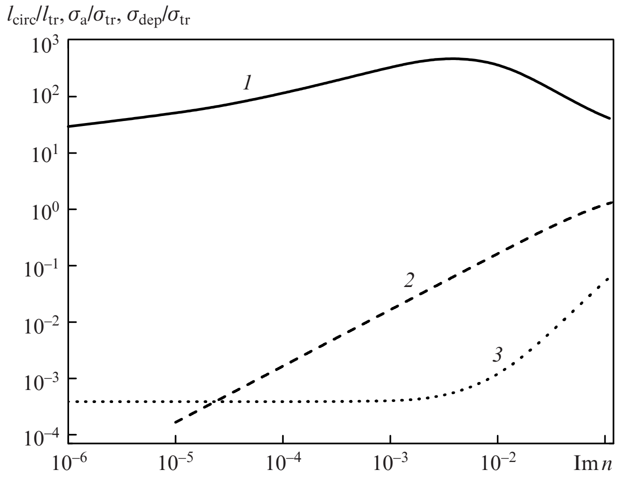

Figure 6. Dependences of the ratios (*1*) $l_{\rm circ}/l_{\rm tr}$, (*2*) $\sigma_{\rm a}/\sigma_{\rm tr}$ and (*3*) $\sigma_{\rm dep}/\sigma_{\rm tr}$ on the imaginary part of the refractive index at a given value of the real part of the refractive index of particles (Ren = 4).

6. Conclusions

We have shown that in multiple scattering of circularly polarised light, the media consisting of scatterers, for which the depolarisation cross section σ_{dep} is much less than the transport cross section σ_{tr}, have abnormal depolarising properties, i.e. circular polarisation attenuates after the establishment of the diffusion regime.

The results of this work may be of interest in the study of highly scattering media by polarisation optical techniques. The effect of circular polarisation retention in a disordered ensemble of resonant Mie particles, when the first Kerker condition is met, can be used, for example, to measure the difference between the absorption coefficients of the right- and left-hand circularly polarised light in media with circular dichroism [26]. Multiple scattering by a disordered ensemble of resonant Mie particles, placed in such a medium, leads to a lengthening of the trajectories of light rays without changing their polarisation. As a consequence, there sharply increase differences in the attenuation of the intensity of left- and right-hand circularly polarised light. In this situation, as in the case of a 'random laser' [27], a disordered ensemble of Mie particles acts as a space distributed resonator.

Acknowledgements. This work was supported by the competitiveness enhancement programme of National Research Nuclear University 'MEPhI' (Contract No. 02.a03.21.0005 dated 08.27.2013).

References

1. MacKintosh F.C., Zhu J.X., Pine D.J., Weitz D.A. *Phys. Rev. B*, **40**, 9342 (1989).
2. Rojas-Ochoa L.F., Lacoste D., Lenke R., Schurtenberger P., Scheffold F. *J. Opt. Soc. Am. A*, **21**, 1799 (2004).
3. Sankaran V., Everett M.J., Maitland D.J., Walsh J.T. *Opt. Lett.*, **24**, 1044 (1999).
4. Ghosh N., Vitkin I.A. *J. Biomed. Opt.*, **16**, 110801 (2011).
5. Andrews D.L. (Ed.) *Photonics: Biomedical Photonics, Spectroscopy, and Microscopy* (New York: Wiley, 2015) Vol.4.
6. Tuchin V.V. *J. Biomed. Opt*., **21**, 071114 (2016).
7. Bicout D., Brosseau C., Martinez A.S., Schmitt J.M. *Phys. Rev. E*, **49**, 1767 (1994).
8. Gorodnichev E.E., Kuzovlev A.I., Rogozkin D.B. *Pis'ma Zh. Eksp. Teor. Fiz.*, **68**, 21 (1998) [*JETP Lett*., **68**, 22 (1998)].
9. Schmidt M.K., Aizpurua J., Zambrana-Puyalto X., Vidal X., Molina-Terriza G., Saenz J. *J. Phys. Rev. Lett*., **114**, 113902 (2015).
10. Gorodnichev E.E., Kuzovlev A.I., Rogozkin D.B. *Pis'ma Zh. Eksp. Teor. Fiz*., **104**, 155 (2016) [*JETP Lett*., **104**, 157 (2016)].
11. Landau L.D., Lifshitz E.M. *Electrodynamics of Continuous Media* (Oxford: Pergamon Press, 1960; Moscow: Nauka, 1982).
12. Gorodnichev E.E., Kuzovlev A.I., Rogozkin D.B. *Zh. Eksp. Teor. Fiz*., **131**, 357 (2007) [*JETP*, **104**, 319 (2007)].
13. Gorodnichev E.E., Kuzovlev A.I., Rogozkin D.B. *Phys. Rev. E*, **90**, 043205 (2014).
14. Garcia-Etxarri A., Gomez-Medina R., Froufe-Perez L.S., Lopez C., Chantada L., Scheffold F., Aizpurua J., Nieto-Vesperinas M., Saenz J.J. *Opt. Express*, **19**, 4815 (2011).
15. Nieto-Vesperinas M., Gomez-Medina R., Saenz J.J. *J. Opt. Soc. Am. A*, **28**, 54 (2011).
16. Geffrin J.M., Garcia-Camara B., Gomez-Medina R., Albella P., Froufe-Perez L.S., Eyraud C., Litman A., Vaillon R., Gonzalez F., Nieto-Vesperinas M., Saenz J.J., Moreno F. *Nat. Commun*., **3**, 1171 (2012).
17. Gomez-Medina R., Froufe-Perez L.S., Yepez M., Scheffold F., Nieto-Vesperinas M., Saenz J.J. *Phys. Rev. A*, **85**, 035802 (2012).
18. Albella P., Poyli M.A., Schmidt M.K., Maier S.A., Moreno F., Saenz J.J., Aizpurua J. *J. Phys. Chem. C*, **117**, 13573 (2013).
19. Kerker M., Wang D.-S., Giles C.L. *J. Opt. Soc. Am*., **73**, 765 (1983).
20. Mishchenko M.I., Travis L.D., Lacis A.A. *Scattering, Absorption and Emission of Light by Small Particles* (Cambridge: Cambridge University Press, 2002).
21. Hovenier J.W., van der Mee C., Domke H. *Transfer of Polarized Light in Planetary Atmospheres* (Dordrecht – Boston – London: Kluwer Acad. Publ., 2004).
22. Gorodnichev E.E., Kuzovlev A.I., Rogozkin D.B. *Zh. Eksp. Teor. Fiz*., **115**, 769 (1999) [*JETP*, **88**, 421 (1999)].
23. Van de Hulst H.C. *Multiple Light Scattering* (New York: Academic, 1980).
24. Garcia-Camara B., Gomez-Medina R., Saenz J.J., Sepulveda B. *Opt. Express*, **21**, 23007 (2013).
25. Gorodnichev E.E., Kuzovlev A.I., Rogozkin D.B. *J. Opt. Soc. Am. A*, **33**, 95 (2016).
26. Berova N., Di Bari L., Pescitelli G. *Chem. Soc. Rev*., **36**, 914 (2007).
27. Gottardo S., Sapienza R., Garcia P.D., Blanco A., Wiersma D.S., López C. *Nat. Photonics*, **2**, 429 (2008).