

PHOTODETECTORS

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Response of a matrix photodetector into components of an optical signal with different spatial frequencies

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Abstract. **We have used the method of direct evaluation of energy incident on each pixel of a photosensitive array to assess the distortions of a matrix photodetector response into a sinusoidal spatial signal. Using the data obtained, we have formulated recommendations for selecting a resolution of a matrix photodetector as a function of a maximum spatial frequency in the recorded interference pattern, as well as for matching of the resolutions of an objective and a photodetector of a digital camera or camcorder.**

Keywords: *matrix photodetector, sinusoidal spatial signal, interference pattern, spatial frequency, modulation transfer function, objective, spatial low-pass filter, digital camera, camcorder.*

1. Introduction

Obvious success and new opportunities offered by a transition from silver halide photographic emulsions to semiconductor matrix photodetectors in registering an optical image have stimulated interest in their use in other fields (for example, in digital holography, holographic interferometry, etc.), which require recording of interference patterns. However, the features of matrix photodetectors, especially the finite size of photosensitive elements (pixels), require the matching of the photodetector resolution and the maximum spatial frequency in the recorded image.

In accordance with the Whittaker – Kotel'nikov – Shannon sampling theorem, the resolution of a matrix photodetector is characterised by its Nyquist frequency [1]

$$N_{\mathrm{N}} = 1/2\Lambda, \tag{1}$$

where Λ is the spatial repetition period of matrix elements, almost coinciding with their size.

This idealised representation of the sampling and reconstruction of continuous images assumes the use of a bipolar interpolation sinc function. In the case of a matrix photodetector, the interpolation function is unipolar, because it is a function of the pixel sensitivity. As a result of differences between the actual interpolation function and the sinc function, the image of a harmonic signal exhibits beats and space-dependent changes in the image characteristics [2].

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The frequency dependence of the contrast with which one or the other device reproduces the applied sinusoidal signal is a modulation transfer function (MTF) of this device. Following paper [3], we obtain the MTF of a matrix photodetector based on the fact that the transfer function of the device is the Fourier transform of its impulse response.

The response of a matrix pixel to a paraxial image of a point light source is determined by the pixel sensitivity function. Neglecting the dependence of sensitivity on the spatial coordinate of the image within the pixel, as well as the possible overlap of the sensitivity functions of neighbouring pixels, we will assume the sensitivity function of the pixel to be rectangular. The Fourier transform of a rectangular function of width Λ is a function

$$T(N) = \mathrm{sinc}(\Lambda N) \tag{2}$$

with a first zero at a frequency of $1/\Lambda$. As a result, at the Nyquist frequency the contrast is

$$T(N_{\mathrm{N}}) = \mathrm{sinc}(\Lambda N_{\mathrm{N}}) = 0.63662 \approx 0.64. \tag{3}$$

Expression (2) for the contrast does not take into account the spatial shift Λ_0 of a sinusoidal signal relative to the array of matrix elements, which with virtually equal probability can take any value in the range $\pm\Lambda$. The authors of Refs [4 – 6] proposed a method of accounting for the influence of Λ_0 on the matrix photodetector response into a sinusoidal spatial signal. This paper presents the results of the analysis of this influence and recommendation for selecting the matrix photodetector resolution as a function of the maximum spatial frequency in the recorded interference pattern. In addition, we report the recommendations for matching the resolutions of the objective and the photodetector, as well as for choosing the cut-off frequency of a spatial low-pass filter in the registration of an optical image formed by the camera or camcorder objective.

2. Evaluation and minimisation of distortions of a matrix photodetector response into a sinusoidal spatial signal

Following papers [4 – 6], we consider the dependence of the contrast in reconstructing a sinusoidal spatial signal by a matrix row on its frequency N. The signal has the form

$$I(x) = 0.5[1 + Q\cos(2\pi Nx)], \tag{4}$$

where $0 \leqslant Q \leqslant 1$ is the modulation coefficient, and x is the coordinate along the row.

Using the expression $N = 1/a\Lambda$, which relates the spatial frequency of the signal and the matrix period via the coefficient a, we determine the energy falling on two nearest matrix elements with the highest and lowest luminance:

$$E_j = \int_{(j-1)\Lambda+\Lambda_0}^{j\Lambda+\Lambda_0} I(x)\,\mathrm{d}x, \quad (5)$$

$$E_k = \int_{(k-1)\Lambda+\Lambda_0}^{k\Lambda+\Lambda_0} I(x)\,\mathrm{d}x, \quad (6)$$

where j and k are the numbers of the elements with the maximum and minimum luminance, respectively.

