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Compensation for nonlinear effects in an optical orthogonal frequency-division multiplexed signal using adaptive modulation

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Abstract. **We study the influence of nonlinear effects on symbol error statistics when a 16-QAM orthogonal frequency-division multiplexed signal is transmitted in a 1000 km length of fibre. A technique of adaptive modulation is proposed for generating signals that are resistant to nonlinear distortions. A considerable improvement of the transmission quality is shown to take effect in using an adaptive modulation scheme.**

Keywords: *optical fibre, nonlinear effects, OFDM signal, adaptive modulation.*

1. Introduction

To date, one of the main factors limiting the capacity of fibre-optic communication lines is the nonlinear effects, the influence of which on an optical signal increases with increasing its power [1]. Because an increase in the signal power is a natural consequence of a more dense use of the link bandwidth, the issue of compensation for nonlinear effects becomes important in the context of modern high-bit-rate communication lines.

For the available bandwidth in modern fibre-optic communication lines to be most fully utilised, advantage is taken, in addition to wavelength division multiplexing techniques, of orthogonal frequency-division multiplexing (OFDM) [2 – 5], which makes use of a large number of closely spaced orthogonal subcarriers that are transmitted in several parallel data sub-channels. The efficiency of OFDM techniques mainly results from minimisation of intersymbol interference in combination with a large number of closely spaced orthogonal sub-channels, due to which a high data transfer rate is achieved [3]. At the same time, along with the positive effects of application of OFDM modulation [6], we should mention the presence of negative factors that restrict the use of this technology. For example, the transmitted OFDM signal is significantly affected by nonlinearities arising, in particular, from four-wave mixing [7], which is especially noticeable at a large number of sub-channels and a large total signal power.

In this paper, we investigate the influence of nonlinearity on the OFDM signal propagation in fibre-optic lines. Each of the frequency OFDM channels (super-channel) is divided into a set of sub-channels; we have studied a signal comprising 100 to 500 sub-channels; as a modulation format for a sub-channel we have chosen the 16-QAM (quadrature amplitude modulation) format.

2. Nonlinear distortion of the 16-QAM signal

The data transfer system used in this paper is shown schematically in Fig. 1. The communication line consists of ten spans of 100-km-long standard single-mode fibres (SSMFs). At the end of each span, all the losses are compensated for by optical EDFA amplifiers. Noise corresponding to EDFA amplified spontaneous emission is added to the optical signal after each amplifier. Chromatic dispersion and nonlinear phase shift are ideally compensated for when the optical signal passes through a receiver.

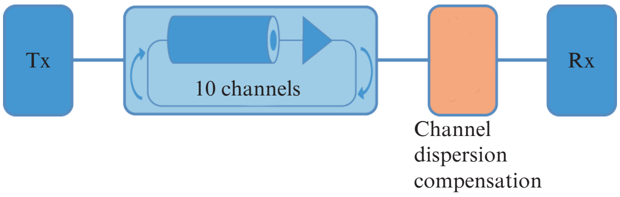

Figure 1. Scheme of the line under study.

The propagation of the signal through optical fibre is described by the nonlinear Schrödinger equation:

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \mathrm{i}\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \mathrm{i}\xi|A|^2 A, \qquad (1)$$

where $A(z,t)$ is the slowly varying envelope of the optical signal; α is the loss in the fibre; β_2 is the chromatic dispersion; and ξ is the fibre nonlinearity. This equation was solved numerically using the symmetric split-step Fourier method. We used the following parameters for numerical simulation: the number of OFDM symbols is $N_s = 2^{12}$, $\alpha = 0.2$ dB km^{-1}, $\xi = 1.4$ W^{-1} km^{-1}, $\beta_2 = -25$ ps^2 km^{-1}, the signal wavelength is $\lambda = 1.55$ μm, the amplifier noise figure is NF = 4.5, the maximum number of OFDM channels is 1024, and the bandwidth is BW = 100 GHz.

