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Generation of attosecond electron bunches upon laser pulse propagation through a sharp plasma boundary*

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Abstract. **The formation of an electron bunch produced due to electron self-injection into a wake wave that is generated by a relativistic-intensity laser pulse propagating through a sharp boundary of semi-bounded plasma is studied in one-dimensional geometry. Analytical expressions are obtained for estimating the bunch length and the energy spread of electrons in the bunch. Numerical simulation is performed which confirms the results of analytical consideration.**

Keywords: *laser pulse, wake wave, semiresticted plasma, electron self-injection, laser-plasma electron acceleration.*

1. Introduction

Laser-plasma driven electron acceleration in low-density plasma is a promising field of high-energy physics development [1]. High expectations are based on the fact that the field intensity of an accelerating plasma wave in a wake-field accelerator may be by several orders higher than in modern accelerators of conventional type. This opens possibilities for designing new-type accelerators that will be small and relatively cheap as compared to classical electron accelerators. Presently, the best experimental result on laser-plasma driven acceleration of electrons was obtained in [2] where the laser pulse with a peak power of 300 TW (the pulse duration was 40 fs at a wavelength of 0.815 μm) accelerated an electron bunch to an energy of 4.2 GeV in a 9-cm-long gas-filled capillary. Electron bunches with such an energy are interesting in various practical applications.

However, the problem of obtaining electron bunches possessing not only a sufficiently high energy (several GeV), but also characteristics required in practical applications is not yet solved. Such requirements are the energy homogeneity of the electron bunch after acceleration, its duration, emittance, and charge. The characteristics of an electron bunch after acceleration are mainly determined by the initial parameters at the instant of injection into an accelerating wake field, which substantially depend on the way of electron injection into the wake wave. In addition, for obtaining an electron bunch of high energy and quality it is necessary to inject electrons at a proper instant, that is, at a certain phase of the wake wave.

Many various injection schemes were suggested over the period of studying laser-plasma electron acceleration: optical injection of electrons into a wake wave [3, 4]; employment of mixture of gases, among which one gas had a high ionisation potential [5–7]; and electron self-injection when the wake wave driver passes through plasma with an inhomogeneous density profile [8–12]. Currently, the most intensive theoretical and experimental investigations are devoted to so-called bubble-regime of electron acceleration in a strongly nonlinear plasma wave, which at certain conditions captures background plasma electrons in a non-stationary regime [13–18].

Further analysis of the suggested scheme and methods of electron injection into a wake wave has shown that methods of optical injection imply extremely exact spatial and temporal matching between several laser pulses. This introduces great technical difficulties to experimental realisation of such methods. Cavern evolution in the bubble-regime, which in its nature is nonstationary, also strongly affects the process of electron self-injection into a wake wave. This, in turn, determines the main characteristics of accelerated motion of an electron bunch such as energy spread of electrons in the bunch, their angular divergence [18], and reproducibility of experimental results. In our opinion, more appropriate are the methods of introducing electrons into an accelerating laser-plasma system based on self-injection of electrons when the laser pulse generating a wake wave passes through a plasma inhomogeneity. In this case, one can choose such profile characteristics of inhomogeneous plasma, which automatically provide self-injection of background plasma electrons into the appropriate phase of the accelerating wake field.

A promising method for injecting background electrons of plasma having a rising gradient of density into the first period of the wake wave was relatively recently suggested in [19], where a numerical modelling demonstrated that under certain conditions the interaction of a laser pulse with plasma generates electron bunches in a localised spatial domain where the plasma density profile flattens out. While realising this method of electron injection into a wake field, the condition of one-dimensionality of this field is substantial. The one-dimensionality of the injection process is provided by the fact that the motion of electrons in the transverse direction, excluding their high-frequency motion in a laser pulse field, can be neglected, because the size of the laser pulse focal spot is very large. In this case, numerical modelling shows that the length of bunches of injected electrons may be very short, tens of attoseconds, at the bunch charge of ~1 nC.

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A theoretical study of electron self-injection into the first period of the wake field was studied in [20, 21] in the case where a plasma inhomogeneity has the form of a sharp vacuum–plasma interface. It was found that this process results from longitudinal breaking of the wave generated by a laser pulse passing through the plasma boundary. The study of this process showed that it is convenient to analyse this phenomenon theoretically by presenting the electron component of plasma as an ensemble of plasma oscillators, whose motion characteristics are determined by the exciting laser pulse. Under the condition that the laser pulse is short enough, so that the process of wake wave breaking occurs after the pulse action on plasma oscillators terminates, it was found that characteristics of plasma oscillators completely determine the process of wake wave breaking, capturing background electrons, and following acceleration of captured electrons. Expressions have been derived for estimating the duration of the electron bunch and the spread of the electron energy in the bunch at long acceleration distances in the wake wave after electron self-injection, when the bunch characteristics of captured electrons approach certain asymptotic limits.

In the present work, we continue studying the physical mechanism of electron self-injection into a wake wave of a laser pulse passing through a sharp plasma boundary. By numerical simulation and theoretical analysis, we will reveal specific features of the process of selecting electrons from the background to a captured bunch and obtain relationships for estimating the bunch duration and the energy spread of electrons in the bunch at the formation stage.

2. Theoretical model of laser pulse interaction with plasma and numerical simulation of electron self-injection

Let us consider a semi-bounded plasma free of external static fields. In its description, we will use the model of cold plasma in which only its electron component is mobile. Ions form an immovable, homogeneous, positively charged background. To simplify the problem, we consider the one-dimensional geometry and assume that the vacuum–plasma interface is sharp.

Let the plasma surface be irradiated along the normal by a short laser pulse, which is a one-dimensional packet of circularly polarised electromagnetic waves with a frequency ω_0 well above the plasma frequency $\omega_{\rm p}$, that is, the density of plasma is low. We assume that the laser pulse propagates at the group velocity $V_{\rm gr}$ from left to right in the positive direction of the z axis with the origin coinciding with the plasma boundary. In the interaction with a laser pulse, each electron from plasma will move along the z axis due to the Miller ponderomotive force stemming from fast high-frequency electron oscillations in the laser field in the transverse direction.

In the one-dimensional geometry, in the case of circular polarisation of laser pulse electromagnetic waves, the longitudinal motion of electrons along the z axis has no high-frequency component and is described by the equations:

$$\frac{{\rm d}P}{{\rm d}t} = |e|\frac{\partial\varphi}{\partial z} - mc^2\frac{\partial}{\partial z}\left(\frac{eA}{mc^2}\right)^2\left[2\sqrt{1+\frac{P^2}{m^2c^2}+\left(\frac{eA}{mc^2}\right)^2}\right]^{-1}, \quad (1)$$

$$\frac{{\rm d}z}{{\rm d}t} = u = \frac{P}{m}\left[\sqrt{1+\frac{P^2}{m^2c^2}+\left(\frac{eA}{mc^2}\right)^2}\right]^{-1}, \quad (2)$$

where $A(z,t)$ is the amplitude of the envelope of the laser pulse vector potential; $\varphi(z,t)$ is the scalar potential for the charge separation field; P and u are the electron momentum and velocity, respectively; and $|e|$ and m are the electron charge and mass. The charge separation field arises due to the action of a laser pulse on electrons, which results in electron shifts from the initial equilibrium position z_0.

