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Measurements of complex coupling coefficients in a ring resonator of a laser gyroscope

A.S. Bessonov, A.P. Makeev, E.A. Petrukhin

Abstract. **A method is proposed for measuring complex coupling coefficients in a ring optical resonator in the absence of an active gas mixture. A setup is described on which measurements are performed in ring resonators of ring He–Ne lasers with a wavelength of 632.8 nm. A model of backscattering field interference between conservative and dissipative sources is presented. Within the framework of this model, the unusual behaviour of backscattering fields in ring resonators observed in experiments is explained: a significant difference in the moduli of coupling coefficients of counterpropagating waves and variation of the magnitude of the total phase shift in a wide range. It is proposed to use this method as a metrological method when assembling and aligning a ring resonator of a laser gyroscope.**

Keywords: *ring laser, ring resonator, laser gyroscope, backscattering, lock-in threshold, coupling coefficients of counterpropagating waves.*

1. Introduction and statement of the problem

Backscattering on mirrors is one of the main sources of errors of a laser gyroscope (LG) based on a ring He–Ne laser with a wavelength of 632.8 nm. This phenomenon leads to the appearance of a gyroscope dead band at low rotation speeds (so-called lock-in threshold), as well as to nonlinear distortions of the frequency characteristic of the gyroscope outside the lock-in band [1, 2].

In describing the effects associated with the influence of backscattering, use is made of the system of equations [2]:

$$\frac{dI_{cw}}{dt} = I_{cw}\frac{c}{L}\Big[\alpha - \beta I_{cw} - \theta I_{ccw} + 2r_{ccw}\Big(\frac{I_{ccw}}{I_{cw}}\Big)^{1/2}\cos(\psi + \varphi_{ccw})\Big], \quad (1)$$

$$\frac{dI_{ccw}}{dt} = I_{ccw}\frac{c}{L}\Big[\alpha - \beta I_{ccw} - \theta I_{cw} + 2r_{cw}\Big(\frac{I_{cw}}{I_{ccw}}\Big)^{1/2}\cos(\psi - \varphi_{cw})\Big], \quad (2)$$

$$\frac{d\psi}{dt} = \Omega + \frac{c}{L}\Big[r_{ccw}\Big(\frac{I_{ccw}}{I_{cw}}\Big)^{1/2}\sin(\psi + \varphi_{ccw}) + r_{cw}\Big(\frac{I_{cw}}{I_{ccw}}\Big)^{1/2}\sin(\psi - \varphi_{cw})\Big], \quad (3)$$

where I_{cw} and I_{ccw} are the intensities of counterpropagating waves of a ring laser (RL) in clockwise and counterclockwise directions; ψ is the phase difference of counterpropagating waves in the RL; Ω is the dither frequency; and α is the difference between the active medium gain and ring resonator (RR) losses. The coefficients β and θ are, respectively, the parameters of self-saturation and mutual nonlinear saturation of counterpropagating waves in the active medium; c is the speed of light; and L is the RR perimeter. The influence of backscattering is described by two complex coupling coefficients (CCs), representing the fractions of the natural oscillation fields scattered in counterpropagating directions:

$$\tilde{r}_{cw} = r_{cw}\exp(i\varphi_{cw}), \quad (4)$$

$$\tilde{r}_{ccw} = r_{ccw}\exp(i\varphi_{ccw}). \quad (5)$$

It is easy to see from the structure of equations (1)–(3) that backscattering is taken into account in them by three parameters rather than four (two moduli r_{cw} and r_{ccw} and two phase shifts φ_{cw} and φ_{ccw}). The phase shifts arising during backscattering of each of counterpropagating waves appear in these equations as a sum $\varphi = \varphi_{cw} + \varphi_{ccw}$. Therefore, by complex CCs we mean three parameters: r_{cw}, r_{ccw} and φ.

The influence of backscattering on the amplitude–frequency response of a ring gas laser has been actively studied since the mid-1960s. For more than half a century of LG history, several hundred articles have been published and many patents have been issued. Special mention should be made of paper [3], where the authors presented the results of model experiments using inverse reflectors installed near the output RL mirror, which revealed the main features of the behaviour of the amplitude–frequency response of RLs as a function of complex CCs. The results of calculations performed in the weak-coupling approximation [4, 5] showed good qualitative agreement with the results of these model experiments. In particular, for the correction to the beat frequency of counterpropagating waves in a RL, the relation was obtained:

A.S. Bessonov Moscow Technological University (MIREA), prosp. Vernadskogo 78, 119454 Moscow, Russia;
A.P. Makeev OJSC ‘M.F. Stel’makh Polyus Research Institute’, ul. Vvedenskogo 3, Bld. 1, 117342 Moscow, Russia;
E.A. Petrukhin JSC Serpukhov Plant ‘Metallist’, ul. Lunacharskogo 32, 142200 Serpukhov, Moscow region, Russia;
e-mail: petruhin53@mail.ru

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$$\frac{\Delta\nu}{\Omega} = 1 - \left(\frac{c}{L}\right)^2 \left(\frac{S_+^2}{2\Omega^2} - \frac{1}{2}\frac{S_-^2}{\Omega_g^2 + \Omega^2}\right), \quad (6)$$

where $\Delta\nu$ is the RL beat frequency;

$$\Omega_g = \Omega_c \frac{\alpha}{\delta}\left(\frac{\beta - \theta}{\beta + \theta}\right); \quad (7)$$

Ω_c is the RR band; and δ is the RR losses. The coefficients in the numerators of the negative and positive corrections to the beat frequency are the following combinations of CCs:

$$S_+ = \sqrt{r_{cw}^2 + r_{ccw}^2 + 2r_{cw}r_{ccw}\cos(\varphi_{cw} + \varphi_{ccw})}\,, \quad (8)$$

$$S_- = \sqrt{r_{cw}^2 + r_{ccw}^2 - 2r_{cw}r_{ccw}\cos(\varphi_{cw} + \varphi_{ccw})}\,. \quad (9)$$

The weak-coupling approximation corresponds to the case when the frequency dithering Ω significantly exceeds the lock-in threshold Ω_L.

