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Acousto-optic gyrotropic-crystal-based modulator with a rotating polarisation vector

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Abstract. **We study the influence of ellipticity of gyrotropic-crystal eigenwaves on the output characteristics of an acousto-optic (AO) modulator based on the interferometer scheme. The schemes of AO modulators are considered, which provide the polarisation vector rotation frequency *fn*/2, where *f* is the frequency of the acoustic wave, and *n* is the integer. Preference is given to the scheme combining cascade and polarisation-independent diffraction. An experimental layout of the AO modulator operating at *f* = 44.5 MHz is described, the modulation frequency of the output laser light intensity being 89 MHz. The frequency of the electrical signal from the photodetector is equal to 179.5 MHz.**

Keywords: *acousto-optic diffraction, Bragg regime, frequency shift, amplitude modulation.*

1. Introduction

Acousto-optics is widely used to control the laser light parameters (see, e.g., [1–3]). One of the important properties of acousto-optic (AO) interaction, i.e. the ease of obtaining a shift of an optical signal frequency with respect to the sound frequency as a result of reflection of light from the travelling acoustic grating [1, 2, 4], is widely used for solving optical heterodyne detection problems [5], in laser Doppler anemometry [6, 7], etc. In paper [8], the effect of the frequency shift in the AO interaction occurring in the polarisation-independent diffraction regime [9–11] was used to generate optical radiation with rotating linear polarisation, while the velocity of rotation was determined by the frequency of the sound wave. Optical radiation at a predetermined linear polarisation rotation frequency is promising, for example, in laser interferometry [7] for fibre-optic communication lines [12], fibre-optic sensors [13], etc. This radiation after passing through a polariser is registered by a photodetector in the form of an electrical signal with a given frequency related to the frequency of sound.

The method proposed in our paper [8] proved to be promising for obtaining amplitude modulation of the light intensity at higher frequencies that are multiples of the frequency of sound. An increase in the modulation frequency is extremely important, e.g., for laser Doppler anemometers, where the modulation frequency determines the range of measurement of velocities of moving particles [6, 7]. We assumed in [8] that the combined optical beams have strictly circular polarisations, and the polarisation analyser at the modulator output can be oriented in any position with the modulation depth being the same. But, strictly speaking, in using single-axis gyrotropic crystals (such as TeO_2), in which the eigenwaves in the general case are no longer circular but elliptical, the modulation depth is reduced, i.e. it begins to depend on the position of the output polariser. This should be taken into account in the fabrication of the specific variants of modulators. Paper [8] also describes schemes, which make it possible to significantly increase the frequency of rotation of the polarisation vector, and compares different scenarios of frequency enhancement.

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2. Superposition of elliptically polarised optical beams

For the analysis of the optical characteristics of the radiation generated at the output of the modulators in question, we consider the summation of two elliptically polarised waves. For simplicity, we assume that the semi-axes of the polarisation ellipse of both waves are oriented in a coordinate system xyz along the x and y axes, and the waves propagate along the z axis. The projections of the electric vectors on the x and y axes will be described by the expressions

$$E_{x1} = a_1\sin(\omega_1 t),$$
$$E_{y1} = b_1\cos(\omega_1 t) \qquad (1)$$

for the first wave and by the expressions

$$E_{x2} = a_2\cos(\omega_2 t),$$
$$E_{y2} = b_2\sin(\omega_2 t) \qquad (2)$$

for the second wave. Here, E_{x1}, E_{y1} and E_{x2}, E_{y2} are the projections of the electric vectors of the first and second waves on the x and y axes; a_1, a_2 and b_1, b_2 are the amplitude of these projections; and ω_1, ω_2 are the frequencies of the waves. The difference in frequencies is thought to arise from the AO interaction. Below we assume that $\omega_2 = \omega_1 + \Omega$, where Ω is the frequency shift of one of the waves with respect to the other frequency ($\omega_1 \gg \Omega$). If the frequency shift is the result of several diffraction events, Ω is the total frequency shift. System (1) describes the propagation of a 'left-hand rotating' wave, while system (2) – 'right-hand rotating' wave (as defined in [14]). Adding fluctuations [equations (1), (2)] separately for x and y axes in accordance with the rule of addition of harmonic oscillations (see, e.g., [15, 16]), we obtain