At the initial stage of interaction between the electron and the laser pulse, the electron shifts in the direction of pulse propagation. The arising charge separation field increases with the shift of the electron and tends to return the latter to the initial position. Hence, at a certain instant, the electron starts to move in the reverse direction and the action of the laser pulse on the electron stops. Then, the electron motion in plasma becomes similar to that of a free relativistic oscillator, when the electron oscillates relative to its equilibrium position (oscillation centre), which coincides with the initial position of the electron prior to the laser pulse action. A combined motion of such plasma oscillators yields a wake wave, which is generated by a laser pulse propagating across plasma.

Assume that at the spatial scale specific for our problem, the laser pulse profile does not change as the pulse propagates through plasma. This corresponds to a quasi-static approximation, in which the driver of the wake wave evolves on sufficiently greater time scales than plasma electrons respond to the action of the driver. From this follows that at a constant velocity $V_{\rm gr}$ of laser pulse propagation in homogeneous plasma, the pulse acts on each successive electron as severe as on the preceding electrons. Hence, all plasma background electrons subjected to the action of the laser pulse are plasma oscillators with the same total energy $E_{\rm os}$ that is determined by the amplitude of the laser pulse vector potential $a_0 = |e|A_0/(mc^2)$, characteristic pulse duration $\tau_{\rm las}$, gamma-factor $\gamma_{\rm ph} = 1/\sqrt{1-V_{\rm gr}^2/c^2}$, and plasma concentration n_0.

Since the laser pulse affects electrons successively as it propagates inside plasma, trajectories of plasma electrons are similar (i.e. equal), but with a certain time delay relative to each other. The principle of trajectory similarity holds true for any plasma electron unless the latter leaves the ion background limits, that is, crosses the boundary of plasma, or its trajectory crosses the trajectory of other electron. The combined electron motion after the action of the laser pulse is a wake wave excited by the pulse in plasma, and in the spatial domain, where trajectories are similar, the phase velocity of the wake wave $V_{\rm ph}$ is equal to the group velocity of the laser pulse $V_{\rm ph} = V_{\rm gr}$.

Theoretical analysis [20, 21] of the method for generating electron bunches by passing a laser beam through a plasma boundary [19] has shown that the physical mechanism of this method is electron self-injection into the wake wave of the laser pulse due to intersections of trajectories of plasma oscillators excited by the pulse. The necessary condition for this physical process is an excess of the total energy of plasma oscillators $E_{\rm os}$ over a certain threshold value $E_{\rm os\,th} = \gamma_{\rm ph}mc^2$. Availability of a plasma boundary is one more important factor that determines the spatial point at which electron self-injection starts and relates the start instant of electron injection into the wake wave with dynamics of laser pulse propagation in plasma.

Numerical simulation is the most illustrative presentation of the features of start and further development of electron self-injection into a wake wave. For this purpose, in the present work, we have simulated numerically the process of interaction between plasma and the laser pulse with an envelope having a time profile at the plasma boundary ($z = 0$) of type

$$a = a_0 \cos^2 \frac{t}{\tau_{\rm las}} {\rm sgn}\Big(\frac{\pi\tau_{\rm las}}{2} - |t|\Big),$$

where $a_0 = 4.982$ is the dimensionless amplitude of the vector potential; and $\tau_{\rm las}$ is the laser pulse duration, corresponding to $\tau_{\rm FWHM} = 1.143\tau_{\rm las} = 12$ fs. It is assumed that the group velocity of the laser pulse in plasma corresponds to a gamma-factor $\gamma_{\rm ph} = 5$. The plasma concentration is determined from the relationship $\omega_0/\omega_{\rm p} = \gamma_{\rm ph} = 5$, where $\omega_{\rm p} = \sqrt{4\pi e^2 n_0/m}$ is the plasma frequency; and ω_0 is the centre frequency of laser radiation corresponding to $\lambda_0 = 1$ μm. From this we obtain $\tau_{\rm las} = 3.956\omega_{\rm p}^{-1}$. These laser pulse parameters provide excitation of plasma oscillators with the energy of $E_{\rm os} = 5.08mc^2$, which is slightly above the threshold value for initiating electron self-injection $E_{\rm os}/(mc^2) - \gamma_{\rm ph} \ll \gamma_{\rm ph}$.

The process of electron self-injection was modelled by calculating motion trajectories of electron macro-particles. For this purpose, the plasma component of plasma was presented as a combination of one-dimensional thin plasma layers of width $k_{\rm p}\Delta z = 0.0005$, where $k_{\rm p} = \omega_{\rm p}/c$. Motion trajectories of such macro-particles were calculated by solving numerically one-dimensional equations (1) and (2) complemented with the Poisson equation:

$$\frac{{\rm d}^2\varphi}{{\rm d}z^2} = 4\pi\,|e|\,(n - n_0), \tag{3}$$

where n is the electron density of plasma perturbed by a laser pulse. Modelling was performed on a time interval sufficient to completely finish the process of electron self-injection into the wake field of the laser pulse and to form a bunch of captured electrons.

In Fig. 1, in the phase space z, P one can see a distribution of an ensemble of plasma electron macro-particles at the start of electron self-injection into the wake wave of the laser pulse and the accelerating force $F = -|e|E_z/(mc\omega_{\rm p})$ acting on electrons at this instant, which is determined by the value of the wake wave electrostatic field E_z. A scheme illustrating the mutual disposition of the laser pulse, plasma electrons, plasma boundary, and variation in the wake field potential along the z axis at the beginning of electron self-injection into the wake wave is presented in [21].