The aim of the study was to investigate the dependence of the number of errors on the position of symbols within the

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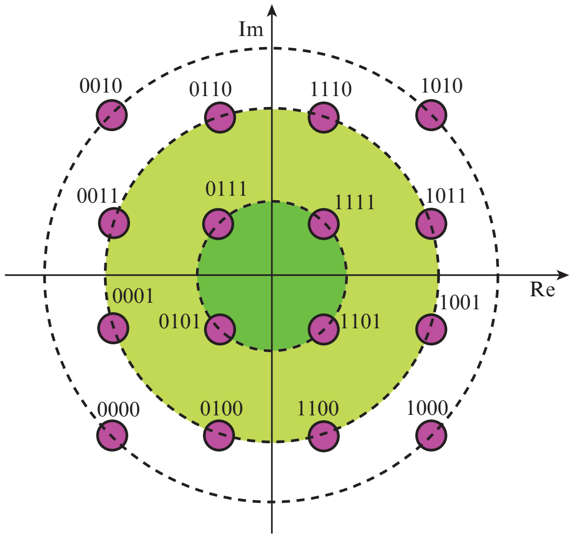

Figure 2. Signal 16-QAM constellation (Re and Im axes indicate real and imaginary signal components, respectively).

16-QAM signal constellation. All the errors are divided into three categories according to the power of the constellation points, corresponding to three 'rings', shown as dashed circles in Fig. 2. All the constellation points have equal probabilities, corresponding to a uniform distribution of the input signal.

Figure 3 shows the dependence of the symbol error rate (SER) on the number of OFDM channels used for the signal propagation in the nonlinear regime. It was assumed that due to nonlinearities the symbols from the outer ring, having a larger power, are subject to greater distortions, and among the symbols from the inner ring there will be fewer errors due to the smaller capacity. However, Fig. 3 shows that the ratio between symbol errors for different signal constellation rings also depends on the number of channels. At a small number of channels, the error distribution is the same as expected; however, with increasing number of channels the amount of error on different rings becomes approximately equal. The observed asymmetry of error probabilities for different rings involves the use of a limited coding range to improve the system performance.

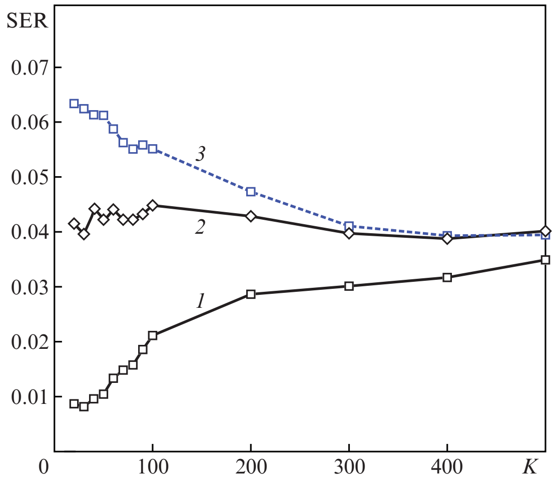

Figure 3. Dependences of the symbol error rate SER on the number K of sub-channels for (*1*) inner, (*2*) central and (*3*) outer rings.

3. Adaptive modulator

As is seen in Fig. 3, the presence of such a nonuniform distribution of the errors gives reason to believe that the use of special signal generation techniques in this case can significantly reduce the number of line errors. From a theoretical point of view, this can be done by reducing the probability of occurrence of symbols from a set with a maximum error probability. This process can generally be referred to as adaptive modulation, because its main goal is to use a special choice of symbols from the constellation diagram. It is also called a quadrature-division hybrid modulation format [8, 9]. The realisation of this approach assumes that various symbols in the data stream are generated using various modulation formats, each of which is a 'truncated' variant of the 16-QAM modulation format (Fig. 2). For example, some symbols can be formed only from the inner ring, and part of the symbols – only from the middle ring.