After integration, we obtain

$$E_j = 0.5\Lambda\left\{1 + \frac{Qa}{\pi}\cos\left[\frac{\pi[(2j-1)\Lambda + 2\Lambda_0]}{a\Lambda}\right]\sin\frac{\pi}{a}\right\}, \quad (7)$$

$$E_k = 0.5\Lambda\left\{1 + \frac{Qa}{\pi}\cos\left[\frac{\pi[(2k-1)\Lambda + 2\Lambda_0]}{a\Lambda}\right]\sin\frac{\pi}{a}\right\}. \quad (8)$$

The image contrast can be found from the formula

$$T_{\mathrm{m}} = \frac{E_j - E_k}{E_j + E_k}. \quad (9)$$

Using relations (7)–(9), it is easy to show that at $Q = 1$, the contrast value at a repetition frequency of pixels, $2N_{\mathrm{N}} = 1/\Lambda$, is identically (regardless of the Λ_0 value) equal to zero, while at the Nyquist frequency (i.e. at $a = 2$), lies in the range $0 \leqslant T_{\mathrm{m}} \leqslant 0.64$, depending on the Λ_0 value, and is equal to 0 and 0.64 at $\Lambda_0 = 0$ and $\Lambda_0 = \pm 0.5\Lambda$, respectively (Fig. 1). Note alsothat only a maximum contrast value at the Nyquist frequency coincides with the value given by formula (3).

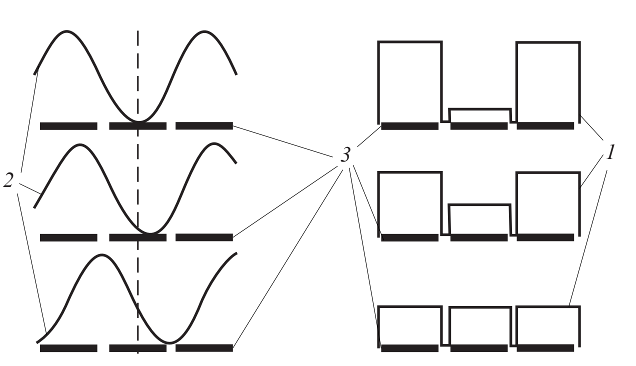

Figure 1. Responses of matrix photodetectors (*1*) to spatial sinusoidal signals (*2*) at $N = N_{\mathrm{N}} = 1/2\Lambda$; (*3*) matrix pixels.

The relative differential in the contrast decreases with increasing coefficient a. Thus, at $a = 3$ ($N = 0.67N_{\mathrm{N}}$), the contrast varies in the range $0.51 \leqslant T_{\mathrm{m}} \leqslant 0.78$ (0.51 at $\Lambda_0 = \pm 0.5\Lambda$ and 0.78 at $\Lambda_0 = 0$), at $a = 4$ ($N = 0.5N_{\mathrm{N}}$) – in the range $0.64 \leqslant T_{\mathrm{m}} \leqslant 0.90$ (0.64 at $\Lambda_0 = 0$ and 0.90 at $\Lambda_0 = \pm 0.5\Lambda$) and, finally, at $a = 8$ ($N = 0.25N_{\mathrm{N}}$) – in the range $0.9 \leqslant T_{\mathrm{m}} \leqslant 0.98$ (0.9 at $\Lambda_0 = 0$ and 0.98 at $\Lambda_0 = \pm 0.5\Lambda$).

Thus, expression (2) does not reflect the dependence of the contrast on the spatial shift of the sinusoidal signal relative to the array of the matrix elements, while the above method of direct assessment of the energy incident on each pixel of the photosensitive array allows one to see how a sinusoidal spatial signal is distorted by the matrix photodetector.

All kinds of arising distortions can be divided into four main groups:

– contrast drop at a constant average signal level and without low-frequency modulation (Fig. 2a);

– row-varying contrast due to low-frequency amplitude modulation (Fig. 2b);

– virtually row-constant contrast, with the average signal level $\overline{L}$ varying according to the sinusoidal law (Fig. 2c);

– simultaneously row-varying contrast and average signal level (Fig. 2d).