First of all, in Fig. 1 one can see that prior to the start of self-injection, oscillating plasma electrons form an accumulation point where their trajectories approach in such a way that the electron concentration is singular at this point. Hence, a further motion of electrons will result in that their trajectories will cross, which is confirmed by numerical simulation. In addition, the theoretical study in [20, 21] has shown that under the condition $E_{\rm os}/(mc^2) - \gamma_{\rm ph} \ll \gamma_{\rm ph}$, electron self-injection into the wake wave or the process of trajectory crossing begins with the electron that was initially separated from the plasma boundary by a distance equal to the amplitude of electron oscillations. Since the value of the restoring force for a free oscillating plasma electron is proportional to its shift from the equilibrium point z_0

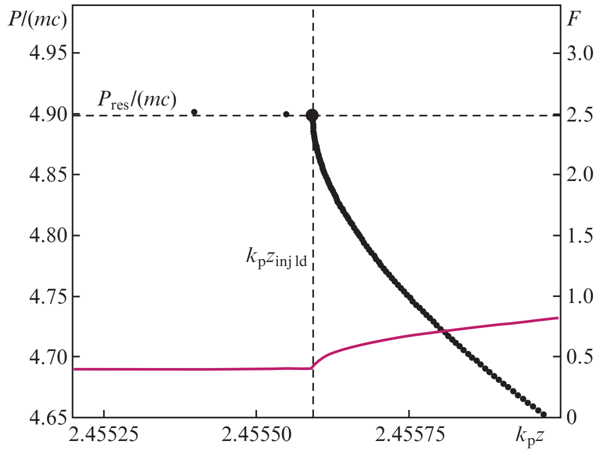

Figure 1. Electron distribution on the phase plane z, P (points) at the beginning of self-injection. The solid curve is the accelerating force $F = -|e|E_z/(mc\omega_{\rm p})$ that affects electrons in the wake wave (see the text).

$$E_z \approx 4\pi|e|n_0(z - z_0), \tag{4}$$

the amplitude of oscillations for a relativistic plasma oscillator with the energy $E_{\rm os}$ is determined by the expression

$$A_{\rm m} = \sqrt{\frac{E_{\rm os} - mc^2}{2\pi e^2 n_0}}.$$

The first electron injected into a wake wave we will call the leader because under the condition $E_{\rm os}/(mc^2) - \gamma_{\rm ph} \ll \gamma_{\rm ph}$ it remains the first at the head of the captured bunch during the entire electron acceleration process. In Fig. 1 and the following figures, the electron-leader is marked by circles of greater diameter than the rest electrons.

It was shown in [20, 21], that the coordinate of leader-electron injection into the wake wave is determined by the expression

$$z_{\rm inj\,ld} = z_{0\,\rm ld} - \sqrt{\frac{E_{\rm os} - \gamma_{\rm ph} mc^2}{2\pi e^2 n_0}}, \tag{5}$$

where $z_{0\,\rm ld} = A_{\rm m}$ is the initial position of the electron-leader prior to the laser pulse action. The electron-leader energy at the instant of self-injection is $E_{\rm inj\,ld} = \gamma_{\rm ph} mc^2 = E_{\rm res}$, that is, its velocity is equal to the phase velocity of the wake wave. The corresponding value of the momentum $P_{\rm res} = mc\sqrt{\gamma_{ph}^2 - 1}$ is marked in Fig. 1 by a horizontal dashed line. Note that the accelerating force acting on the electron-leader at this instant is positive. Hence, immediately after self-injection into the wake wave, the electron-leader transfers to the acceleration regime in the wake field of laser pulse.

Analysis of Fig. 1 allows for a conclusion that in the considered case when the energy of plasma oscillators is slightly above the threshold energy $E_{\rm os}/(mc^2) - \gamma_{\rm ph} \ll \gamma_{\rm ph}$, the accumulation point prior to electron self-injection into the wake wave is formed by the electrons having the initial coordinate $z_0 > z_{0\,\rm ld}$. These electrons are disposed deeper in plasma than the electron-leader and, hence, are self-injected into the wake wave later because they begin to interact later with the laser pulse propagating in plasma from left to right. Electrons that were initially to the left from the electron-leader, $z_0 < z_{0\,\rm ld}$, do not form an obvious accumulation point. Hence, in theoretical analysis of the considered case of self-injection of electrons $E_{os}/(mc^2) - \gamma_{ph} \ll \gamma_{ph}$, they may be neglected.

The reason of the qualitative distinction in the behaviour of electrons with the initial coordinates $z_0 < z_{0\,ld}$ from electrons with $z_0 > z_{0\,ld}$ is that the trajectory of the former prior to self-injection passes partially through vacuum. Since the restoring force for the electron in vacuum is less [and is not determined by expression (4) due to the absence of an ion background at that place], electrons move in vacuum slower, which leads to trajectory dithering in the phase space as electrons approach the point $z_{inj\,ld}$ of the start of background plasma electron self-injection into the wake wave (the vertical dashed line in Fig. 1).

Further evolution of electron self-injection into the wake wave is illustrated in Fig. 2, where the distribution of plasma electron macroscopic particles is shown in the phase space z, P at the instant t_b of termination of the main regime of electron self-injection into the wake wave. We will consider this case in more details below. Analysis of Fig. 2 yields several important conclusions concerning electron self-injection into a wake wave.

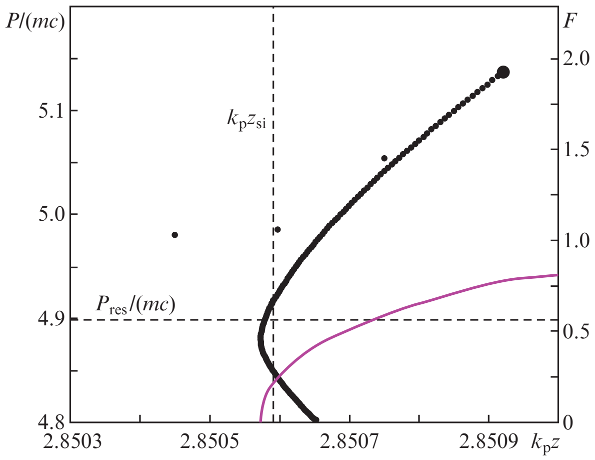

Figure 2. Electron distribution on the phase plane z, P (points) at the instant t_b of termination of the main self-injection regime. The solid curve is the accelerating force $F = -|e|E_z/(mc\omega_p)$, affecting electrons in the wake wave (see the text).

Trajectories of background plasma electrons that were initially to the right from the electron-leader, cross each other as the laser pulse propagates from left to right in plasma. The electron-leader is the first electron whose trajectory is crossed by the trajectory of a background electron. However, this does not imply that the background electron starts a self-injection process. Self-injection of a background electron into the wake wave occurs when its trajectory is crossed at a certain spatial point by a trajectory of a neighbouring electron. One can see from Fig. 2 that the coordinate of the electron self-injection point z_{si} shifts following the laser pulse approximately according to the relationship

$$z_{si}(t) = z_{inj\,ld} + V_{gr}(t - t_{inj\,ld}). \tag{6}$$

The coordinate z_{si} at the instant t_b is marked in Fig. 2 by a vertical dashed line. Real position of the self-injection point is slightly behind. This is related with the fact that electrons injected earlier into the wake wave have formed a bunch with a charge that affects the motion of following background electrons approaching the self-injection point.