for the direction along the x axis

$$E_x = E_{x1} + E_{x2} = a_1\sin(\omega_1 t) + a_2\cos(\omega_1 t + \varphi_1)$$

$$= A\sin(\omega_1 t + \varphi'), \quad (3)$$

where

$$A = \sqrt{a_1^2 + a_2^2 - 2a_1a_2\sin\varphi_1}; \ \tan\varphi' = \frac{a_2\cos\varphi_1}{a_1 + a_2\sin\varphi_1}; \quad (4)$$

for the direction along the y axis

$$E_y = E_{y1} + E_{y2} = b_1\cos(\omega_1 t) + b_2\sin(\omega_1 t + \varphi_1)$$

$$= B\sin(\omega_1 t + \varphi''), \quad (5)$$

where

$$B = \sqrt{b_1^2 + b_2^2 - 2b_1b_2\sin\varphi_1}; \ \tan\varphi'' = \frac{b_1 + b_2\cos\varphi_1}{b_2\cos\varphi_1}. \quad (6)$$

In expressions (3)–(6), $\varphi_1 = (\omega_2 - \omega_1)t = \Omega t$ is the initial phase slowly varying in time. The above expressions show that the projections of the total electric field $\boldsymbol{E}$ on the x and y axes vary harmonically with the same frequency ω_1, and the amplitudes of the projections A and B, as well as the initial phases φ' and φ'' slowly vary in time. We will consider the process of forming the polarisation for a small period of time. Then, the values of A, B, φ' and φ'' can be considered constant. In this case, two waves propagating along the z axis and having the same frequency, the polarisation vector of one of which being directed along the x axis and the other along the y axis, form in total an elliptically polarised wave [14]. Let the electric fields of the waves be described by the expression $E_x = A\sin(\omega_1 t + \varphi')$ and $E_y = B\sin(\omega_1 t + \varphi'')$. Then, the end of the electric field vector describes an ellipse of form [14, 17]

$$\left(\frac{E_x}{A}\right)^2 + \left(\frac{E_y}{B}\right)^2 - 2\left(\frac{E_x}{A}\right)\left(\frac{E_y}{B}\right)\cos\Delta\varphi = \sin^2\Delta\varphi, \quad (7)$$

where $\Delta\varphi = \varphi' - \varphi''$.

Figure 1 illustrates an example of the summation of two elliptically polarised waves. Calculations are performed based on expressions (3)–(7). We assumed that the major semi-axes (*1*) and (*2*) of the initial ellipses are oriented orthogonally to each other, the lengths of the major semi-axes are equal to five standard units, and the lengths of the minor semi-axes are equal to two standard units. Ellipse (*1*) is described by equation (1), and ellipse (*2*) – by expression (2). Figure 1b shows the results of adding ellipses for $\Delta\varphi$ equal to 0, 0.25π, 0.5π, 0.75π, π, 1.25π, 1.5π and 1.75π [ellipses (*1*–*8*), respectively]. One can see that at $\Delta\varphi = 0$ [line (*1*)], the end of the resulting electric vector moves along a straight line at an angle of 45° with the axes x and y, which corresponds to linear polarisation. With changing $\Delta\varphi$, the polarisation vector begins to describe ellipses. At $\Delta\varphi = \pi$ [line (*5*)], the polarisation again becomes linear, but directed orthogonally to polarisation (*1*). For linear polarisations the field amplitudes have the form $E = a_1b_1\sqrt{2/(a_1^2 + b_1^2)}$. At $\Delta\varphi \neq \kappa\pi$, where κ is the integer, the electric vector describes ellipses. The greatest ellipticity is reached at $\Delta\varphi = 0.5\pi + \kappa\pi$. In this case, the semi-axes are oriented along the x and y axes [ellipses (*3*) and (*7*)], and the lengths of the major and minor semi-axes are equal to $a_1 + b_1$ and $a_1 - b_1$, respectively.