We have already mentioned that we divide all the points of the signal 16-QAM constellation into three sets with the same power, i.e. three rings, as shown in Fig. 2. Let us enumerate these sets in order of increasing amplitude. Then, the first set will be composed of the points of the inner ring, and the third set – of the points of the outer ring.

To assess the impact of nonlinear effects on the 16-QAM signal, we assume that the frequency of occurrence of symbol errors in a particular ring depends only on its capacity and the number of channels used, i.e. in each case, this frequency is considered constant. The symbol error rate of the ith set is denoted by q_i and the probability of occurrence of a symbol from the ith set in a data stream – by P_i. Then, the symbol error rate may be found from the formula

$$\mathrm{SER}(P_1, P_2, P_3) = P_1 q_1 + P_2 q_2 + P_3 q_3. \qquad (2)$$

Since $P_3 = 1 - P_1 - P_2$, then in the case under study, the SER depends only on two unknown probabilities.

It should be noted that by changing the probability of occurrence of symbols in different rings, we will generally reduce the information entropy of the data stream, which in turn increases the redundancy of the transmitted message. This leads to a decrease in the actual link data rate. Entropy H of the data stream per symbol for the 16-QAM modulation format can be found as follows:

$$H(P_1, P_2) = -P_1 \log_{16}\frac{P_1}{4} - P_2 \log_{16}\frac{P_2}{8} - (1 - P_1 - P_2) \times \log_{16}\frac{1 - P_1 - P_2}{4}. \qquad (3)$$

We can find from equation (2) that the initial symbol error rate (i.e. SER without any encoding) has the form

$$\mathrm{SER}_0 = \frac{1}{4}q_1 + \frac{1}{2}q_2 + \frac{1}{4}q_3. \qquad (4)$$

In the general case, the encoded signal has a different symbol error rate, namely, SER_c. Let us denote by γ the degree of SER reduction ($0 \leqslant \gamma \leqslant 1$), which is given by the expression

$$\gamma = \frac{\mathrm{SER_c}}{\mathrm{SER_0}}. \qquad (5)$$

This ratio can be regarded as a measure of encoding performance. Nevertheless, there is always a compromise between SER reduction and data stream redundancy required to implement this encoding.

This problem can be considered from the point of view of finding the input signal distribution probabilities $\boldsymbol{P} = (P_1, P_2, P_3)$, which correspond to the maximum information entropy for a given degree of error rate reduction γ and a given set $\boldsymbol{Q} = (q_1, q_2, q_3)$. Thus, the desired probabilities can be found by the search for each input vector $\boldsymbol{Q}$. Once they are found, the signal is formed in such a way that in each frequency sub-channel we observe one and the same distribution of the symbols used. This can be achieved through the use of a block-adaptive approach for obtaining modulated data.

4. Numerical results

The improved performance of the data transmission system was estimated by the adaptive approach for 20, 100, and 500 channels. Using the error statistics (Fig. 3), we found the optimal probability vector $\boldsymbol{P}$, providing maximum entropy. For the modulator operation to be evaluated at various γ, we changed the degree of SER reduction from 0.5 to 1.

Figure 4 shows the numerically obtained values of γ as a code redundancy function for a different number of channels used. The code redundancy is $R = 1 - C$, where C is the code rate, i.e. the proportion of symbols with respect to the 16-QAM format, which is involved in the signal formation. Here we see that the use of adaptive modulation can significantly reduce the SER for small values of γ.

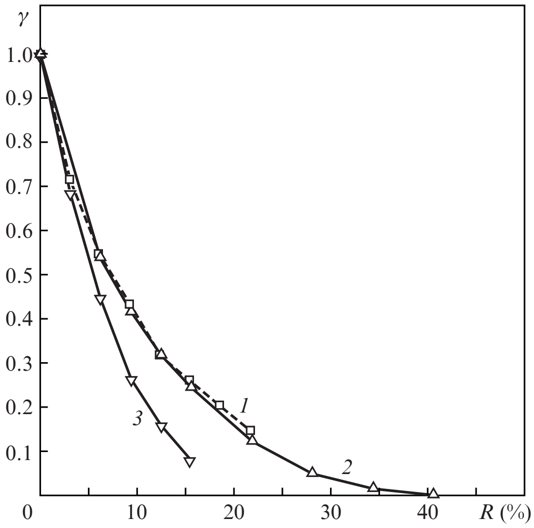

Figure 4. Dependences of γ on the code redundancy R for the number of sub-channels K = (*1*) 20, (*2*) 100, (*3*) and 500.