In Fig. 2 one can also see that the energy of some self-injected electrons is higher than the resonance energy $E_{res} = \gamma_{ph} mc^2$ and such electrons can be considered captured for acceleration in the wake wave. The energy of the rest of the electrons in the bunch is less than the resonance energy. In Fig. 2, these groups of electrons are separated by a horisontal dashed line corresponding to $P_{res} = mc\sqrt{\gamma_{ph}^2 - 1}$. Such a distinction in the electron energies of the self-injected bunch is related to the fact that, except for the electron-leader, the momentum of other background electrons at the instant of their self-injection is always less than the momentum corresponding to the resonance energy $E_{res} = \gamma_{ph} mc^2$, which guarantees electron capturing into the wake wave. In order to be captured by the wake wave, the electron after self-injection should acquire an additional energy from the wake wave field. It is possible if at the instant of self-injection the electron gets into the accelerating phase of the wake wave.

Thus, the condition that the electron trajectory crosses the trajectory of the neighbouring electron is the only necessary condition for capturing into the wake wave. Further behaviour of the electron also depends on the electric field direction of the wake wave at the point of electron self-injection. Comparing Figs 1 and 2 one can see that the accelerating force at the point of electron self-injection gradually falls as the self-injection process evolves. Modelling shows that the main part (more than 90% at the laser pulse parameters specified above) of electrons in the captured bunch comprises electrons that fit the accelerating phase of the wake wave at the instant of self-injection. Under the condition that the total energy of plasma oscillators is slightly above the threshold energy $E_{os}/(mc^2) - \gamma_{ph} \ll \gamma_{ph}$ one may assume that fitting the accelerating phase of the wake field is the sufficient condition for electron capturing.

In Fig. 2, the instant t_b of self-injection evolution is shown, at which the self-injected electron fits into the wake wave phase with a zero accelerating force. We can say that in the time interval $t_{inj\,ld}$, t_b, the main self-injection regime is realised for background plasma electrons interacting with a laser pulse. Modelling yields that for the parameters of the laser pulse specified above this instant is $t_b \cong t_{inj\,ld} + 0.4\omega_p^{-1}$.

The formula for estimating the time interval needed for injecting bunch electrons into the wake wave in the main regime can be obtained from the condition that at the moment t_b the charge of all electrons to the right from the self-injection point is equal to a total charge of all ions from the same side. At the instant of self-injection of the electron-leader, the electric field at the self-injection point is calculated by formulae (4) and (5):

$$E_{z\,ld} = -\sqrt{8\pi n_0 (E_{os} - \gamma_{ph} mc^2)}. \tag{7}$$

According to the estimate, the wake field at the self-injection point will turn to zero when it shifts by the distance

$$z_b - z_{inj\,ld} = \sqrt{\frac{E_{os} - \gamma_{ph} mc^2}{2\pi e^2 n_0}}. \tag{8}$$

In Fig. 2, one can see that the point z_b is actually slightly to the left from the position $z_b = A_m$ calculated by formula (8). This is related to the fact that the modelling takes into account a small contribution into the charge from electrons that were initially to the left from the electron-leader: $z_0 < z_{0\,ld}$.

Since the group velocity of the laser pulse at $\gamma_{ph} \gg 1$ is close to the velocity of light, in view of approximate relationship (6) one can obtain the following estimate for the time interval during which electron self-injection occurs in the main regime:

$$t_{\mathrm{b}} - t_{\mathrm{inj\,ld}} \approx \sqrt{\frac{E_{\mathrm{os}} - \gamma_{\mathrm{ph}} mc^2}{2\pi e^2 n_0 c^2}}. \tag{9}$$

Calculation by this formula at the laser pulse parameters specified above yields the value that actually coincides with results of modelling. Note that formula (8) gives an estimate for the charge of the captured electron bunch per unit cross-section area:

$$\sigma_{\mathrm{tr}} \approx -|e| n_0 (z_{\mathrm{b}} - z_{\mathrm{inj\,ld}}) \approx -|e| n_0 \sqrt{\frac{E_{\mathrm{os}} - \gamma_{\mathrm{ph}} mc^2}{2\pi e^2 n_0}}.$$

Modelling shows that other electrons that constitute a small part of the captured bunch fit into the bunch by a complicated trajectory, which passes to a deceleration phase domain of the wake wave. Their return to the accelerating domain of the wake wave phase is possible due to the nonstationary character of the wake wave itself, which is caused by the mass overflow of background plasma electrons from right to left when passing through the laser-pulse-induced bunch of electrons captured previously.

Figure 3 illustrates this situation at the instant $t_{\mathrm{ex}} = t_{\mathrm{inj\,ld}} + 1.012\omega_{\mathrm{p}}^{-1}$. Here, a multitude of plasma background electrons passing through the bunch without self-injection looks like a solid curve because separate macro-particles are not distinguished due to high concentration. These electrons did not participate in the self-injection process, that is, their trajectories did not cross with that of the neighbouring electron. However, a large electron charge transferred by them to the opposite side from the bunch assists in that a small part of self-injected electrons, which at the instant of self-injection were not in the conditions favourable for capturing by the wake wave, return afterward to the accelerating phase domain of the wake wave and are captured. Diamond in Fig. 3 marks the electron that, according to modelling, is the last in the captured bunch. At instant t_{ex}, this electron from the deceleration phase of the wake wave approaches the boundary of the acceleration phase. Then, separation of electrons, which form the laser-pulse-induced bunch from the total number of background electrons stops.

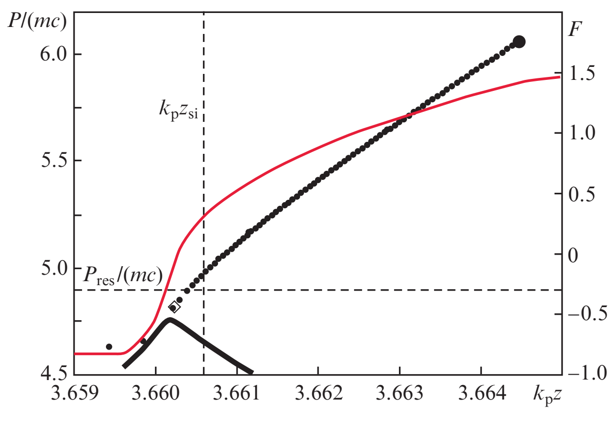

Figure 3. Electron distribution on the phase plane z, P (points) at the instant t_{ex} of terminated separation of self-injected electrons in the energy space. The solid curve is the accelerating force $F = -|e|E_z/(mc\omega_{\mathrm{p}})$, affecting electrons in the wake wave (see the text).