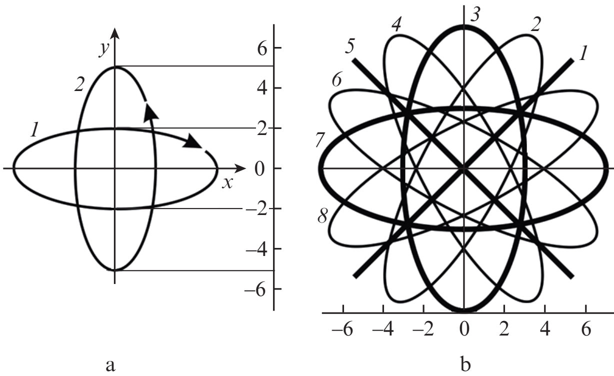

Figure 1. Summation of two elliptically polarised waves: (a) (*1*) and (*2*) polarisation ellipses of initial waves described by expressions (1) and (2), respectively, and (b) resulting waves as functions of the phase difference $\Delta\varphi$ between them (see the text). The numbers on the axes determine the lengths of the semi-axes of the ellipses.

Summation of two circularly polarised waves having the same amplitude [in expressions (1) and (2) $a_1 = a_2 = b_1 = b_2$), as the analysis shows, always results in the formation of a linearly polarised wave, the angle of rotation of the polarisation vector being equal to $\Delta\varphi/2$. Let us explain this with an example of light propagation through a gyrotropic crystal. Figure 2 shows linearly polarised optical radiation with the wave vector $\boldsymbol{K}$ and polarisation $\boldsymbol{E}_1$ propagating in the direction of the z axis and incident onto the face P_1 of a gyrotropic crystal. Inside the crystal, the radiation is split into two circularly polarised eigenwaves (*1*) and (*2*), whose polarisation vectors rotate in opposite directions. The waves travel at different velocities, and the phase difference between them is

$$\Delta\varphi = \frac{\omega}{c}L(n_R - n_L), \quad (8)$$

where ω and c are the frequency of a light wave and the velocity of light; L is the length of path rays in the crystal; and n_R and n_L are the refractive indices for eigenwaves. At the exit of the crystal in the plane P_2, the waves are summed, forming linearly polarised light with the polarisation vector $\boldsymbol{E}_2$. The angle of rotation $\boldsymbol{E}_2$ relative to $\boldsymbol{E}_1$ is $\psi = \Delta\varphi/2$ [18]. We now assume that circularly polarised waves propagate within the crystal at the same velocity ($n_R = n_L$), but have different frequencies (ω_1 and ω_2). Then,

$$\Delta\varphi = \frac{L}{c}(\omega_1 - \omega_2) = \frac{L}{c}\Omega = \Omega t, \quad (9)$$

where t is the time. Assuming that $\Omega \ll \omega_1, \omega_2$, the value of $\Delta\varphi$ changes slowly with time. In this case, the polarisation vector $\boldsymbol{E}_2$ changes the angle of incidence with the angular velocity $\psi/t = \Delta\varphi/2t = \Omega/2$. In other words, the vector $\boldsymbol{E}_2$ does not change its modulus and evenly rotates around the direction of the beam propagation. For elliptical polarisations, as can be inferred on the basis of Fig. 1b, the situation is much more complicated. Here, the total field will be generally represented by an ellipse whose parameters (ellipticity and tilt of the major semi-axis) change slowly with time. Thus, strictly speaking, the major semi-axis rotates unevenly. Figure 3 shows the changes in the angle ψ as functions of Ωt for the wave, formed by the summation of two circularly polarised waves [dashed line (*1*)], and the rotation angle ψ' [solid curve (*2*)] of the major semi-axis upon summation of waves with elliptical polarisations (*1*) and (*2*) (Fig. 1a). One can see that with changing Ωt curve (*2*) varies nonlinearly, although passes close enough to line (*1*). However, with a good approximation, it can be assumed that the rotation angle of the major semi-axis is equal to half the phase difference between the eigenwaves. With increasing ellipticity of the eigenwaves, curve (*2*) approaches line (*1*).

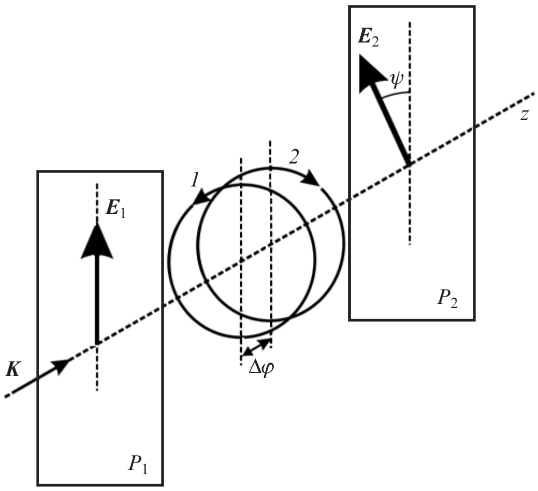

Figure 2. Formation of a linearly polarised wave upon summation of two circularly polarised waves.