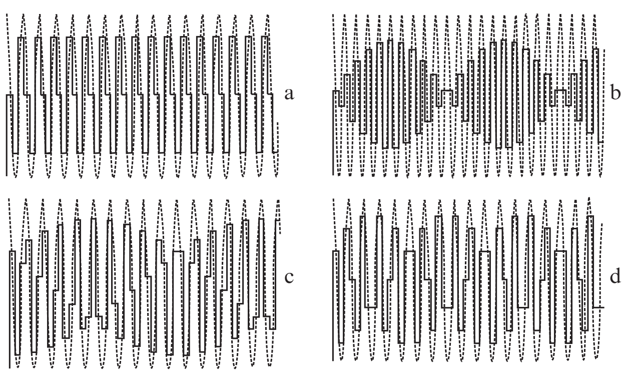

Figure 2. Responses of a matrix photodetector (solid curves) to a sinusoidal spatial signal (dashed curves) at (a) $N = 1/3\Lambda$, $\Lambda_0 = 0.25\Lambda$ ($T_{\mathrm{m}} = 0.72$), (b) $N = 1/2.1\Lambda$, $\Lambda_0 = 0$ ($0.10 \leqslant T_{\mathrm{m}} \leqslant 0.66$), (c) $N = 1/3.1\Lambda$, $\Lambda_0 = 0$ ($0.70 \leqslant T_{\mathrm{m}} \leqslant 0.79$, $0.50 \leqslant \overline{L} \leqslant 0.73$) and (d) $N = 1/2.8\Lambda$, $\Lambda_0 = 0$ ($0.10 \leqslant T_{\mathrm{m}} \leqslant 0.6$, $0.50 \leqslant \overline{L} \leqslant 0.70$).

Similar distortions in the popular photography literature are called maze-like artifacts [7].

The presented analysis shows that distortions of the photodetector matrix response are virtually absent (regardless of the spatial shift of the sinusoidal signal relative to the array of the matrix elements) only at $N \leqslant 1/8\Lambda$ ($N \leqslant 0.25N_{\mathrm{N}}$). If $N \geqslant 1/4\Lambda$ ($N \geqslant 0.5N_{\mathrm{N}}$), sampling distortions begin to adversely affect the image quality, and, in addition to a contrast drop, there may be a parasitic low-frequency modulation and changes in the average level of the signal observed in the form of a moiré or smooth parasitic changes in the image brightness.

The dependence of the contrast on the relationship between the spatial frequencies and the signal shift relative to the array of the matrix elements has been also investigated when determining the ultimate resolution of optoelectronic surveillance systems by the probabilistic method [2]. The calculation results of the probability of recognition of selected objects showed its sufficient invariance to the phase shift with respect to the matrix in the case when $N \leqslant 0.75N_{\mathrm{N}}$. However, a sharp drop in the probability is already observed at $N > 0.5N_{\mathrm{N}}$.

Generalisation of the results obtained by the method of direct evaluation of the energy falling on each pixel of a photosensitive matrix and by the probabilistic method shows that for a high-quality image to be obtained, the width of the spectrum of spatial frequencies in the image applied to the photodetector array should not exceed $N_b = (0.25-0.5)N_N$. In this case, it is desirable that the components of the optical signal with spatial frequencies $N < N_b$ have a maximum possible contrast.

Next, we focus on the implementation of these recommendations in the case of registration of an optical image formed by an objective of a camera or camcorder. A monochromatic MTF of a diffraction-limited (aberration-free) objective is described by the expression

$$T_{DL} = \begin{cases} \frac{1}{\pi}(2\alpha - \sin 2\alpha) \text{ at } |N| \leqslant \frac{2A'}{\lambda}, \\ 0 \text{ at } |N| > \frac{2A'}{\lambda}, \end{cases} \quad (10)$$

where $\alpha = \arccos(\lambda N/2A')$; λ is the wavelength, at which the MTF is calculated; and A' is the numerical aperture of the objective in the image space. At a small numerical aperture, it can be related with the aperture value P (the denominator of the relative aperture) by a simple expression: $A' \approx 1/2P$. A polychromatic MTF in the working spectral range $\lambda_{min} \leqslant \lambda \leqslant \lambda_{max}$ is obtained using several monochromatic MTFs [8].

Figure 3 shows typical polychromatic MTFs of a model objective with a given level of aberrations: a diffraction-limited MTF and an aberration-free MTF of the same objective. It is easy to see that aberrations, depending on their values, lead to a drop in the contrast (down to zero) at average frequencies.