Variation dynamics of the energy spectrum $n(E)$ of the ensemble of electrons participated in self-injection including both captured to the bunch and uncaptured electrons is interesting. The corresponding energy spectra are presented in Fig. 4 for the time instants $t_i = t_{\mathrm{b}} + (i-1)\Delta t$ with a step $\Delta t = 0.153\omega_{\mathrm{p}}^{-1}$. The spectrum with the index $i = 1$ corresponds to Fig. 2 and, consequently, indicates a singularity in the vicinity of the energy value, at which the main regime of electron self-injection stops. In time, the singularity of the energy spectrum gradually spreads and transfers to the maximum density (spectra $i = 3, 4$) due to the difference in the force of the wake field acting on electrons. In this case, the left boundary of the electron energy spectrum shifts to a domain of lower energies. Low-energy self-injected electrons arisen in the spectrum are related with that they are in the domain of the decelerating phase of the wake wave. Finally, the spectrum with the index $i = 5$, which corresponds to Fig. 3, illustrates how a small part of electrons with a relatively low energy still continue to loose energy and separate from the main part of the spectrum which is comprised of electrons that, on the contrary, are accelerated. Finally, electrons are separated in the energy space and formation of the bunch of captured electrons terminates.

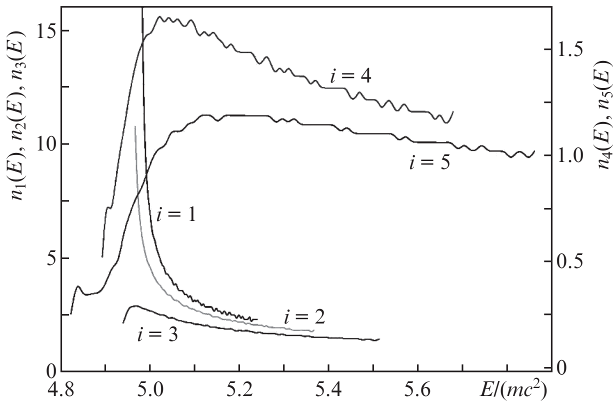

Figure 4. Energy spectra $n_i(E)$ of electrons self-injected into the wake wave at the instants $t_i = t_{\mathrm{b}} + 0.153\omega_{\mathrm{p}}^{-1}(i-1)$.

The time interval $t_{\mathrm{ex}} - t_{\mathrm{inj\,ld}}$ needed for separation of self-injected electrons in the energy space is several times longer than the interval $t_{\mathrm{b}} - t_{\mathrm{inj\,ld}}$ during which self-injection of the main part of electrons occurs; however, these intervals are of the same order. Hence, for estimates one may assume that the characteristic time of electron bunch generation by a laser pulse is, by the order of magnitude, as follows

$$\Delta T_{\mathrm{tr}} \approx t_{\mathrm{b}} - t_{\mathrm{inj\,ld}} \approx \sqrt{\frac{E_{\mathrm{os}} - \gamma_{\mathrm{ph}} mc^2}{2\pi e^2 c^2 n_0}} \tag{10}$$

(taking into account that the group velocity of the laser pulse is close to the velocity of light).

3. Determining characteristics of the bunch of self-injected electrons

Analysis of Figs 1–3 shows that the spread of the electron energy and the longitudinal length of the bunch of self-injected electrons at the formation stage can be estimated from the energy time dependence and spatial position of the electron-leader. The energy and the current coordinate of the electron-leader determine upper limits of the energy interval and interval along the z axis, in which all the rest electrons, which are self-injected into the wake wave, are distributed. For the lower limit of the energy interval one may take the value $\gamma_{\rm ph}mc^2$; the coordinate of the electron that closes the bunch can be estimated from relationship (6).

The trajectory of the electron-leader after its self-injection into the wake wave can be found by integrating the equation of motion in the field of the wave E_z:

$$\frac{dP}{dt} = -|e|\,E_z. \tag{11}$$

Since the electron-leader is the first one injected into the wake wave, it accelerates in the wake field not perturbed by electron self-injection. This wake field at each instant is determined by the dynamic distribution along the z axis of electrons – plasma oscillators. Each of them performs free oscillations around its equilibrium position under the action of restoring force (4). Oscillations of neighbouring oscillators have a phase shift. A distinction in trajectories of electrons initially separated by a distance Δz_0 from each other arises due to the time delay $\Delta z_0/V_{\rm gr}$ of the laser pulse action.

Taking into account this circumstance, the set of trajectories for all background electrons residing in front of the electron-leader after the laser pulse action terminates can be written in the integral form [21]:

$$ct - ct_0 - \frac{cz_0}{V_{\rm gr}} = I(z,z_0), \tag{12}$$

where

$$I(z,z_0) = \int_{z_0}^{z} \frac{\mathrm{d}\eta}{\sqrt{1 - m^2c^4/[E_{\rm os} - 2\pi e^2 n_0(\eta - z_0)^2]^2}};$$

$ct_0 = I(-A_{\rm m}, A_{\rm m})$; and $A_{\rm m} = \sqrt{(E_{\rm os} - mc^2)/(2\pi e^2 n_0)}$ is the amplitude of electron oscillations. The integration constant for the set of electron trajectories in expression (12) is determined from the condition that the electron whose oscillation centre coincides with the origin of coordinates passes the centre at the instant $t = 0$ with the velocity vector directed from plasma to vacuum. Representation (12) of plasma oscillator trajectories is suitable for each background electron from plasma until this electron crosses the plasma boundary or its trajectory crosses the trajectory of another electron.

It was shown [21] that from relationship (12) one can find the coordinate and instant of electron-leader self-injection (5):

$$t_{\rm inj\,ld} = t_0 + \frac{A_{\rm m}}{V_{\rm ph}} + \int_{A_{\rm m}}^{z_{\rm inj\,ld}} \frac{\mathrm{d}\eta}{\sqrt{1 - m^2c^4/[E_{\rm os} - 2\pi e^2 n_0(\eta - A_{\rm m})^2]^2}}. \tag{13}$$

Combined relationships (5), (12), and (13) under the conditions

$$0 < k_{\rm p}(z_0 - A_{\rm m}) \ll 1,\ 0 < k_{\rm p}(z - A_{\rm m}) + \sqrt{2[E_{\rm os}/(mc^2) - \gamma_{\rm ph}]} \ll 1$$

allow one to find the distribution of background electrons in the wake wave at the instant of electron-leader self-injection:

$$n = n_0\left[\frac{\gamma_{\rm ph}^3}{2\sqrt{2[E_{\rm os}/(mc^2) - \gamma_{\rm ph}]}}\right]^{1/2} \times \frac{V_{\rm ph}}{c\sqrt{k_{\rm p}(z - A_{\rm m}) + \sqrt{2[E_{\rm os}/(mc^2) - \gamma_{\rm ph}]}}}. \tag{14}$$

Then, by integrating Poisson equation (3) with allowance for the value of the wake wave field at the point of electron-leader self-injection (7) one can find the expression for this field at the instant of self-injection:

$$E_z(z) = E_{z\,\rm ld} - \frac{mc\omega_{\rm p}}{|e|}\left[\frac{2\gamma_{\rm ph}^3}{\sqrt{2[E_{\rm os}/(mc^2) - \gamma_{\rm ph}]}}\right]^{1/2} \times \frac{V_{\rm ph}}{c}\sqrt{k_{\rm p}(z - z_{\rm inj\,ld})} + k_{\rm p}(z - z_{\rm inj\,ld}). \tag{15}$$

Since the phase velocity of the wake wave is equal to the group velocity of the laser pulse $V_{\rm ph} = V_{\rm gr}$, by substituting the variable $k_{\rm p}(z - z_{\rm inj\,ld})$ in expression (15) for $\xi - \xi_{\rm inj}$, where $\xi = k_{\rm p}(z - V_{\rm ph}t)$ and $\xi_{\rm inj} = k_{\rm p}(z_{\rm inj\,ld} - V_{\rm ph}\,t_{\rm inj\,ld})$, we find the expression for the wake wave field, in which the electron-leader accelerates, for arbitrary time.