Apart from evaluating the adaptive modulation efficiency by reducing the number of errors, this efficiency has also been assessed for conventional telecommunication systems way by finding the transmission quality factor (Q-factor) for the OFDM system with and without using adaptive modulation. Figures 5 and 6 show the dependences of the Q-factor on the initial peak power for 100 and 500 channels, respectively. One can see that in the linear regime, the Q-factor is independent of γ. Nevertheless, the Q-factor in the nonlinear regime is markedly improved by using adaptive modulation.

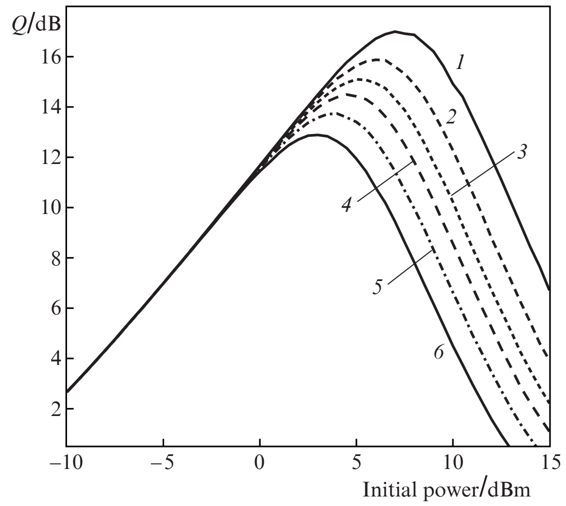

Figure 5. Dependences of the Q-factor on the initial power for 100 channels at γ = (*1*) 0.55, (*2*) 0.60, (*3*) 0.65, (*4*) 0.70, (*5*) 0.80, and (*6*) 1.00.

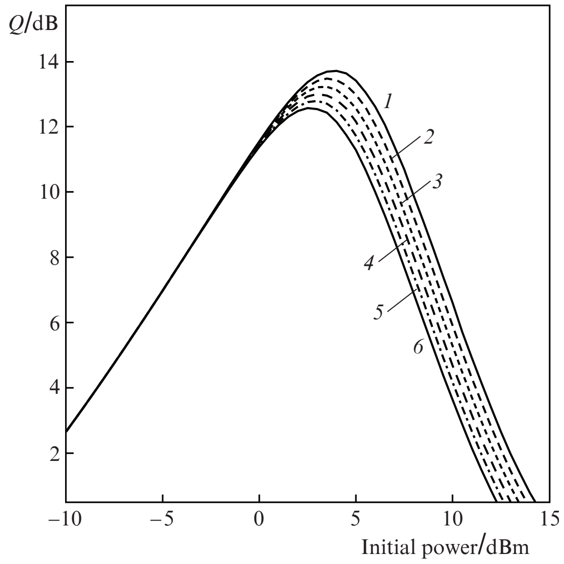

Figure 6. Dependences of the Q-factor on the initial power for 500 channels at γ = (*1*) 0.95, (*2*) 0.96, (*3*) 0.97, (*4*) 0.98, (*5*) 0.99, and (*6*) 1.00.

5. Conclusions

We have examined the influence of nonlinear effects on the statistics of symbol errors in the transmission of a 16-QAM OFDM signal along a 1000 km length of fibre. An adaptive modulation technique is proposed for generating the signal that is resistant to nonlinear distortions. We have demonstrated a significant increase in the quality of transmission using an adaptive modulation scheme.

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