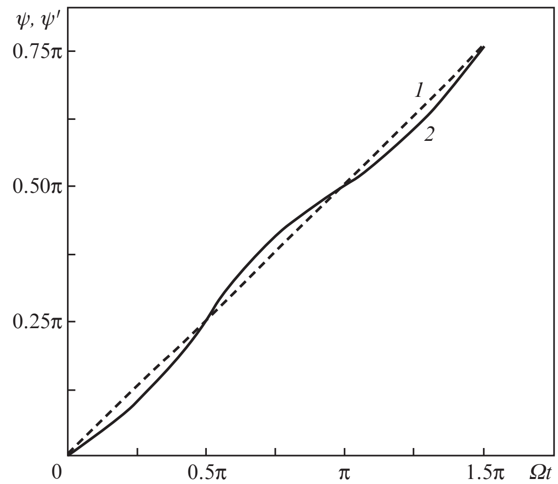

Figure 3. Dependences of angles (*1*) ψ and (*2*) ψ' on Ωt.

For the sake of completeness we should add that if we consider the summation of two equal-amplitude waves linearly polarised along the x and y axes, then their summation results in the formation of an elliptically polarised wave. Semi-axes of the polarisation ellipse do not rotate; it is always oriented so that the major semi-axis makes an angle of 45° with the axes of x and y [14, 17]. When changing $\Delta\varphi$, the length of the major semi-axis decreases and the length of the minor semi-axis increases; the ellipse becomes a circle, which is transformed again into an ellipse, but rotated by 90° relative to the initial ellipse.

It follows from the foregoing that by summing the waves with elliptic polarisations close to circular ones, the frequency of the rotation of the total polarisation vector is equal to half the difference between the sum frequencies of the waves. However, in measuring the light intensity, use is commonly made of quadratic photodetectors. Therefore, the frequency of the electrical signal from the photodetector is equal not to $\Omega/2$ but to Ω. This issue must be considered when using in practice the proposed method of summation of two circularly polarised waves.

We should consider one more issue that relates to the characteristics of the modulator described in [8]. The modulator uses a mirror, which has two functions: light reflection strictly in the opposite direction and transformation of optical radiation polarisation into orthogonal polarisation. This is fully realised only in the case if the light is circularly polarised. If it has elliptical polarisation, the polarisation ellipse after reflection rotates in the opposite direction, but its axis is not rotated by 90°. As stated in [9, 10], this reduces the fraction of the light involved in the re-diffraction, whereby, in general, the summed waves at the modulator output have different amplitudes. Therefore, the closer the optical beams of light propagate to the optical axis of the crystal, the closer the polarisation of the waves to circular and the better the conditions of the formation of a linearly polarsed wave with a rotating polarisation vector. Our experiments have shown that the optimal conditions for producing a wave with a rotating polarisation vector based on the scheme described [8] at a wavelength of 0.63 μm are achieved in the sound frequency range $f = 20-45$ MHz. For modulation at higher frequencies, we propose to use a frequency shift multiplication scheme.

3. Scheme of multiplication of the polarisation vector rotation velocity

One of possible frequency multiplication schemes was already described in [19], where we proposed to use cascade diffraction, when one of the eigenwaves of the incident light undergoes multiple acts of diffraction on the same acoustic wave. At the device output, diffracted optical radiation was added to the other eigenwave, which does not participate in diffraction. Four cascades of diffraction on a sound wave with a frequency of 45 MHz were obtained experimentally. The total optical radiation was directed to the photodetector. The frequency of the electrical signal at the output of the photodetector was 180 MHz.