The equation of motion for the electron-leader (11) comprises the integral, which in the coordinate system related to the wave corresponds to the energy conservation law for the electron-leader moving in the wake potential:

$$\frac{E'}{mc^2} = 1 + \gamma_{\rm ph}^2 R k_{\rm p}'(z' - z'_{\rm inj\,ld}) + \frac{2^{3/2}\beta}{3}\frac{\gamma_{\rm ph}^2}{\sqrt{R}}[k_{\rm p}'(z' - z'_{\rm inj\,ld})]^{3/2} - \frac{\gamma_{\rm ph}}{2}[k_{\rm p}'(z' - z'_{\rm inj\,ld})]^2, \tag{16}$$

where $R = \sqrt{2[E_{\rm os}/(mc^2) - \gamma_{\rm ph}]}/\gamma_{\rm ph}$; and $\beta = V_{\rm ph}/c$. In what follows, the prime marks values in the coordinate system related to the wave.

Subsequent consideration will show that the most interesting range of variation in the parameter $k_{\rm p}'(z' - z'_{\rm inj\,ld})$ is given by the values of $\sim R^3$. Under this condition, the first two summands in (16) can reach $\sim\gamma_{\rm ph}^2 R^4$, and the third summand $\gamma_{\rm ph}R^6$. If $\gamma_{\rm ph}R^2 \ll 1$, then the third summand in (16) can be omitted. The physical sense of this approximation is that in the time interval of the electron bunch formation, the electron-leader shifts in phase from the phase of its self-injection into the wake wave so negligibly that a variation of the wake field due to the ion charge at such scales can be ignored. Thus, the change of the filed value in Poisson equation (3) is only determined by a contribution of background electrons, which form the accumulation density point near the coordinate of electron-leader self-injection.

Relationship (16) is the equation for a trajectory of the electron-leader in the coordinate system related to the wave. At $\gamma_{\rm ph}R^2 \ll 1$, the trajectory can be defined as

$$k_{\rm p}'(z' - z'_{\rm inj\,ld}) = R^3 \times$$

$$\times\frac{1}{2}\left\{\frac{\gamma_{\rm ph}\omega'_{\rm p}\,(t'-t'_{\rm inj\,ld})}{R}+\frac{\beta}{6}\left[\frac{\gamma_{\rm ph}\omega'_{\rm p}\,(t'-t'_{\rm inj\,ld})}{R}\right]^2\right\}. \quad (17)$$

By passing to the laboratory system of coordinates in (17), one can find the variation of the electron-leader phase in the wake wave as a function of time:

$$\xi-\xi_{\rm inj\,ld}=\frac{R^3}{2}\left(\tau+\frac{\beta}{6}\tau^2\right)\left[1-\gamma_{\rm ph}R^2\tau\left(1+\frac{\tau}{6}\right)\left(1+\frac{\tau}{3}\right)\right], \quad (18)$$

where

$$\tau=\frac{\omega_{\rm p}(t-t_{\rm inj\,ld})}{\gamma_{\rm ph}R}=\frac{\omega_{\rm p}(t-t_{\rm inj\,ld})}{\sqrt{2[E_{\rm os}/(mc^2)-\gamma_{\rm ph}]}}.$$

In Fig. 5, one can see good agreement of modelling results with calculation by formula (18). Taking expression (6) into account one can obtain that the length $L_{\rm b}$ of the electron bunch at the formation stage is estimated as

$$k_{\rm p}L_{\rm b}\approx k_{\rm p}(z_{\rm ld}(t)-z_{\rm si}(t))=\xi-\xi_{\rm inj\,ld}, \quad (19)$$

where $z_{\rm ld}(t)$ is the time dependence of the coordinate of the electron-leader accelerated in the wake field.

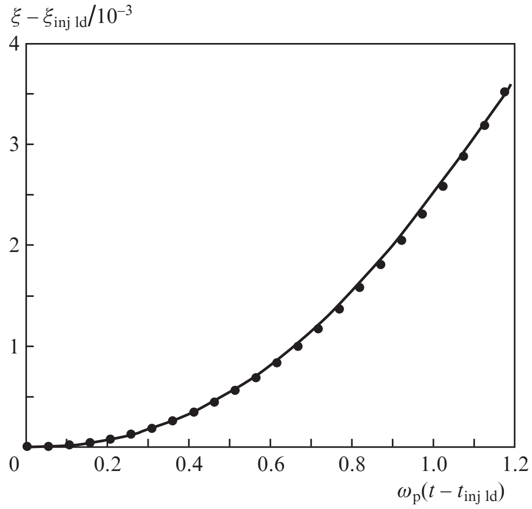

Figure 5. Time dependence of the phase shift of the accelerated electron-leader in the wake field relative to the phase of its injection into the wake wave. Points refer to modelling, and the solid curve is calculation by formula (18).

Under the condition $\gamma_{\rm ph}R^2\ll 1$, formula (16) gives an estimate for the electron-leader energy excess over the resonance energy $\Delta E=E-\gamma_{\rm ph}mc^2$, that is, the energy spread in the bunch:

$$\Delta E=mcV_{\rm ph}\sqrt{2\gamma_{\rm ph}^3\sqrt{2[E_{\rm os}/(mc^2)-\gamma_{\rm ph}]}}$$

$$\times\sqrt{\xi-\xi_{\rm inj\,ld}+\frac{2^{3/2}V_{\rm ph}}{3c}\left[\frac{(\xi-\xi_{\rm inj\,ld})\gamma_{\rm ph}}{\sqrt{2[E_{\rm os}/(mc^2)-\gamma_{\rm ph}]}}\right]^{3/2}}. \quad (20)$$

Figure 6 illustrates good agreement of modelling results with calculation by formula (20).