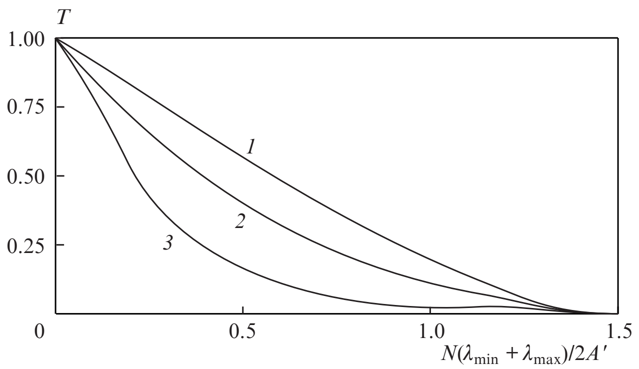

Figure 3. Example of polychromatic MTFs of the objective: (*1*) diffraction limited, (*2*) at small aberrations and (*3*) at aberrations significantly distorting the image formed.

It is obvious that practically visible distortions of the matrix photodetector response can be totally eliminated by a simple limitation of the numerical aperture of the objective, which is obtained from the condition

$$\frac{2A'}{\lambda} \leqslant (0.25 - 0.5) N_N. \quad (11)$$

However, for an acceptable response of the matrix photodetector to be obtained under low-light conditions, the camera objective must be wide aperture, i.e., must have a sufficient numerical aperture. At the same time, aberrations dramatically increase with increasing numerical aperture, which limits the resolution of the objective. Typically, the resolution of the objective is characterised by the spatial frequencies at which the contrast does not drop below a predetermined level. Depending on the features and price of the camera, this level is usually given in the range of 0.2–0.5.

As a result, to ensure the required light-collecting power and high resolution while minimising image distortions caused by the photodetector sensitivity, it is reasonable:

– to choose a matrix photodetector with the smallest possible pixel size;

– to choose a numerical aperture of the camera objective following the conditions of achieving both the desired illuminance and the degree of aberration correction, at which the contrast at the boundary frequency N_b will not drop below a predetermined level [$T(N_b) \geqslant 0.2-0.5$];

– to achieve an excess contrast of at least one and a half to two times at a frequency $0.5N_b$ over the contrast at a frequency N_b during aberration correction; and

– to eliminate incidence of spatial frequencies $N > N_b$ onto the matrix photodetector.

The latter condition can be satisfied only by introducing a spatial low-pass filter between the objective and the photodetector of the camera or camcorder [9]. Such a filter is composed of birefringent media. It splits the image formed by the objective, and thus focuses not one but four images, spatially shifted relative to each other in two mutually perpendicular directions, onto the photodetector matrix. Lenhardt [3] showed that the MTF of such a filter is described by the function

$$T_{LPF} = |\cos(\pi l N)|, \quad (12)$$

where l is the distance to which the images are shifted relative to each other in two mutually perpendicular directions. The shift by a pixel ($l = \Lambda$) resets the contrast at the Nyquist frequency. The contrast at the frequency $N_b = 0.5N_N$ in this case is equal to 0.707. If, for example, the contrast at the frequency $0.5N_N$ is reduced by at least to 0.4, we obtain a shift equal to one and a half pixel ($l = 1.5\Lambda$).

3. Conclusions

Using the method of direct evaluation of the energy incident on each pixel of a photosensitive array, we have shown that the distortions of the matrix photodetector response into a spatial sinusoidal signal, caused by a spatial shift of the signal relative to the array of the matrix elements, depend on the relation between the spatial frequency of the signal and the Nyquist frequency of the array. All kinds of arising distortions are divided by their characteristic features into four main groups.

It follows from the generalisation of the results obtained by direct estimation of the energy falling on each pixel of the photosensitive array and by the probabilistic method that in order to produce high-quality images, one needs to limit the range of spatial frequencies in it by no more than half the Nyquist frequency. Using the data obtained, we have formulated recommendations for selecting the resolution of the matrix photodetector as a function of maximum spatial frequency in the recorded interference pattern, as well as for matching the resolutions of the objective and photodetector of a digital camera or camcorder. In addition, we have determined the requirements imposed on the spatial low-pass filter mounted between the objective and the photodetector of the camera.

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