In this work we proposed another scheme, which has, in our opinion, substantial advantages in comparison with the scheme from paper [19]. The proposed method is in some sense a 'hybrid' technique based on a combination of the methods from [8] and [19]. It allows one to increase the modulation depth of the light intensity by approximately five times as compared with the modulation depth of cascade diffraction [18]. The optical scheme of the method is shown in Fig. 4. This is a scheme for obtaining the amplitude-modulated optical signal at a frequency $2f$; however, it can easily be generalised for modulation of the light at a frequency nf (n is an integer). Initial radiation with the intensity I_0 is coupled to the

AO modulator at an angle θ_1 to the front of the acoustic wave. It is assumed that the modulator is made of a gyrotropic material whose eigenwaves are circularly polarised. For convenience, right-hand circular polarisation will be called R-polarisation, and left-hand circular polarisation will be called L-polarisation. We assume that anisotropic diffraction of light by the sound wave is accompanied by a change in polarisation, i.e., R-polarisation transforms into L-polarisation, and vice versa. In the general case of anisotropic diffraction, the angle of incidence onto the sound wave and the angle of reflection from it are not equal to each other [1, 2]. We assume that the light is incident at an angle θ_1, with the R-polarised wave being diffracted into the L-polarised wave (R $\rightarrow$ L). The angle of reflection of the light wave is θ_2.

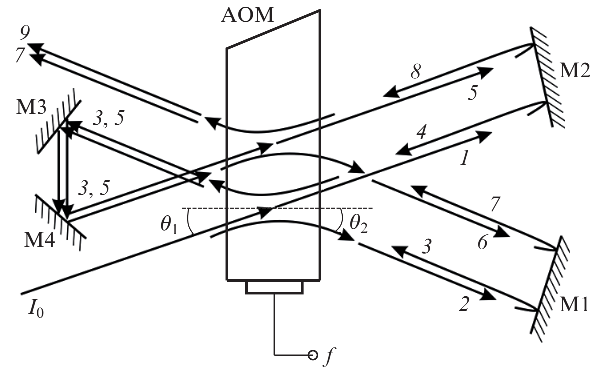

Figure 4. Optical scheme of formation of amplitude-modulated optical radiation.

In Fig. 4 the initial linearly polarised radiation with a frequency ω is shown to split into R- and L-polarised waves in the crystal. The wave with L-polarisation passes through the crystal without diffraction in the direction of beam (*1*), and the wave with the R-polarisation is diffracted by the sound wave in the direction of beam (*2*), thereby undergoing a transition into the L-polarised wave. As a result of diffraction, the wave frequency becomes equal to $\omega - f$. Wave (*2*) reflected from mirrors M1 strictly backwards transforms into wave (*3*), which becomes the R-polarised wave due to reflection. Since wave (*3*) is incident on the sound wave at an angle θ_2, it does not participate in diffraction and propagates in the direction mirror M3. Wave (*1*), reflected from mirror M2 strictly backwards [beam (*4*)] becomes R-polarised. This wave is incident on the sound wave at an angle θ_1 and diffracts, becoming wave (*5*). It is L-polarised, and its frequency is equal to $\omega + f$. Waves (*3*) and (*5*), propagating collinearly to each other, are reflected by mirrors M3 and M4. After reflection from these two mirrors, the waves preserve their polarisation [R-polarisation for wave (*3*) and L-polarisation for wave (*5*)]. Only when waves (*3*) and (*5*) preserve the same polarisation after reflection from the mirrors, as the analysis shows, the effect of accumulation of the frequency difference is achieved. Mirror M4 directs both waves in the AO modulator (AOM) at an angle θ_1. Wave (*3*) is diffracted by the sound wave in the direction of beam (*6*), and its frequency becomes equal to $\omega - 2f$. After reflection from mirror M1, it is transformed into R-polarised wave (*7*) and passes through the AOM without diffraction. Wave (*5*) also crosses the AOM from M4 to M2 without diffraction, but after reflection from mirror M2 it changes L-polarisation for R-polarisation, becoming wave (*8*). It is diffracted by the sound wave in the direction of beam (*9*). The frequency of wave (*9*) is equal to $\omega + 2f$, and it has L-polarisation. Thus, at the device output, waves (*7*) and (*9*) are produced with R- and L-polarisations and frequencies $\omega - 2f$ and $\omega + 2f$, respectively. These waves form in total a linearly polarised wave whose polarisation vector rotates with a frequency equal to half the difference between the sum frequencies of the waves, i.e. $2f$. If a polariser, followed by a photodetector, is installed along the path of such a wave, at its output an electrical signal with a frequency $4f$ is registered.

The proposed scheme significantly surpasses the scheme utilising only cascade AO diffraction [19]: two interfering waves are involved in diffraction in this scheme, i.e. their fronts 'deform' in the same manner, and hence the interference conditions are much better than in the case of cascade diffraction; the waves pass almost the same way, and so there is no need to control the coherence length; a beam splitter is not used, which significantly reduces optical losses of the entire device.