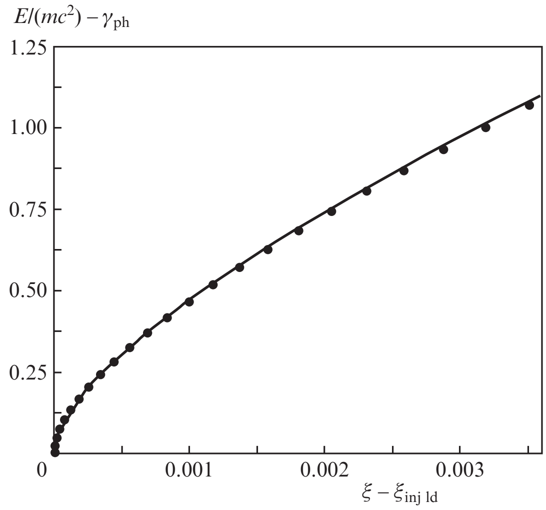

Figure 6. Dependence of the energy of the electron-leader accelerated in the wake field on the current value of its phase in the wake wave. Points refer to modelling, and the solid curve is calculation by formula (20).

One can estimate the parameters of the electron bunch at the end of the main regime of electron self-injection by formulae (18)–(20). In view of (9) it follows from (18) that the bunch length at this moment $t_{\rm b}$ is determined by the expression

$$k_{\rm p}L_{\rm b}\approx 0.68R^3=1.92\left(\sqrt{E_{\rm os}/(mc^2)-\gamma_{\rm ph}}/\gamma_{\rm ph}\right)^3, \quad (21)$$

and, according to (20), the energy spread in the bunch is

$$\Delta E\approx 3.1(E_{\rm os}-\gamma_{\rm ph}mc^2). \quad (22)$$

Expressions for the length of the electron bunch (21) and the energy spread between electrons in the bunch (22) clearly show the obvious physical property of the electron bunch generation by the laser pulse. Namely, under the condition that the electron bunch is generated outside the space domain occupied by the laser pulse, all characteristics and features of this process, including parameters of the electron bunch obtained, directly depend only on the set of parameters specific for plasma oscillators (see also [21]). These parameters comprise their total oscillation energy $E_{\rm os}$ and the phase shift between oscillations determined by the phase velocity $V_{\rm ph}$ (or gamma-factor $\gamma_{\rm ph}$). The role of the driver of theses oscillations in formulae (20), (21) is minor. This means that the laser pulse profile is not principally important for generating electron bunches if the pulse, while passing through plasma, leaves in its trace plasma oscillators having the same energy and phase delay relative to each other. Moreover, the driver type itself also is not important for exciting plasma oscillations. For example, a laser pulse as an exciter of plasma oscillations can be replaced with an electron bunch possessing characteristics capable of exciting longitudinal electron oscillations in plasma with the same $E_{\rm os}$ and $\gamma_{\rm ph}$.

Thus, the relationship between the total energy of plasma oscillators excited by the laser pulse and the profile or parameters of the pulse is not of unique character. For obtaining the required value of $E_{\rm os}$, these parameters can be varied within possibilities of the experiment under the condition that assumptions made in the present work hold. The problem of the energy of plasma oscillations excited in a trace of the laser pulse propagating across plasma is equivalent to a long-history problem [22] concerning the amplitude of a wake field excited by a laser pulse in plasma. Unfortunately, this problem has no simple analytical solution in the case of generating a high-amplitude nonlinear plasma wave needed in the considered case. This is why it is impossible to express analytically the value of the total energy of the plasma oscillator in terms of characteristics describing the laser pulse profile in order to substitute it into formulae (21), (22).

Thus, let us estimate the length of the electron bunch and energy spread for electrons in the bunch for a particular laser pulse with the parameters specified above. By the time t_b, when the main part of electrons is accumulated in the bunch generated by the laser pulse, the bunch duration, according to (21), (22), will be ~ 1 as, and the relative energy spread $\Delta E/E$ between electrons will be $\sim 5\%$. The average bunch electron energy will be $\sim \gamma_{ph} mc^2$. If for fixing the bunch length one takes the moment t_{ex}, when the process of the bunch electron separation from a multitude of plasma background electrons terminates, then formulae (18), (19) yield the estimate for the bunch length ~ 6 as. The reason for the elongated bunch length is that different bunch electrons are subjected to different accelerating forces acting from the wake field, which include repulsion of electrons in the bunch due to its own charge.

Note that influence of the electron repulsion effect at the stage of separation from a multitude of plasma background electrons on the bunch length is automatically taken into account, because the bunch length is only determined by the trajectory of the electron-leader. After the bunch electrons have been separated from the main group of plasma background electrons, this approach is no more applicable. It is necessary to additionally determine the trajectory of the electron closing the bunch, which moves in the wake field; however, the wake field cannot be found in such a simple way as in the case of the electron-leader. One approach to determining the length of the electron bunch in this case is presented in [21].

Finally, let us discuss applicability of the one-dimensional model of plasma with a sharp boundary used in the present work for studying generation of electron bunches by a non-unidimensional laser pulse that crosses the boundary of inhomogeneous plasma with a transition layer.

From the general methodological point of view, the transition layer of plasma can be neglected if the layer thickness d is much less than any other parameters characterising the considered physical phenomenon in the z axis (for example, the oscillation amplitude of the plasma oscillator A_m, the length of the wake wave λ_p, and the characteristic length of the laser pulse $c\tau_{las}$). From the practical point of view, this is a very strict condition.

Let us analyse qualitatively consequences from the presence of the transition layer between vacuum and homogeneous plasma at a boundary of inhomogeneous plasma. Some conclusions can be made from the analysis of Fig. 2, where results of numerical simulation demonstrate the plasma electron distribution at the instant t_b corresponding to termination of the main regime of electron self-injection into the wake wave. It is important that numerical simulation takes into account all plasma electrons including those which get out to vacuum and cross the sharp boundary of plasma at $z = 0$ in the process of the electron bunch generation by the laser pulse.

It follows from Fig. 2 that the coordinate of the point $k_p z_b = 2.8505$, where the wake field turns to zero, is, although close to, however, distinct from the coordinate $k_p A_m = 2.8565$ calculated theoretically. This is related to the fact that, in contrast to modelling, electrons disposed initially to the left from the electron-leader ($z_0 < z_{0\,ld}$) were neglected and their trajectories partially passed through vacuum. Analysis of the electron state of the bunch at this instant shows that the distinction between z_b and A_m with a sufficient accuracy is explained by a small (less than 2%) part of electrons fitting the bunch from the domain $z_0 < z_{0\,ld}$. As mentioned, so small a part of such electrons in the bunch is explained by the fact that they do not form the well expressed accumulation point to the left from the electron-leader at the instant of self-injection of the latter $t_{inj\,ld}$. This circumstance, in turn, is a consequence from that in vacuum, due to the absence of the ion background, the force returning the electron to its centre of oscillations is substantially weaker than the restoring force acting on the electron in plasma.

It is clear that a transition layer will increase the frequency of electron oscillations with the initial position $z_0 < z_{0\,ld}$ and will result in a greater part of these electrons in the bunch generated by the laser pulse. However, one may expect that the presence of electrons in the generated bunch will not change the physical picture of the phenomenon considered as long as electrons with the initial coordinates $z_0 > z_{0\,ld}$ play a dominating role in the bunch. Thus, the simplification used in the present work, namely, substitution of the transition layer for a sharp boundary, can be used for studying mechanisms of the electron bunch formation when a laser pulse crosses a boundary of inhomogeneous plasma. Qualitative analysis of the influence of the transition layer on generation of electron bunches by a laser pulse in inhomogeneous plasma will be performed later.

Applicability of the one-dimensional geometry for studying the mechanism of electron self-injection into the wake field generated by a non-unidimensional laser pulse is determined by the condition that the motion of background plasma electrons is close to unidimensional. It is known [13, 16] that dynamics of background plasma electrons in the so-called bubble-regime of laser pulse propagation is strongly determined by the transverse size of the pulse. In this case, the size of the arising cavern w_0 is estimated by the formula $k_p w_0 \sim \sqrt{a_0}$, i.e., it depends on the laser pulse amplitude. In view of the correspondence between the cavern size w_0 and the width σ of the laser pulse ($k_p w_0 \sim k_p \sigma$), in a stable bubble-regime the amplitude of the laser pulse and its width should be matched: $k_p \sigma \sim \sqrt{a_0}$. Numerical modelling performed in [23] refined this relationship for the case of linearly polarised laser radiation: $k_p \sigma = 2\sqrt{a_0}$.

The regime of one-dimensional interaction of laser radiation with plasma considered in the present work excludes the possibility of producing a cavern, which is not a one-dimensional object. For generating a cavern, the width of the laser pulse having the amplitude a_0 should satisfy the condition $k_p \sigma \gg 2\sqrt{a_0}$. This estimate was verified by the authors of this method of injecting electrons into a wake wave [19] by means of two-dimensional numerical simulation of the process of plasma interaction with a linearly polarised laser pulse having the profile

$$a = a_0\exp\left(-\frac{r^2}{\sigma^2}\right)\cos^2\frac{t}{\tau_{\rm las}}\,\mathrm{sgn}\left(\frac{\pi\tau_{\rm las}}{2}-|t|\right)$$

and duration $\tau_{\rm FWHM} = 12.13$ fs at $\gamma_{\rm ph} = 5-7$, $\lambda_0 = 1$ μm, $\sigma = 20\lambda_0$; the results obtained were compared with the modelling results in the one-dimensional geometry. It was shown that fulfilment of the condition $k_{\rm p}\sigma \gg 2\sqrt{a_0}$ provides good agreement for energy characteristics of the electron bunch generated by a laser pulse in the interaction with semirestricted plasma as in the one-dimensional, so and in two-dimensional geometry. Appropriateness of using the one-dimensional approximation was more thoroughly considered in [21].

At a given laser pulse width σ one can also estimate the value of the electron bunch charge. By using formula (8) for a layer thickness of background plasma electrons and assuming that the amplitude distribution of the laser pulse in the transverse direction is $a = a_0\exp(-r^2/\sigma^2)$, one can obtain the formula for estimating the bunch charge:

$$Q_{\rm tr} \approx -|e|\,k_{\rm p}^{-1}n_0\sigma^2\sqrt{2[E_{\rm os}(a_0)/(mc^2)-\gamma_{\rm ph}]}\,\ln(a_0/a_{\rm th}), \quad (23)$$

where, at a prescribed laser pulse duration $\tau_{\rm FWHM}$, the threshold amplitude of the laser pulse $a_{0\,\rm th}$ corresponds to the energy of plasma oscillators $E_{\rm os\,th}$. Calculation shows that for a circularly polarised laser pulse radiation with $\lambda_0 = 1$ μm and $\tau_{\rm FWHM} = 12$ fs at $\gamma_{\rm ph} = 5$, the threshold amplitude is $a_{0\,\rm th} \cong 4.919$. Then, according to the estimate by formula (23), one obtains that the laser pulse with the amplitude $a_0 = 4.982$ ($E_{\rm os}/(mc^2) \cong 5.08$), $\sigma = 20\lambda_0$ (i.e., power $P \approx 0.427$ PW), generates the electron bunch with a charge $Q_{\rm tr} \approx 11.6$ pC. Calculations of the bunch charge at other parameters of plasma and laser pulse are given in [20].

4. Conclusions

Investigation of the generation of short electron bunches when a relativistic-intensity laser pulse propagates through a sharp plasma boundary revealed the main features of the physical mechanism underlying this process.

Numerical simulation has clearly demonstrated that the generation of electron bunches by a laser pulse is a consequence of multithread motion of the electron component of plasma. In certain conditions, such a motion can be presented in the form of mutually intersecting trajectories of plasma electrons, each electron being initially a plasma oscillator excited by laser pulse, which executes free oscillations around the initial position, which this oscillator occupied prior to the action of the laser pulse. The necessary condition for oscillator trajectory intersection is an excess of their total energy $E_{\rm os}$ over the threshold value $E_{\rm os\,th} = mc^2/\sqrt{1-V_{\rm gr}^2/c^2}$, determined by the group velocity of the laser pulse.

Intersection of the trajectory of the plasma oscillator with that of the neighbouring oscillator results in self-injection of this electron into the wake wave produced by the laser pulse, which is the physical mechanism of generating an electron bunch. However, not all electrons after self-injection are captured in the bunch produced, because the critical item is the position of the point of electron self-injection in the wake wave. If the total energy of plasma oscillators insignificantly exceeds the threshold value $E - E_{\rm os\,th} \ll E_{\rm os\,th}$ then the electrons whose self-injection point fits into the accelerating phase domain of the wake wave are kept in the generated bunch and captured by the wake field. Such electrons constitute the main ensemble of the electron bunch generated by a laser pulse. Other electrons, which perform self-injection near the boundary of the accelerating phase or in the decelerating domain of the wake field, are only partially captured by the wake wave. Those can be captured only after the plasma background electrons, not participating in the process of electron self-injection, start intensively flow through the bunch of captured electrons, which follows the laser pulse.

For determining the parameters of the electron bunch generated by a laser pulse, we have analytically obtained the trajectory for the electron-leader, i.e., for the first electron injected into a wake wave, which resides in the head of the bunch. It was shown that at the formation stage of the electron bunch when background plasma electrons are self-injected into the wake wave, the duration of the electron bunch and the spread of the electron energy in the bunch are determined by the trajectory of the electron-leader and by variation of its energy along this trajectory.

Simple formulae are obtained for estimating the duration of the electron bunch and the energy spread of bunch electrons after the process of the bunch formation terminates. It is shown that characteristics of the generated electron bunch are determined by the group velocity of the laser pulse and by the energy of plasma oscillators excited by the pulse. It was found that the electron bunch duration may be ~10 as at the relative spread of the electron energy less than 10%.

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