4. Experiment and discussion of the results

For verification of the results we performed an experiment, the optical scheme of which is shown in Fig. 5. Optical radiation with an intensity I_{inc} and a wavelength of 0.63 μm generated by laser (L) is reflected by mirrors M1 and directed to the AOM. The sound frequency was 44.5 MHz. At the output of the modulator, two waves, i.e. one that passed with intensity I_0 and the other that was reflected with intensity I_1 by the sound wave, are formed. Exterior mirrors M2–M5 ensured the path of rays, shown in Fig. 4. Radiation from the device was directed to polariser P, after which – to photodetector PD. The electrical signal from the photodetector was fed to an oscilloscope. Figure 6 shows the oscillogram of the signal. Polariser P was set in position ensuring a maximum modulation depth of the signal. One can see that it is a sinusoidal polariser, and its frequency (177.9 MHz) coincides with the quadruple frequency of the sound wave. It is also seen that the electric signal is 'noisy'. This noise is caused by the noise of the photodetector tract: in the absence of an optical signal in the photodetector, the electrical signal on the oscilloscope is a horizontal line with the same noise value. The modulation depth of the signal reaches 50%. Note that in [19] the modulation depth was only 10%. If the orientation of polariser P in Fig. 5 is changed, the modulation depth will decrease and with changing orientation by 45° it will amount to ~10%, i.e. depending on the orientation of the polariser the modulation depth is reduced from 50% to 10%. It is in good qualitative agreement with Fig. 1b. The modulation depth of more than

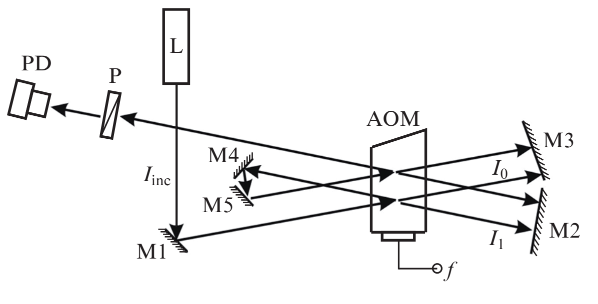

Figure 5. Optical scheme of the experimental setup.

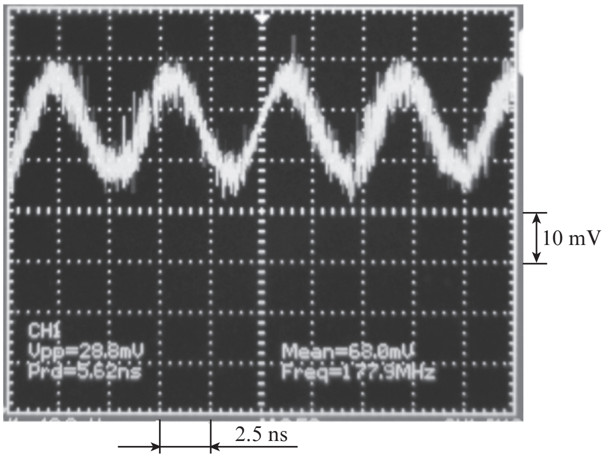

Figure 6. Oscillogram of the electric signal from the photodetector.

50% could not be obtained, which, in our opinion, is due mainly to distortions of the fronts of optical waves interfering with each other. In any case, a good amplitude-modulated signal is observed at a frequency of $4f$.

The described method can be used to obtain an amplitude-modulated optical wave with a high modulation frequency nf ($n > 2$), while on the oscilloscope screen an electrical signal with a frequency $2nf$ is observed.

5. Conclusions

Based on the above we can formulate the main results.

1 We have presented a general technique for calculating the polarisation state of the radiation generated as a result of the summation of two waves with arbitrary (in particular, mutually orthogonal) polarisations. It is shown that during the summation of two waves with circularly mutually orthogonal polarisations and different frequencies, the total radiation is linearly polarised and the polarisation vector rotates with a frequency $\Omega/2$, where Ω is the difference in the frequencies of the summed waves.

2. We have described schemes, which make it possible to significantly increase the frequency of the polarisation vector rotation. The best is a 'hybrid' scheme combining cascade and polarisation-independent diffraction schemes.

The results obtained can be used in various devices utilising the rotation of the polarisation vector with a predetermined frequency.

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