Equation (6) can be used for the experimental estimation of the magnitudes of complex CCs. To do this, the RL is placed on a rotating stage and the dependence of the scale factor on the rotation speed (frequency dithering Ω) is measured. The parameter S_+ determines the RL lock-in threshold Ω_L [$\Omega_L = (c/L)S_+$] (in the framework of the weak-coupling approximation). The ratio of the parameters S_+ and S_- determines the sign of the LG scale factor correction. Typically, in ring He–Ne lasers with a wavelength of 632.8 nm, the parameter S_- is 3–5 times higher than S_+. Therefore, at high rotation speeds, a positive scale factor correction is observed [2].

Of course, the results of measurements of the parameters S_+ and S_- [see relations (8) and (9)] make it impossible to determine the values of all three backscattering parameters: r_{cw}, r_{ccw} and φ. In order to carry out their estimation, it is assumed (see, for example, [6]) that the CC moduli for counterpropagating directions are equal. In a number of cases (in particular, when analysing sources of error of an extra-large LG [7]), complex CCs are found from the results of measurements of the variable components of the counterpropagating wave intensities at the beat frequency. With such an estimation method, it is not necessary to assume the equality of the CC moduli, since use is made of the values of three measured parameters: the modulation depth of the counterpropagating wave intensities and the phase shift between the variable components of the intensities at the beat frequency.

The above methods can be attributed to indirect methods of measuring complex CCs in RLs. Without focusing attention on the obvious shortcomings of the indirect methods associated with inaccurate knowledge of the values of the parameters of the nonlinear interaction of counterpropagating waves (θ and β) and with the use of approximate solutions of the system of equations (1)–(3), we point out a more significant and fundamental disadvantage: Complex CCs are determined only after the RR is assembled and filled with a working gas mixture.

Practice shows that in mass assembly, the spread in the values of complex CCs is significant. As a result, the difference between the minimum and maximum values of the lock-in threshold reaches 20 to 30 times. It is not always possible to explain this by the influence of large dust-like particles in the working region of mirrors. Such a large spread in the parameter values is due, first of all, to the physical nature of the formation of the backscattering fields in the RR (so-called speckle pattern [8, 9]). Therefore, the assembly and alignment of a laser gyro is today a ‘lottery’, the result of which becomes known after a time-consuming procedure of vacuum processing of a monoblock sensor. For this reason, measurements of complex CCs at the stage of assembly and alignment of the RR is not only an extremely important means of control, but, more significantly, it makes it possible to increase the LG accuracy.

The present paper consists of two main parts. The first part (Sections 2, 3) describes the methods for measuring complex CCs in a RR. The results of measurements for ring resonators of a He–Ne laser with a wavelength λ = 632.8 nm are presented. The effect of the backscattering field interference of counterpropagating waves on complex CCs is analysed. The second part of the paper (Section 4) considers the interference model of waves of dissipative and conservative point sources, which makes it possible to describe correctly the regularities observed in the experiments.

2. Methods for measuring complex CCs in the RR

We start with the simplest optical scheme, which allows the CC modulus to be measured for one of the counterpropagating waves in the RR (Fig. 1a). When the generation frequency of a probe laser (PL) coincides with the RR natural oscillation frequency, the intensities of the forward (I) and backscattered (I_{bs}) waves are determined by the relations [10]:

$$I = \frac{4T_1T_2}{\delta^2}I_0, \quad (10)$$

$$I_{bs} = \frac{4r^2}{\delta^2}I\,. \quad (11)$$

Here, T_1 and T_2 are the transmission coefficients of the input and output RR mirrors; I_0 is the intensity of laser radiation (we assume an ideal matching of the amplitude–phase front of the laser and the RR); and r is the CC modulus (the index of the direction is not specified). It should be noted that these relations were obtained on the assumption that the RR losses are small. For reference, the total RR loss of modern LGs is hundreds of ppm (1 ppm is one part per million or 0.0001 %).

Let us carry out numerical estimates. For typical values of the LG RR parameters (δ = 400 ppm, $T_1 = T_2$ = 150 ppm, r = 0.1–1 ppm), the ratio of backscattered and forward wave intensities is $I_{bs}/I = 2.5 \times 10^{-7}$–$2.5 \times 10^{-5}$. Or to estimate the absolute value of the power of the backscattered wave emerging from the RR (we assume that the PL has an output power of 1 mW), we have 1.4×10^{-13}–1.4×10^{-11} W. Registration of such powers of optical radiation does not cause significant technical difficulties.

Of course, with this method of measurement, it is not possible to obtain information about the magnitude of the phase shift φ of counterpropagating waves arising due to backscattering. The phase shift measurement should rely on a fundamentally different optical scheme. In our experiments, two such schemes were used (Figs 1b and 1c), where the natural oscillations of the RR were excited in opposite directions. In this case, the fields of counterpropagating waves represent a superposition of natural oscillations of the resonator and the fields of backscattered waves:

$$E_{cw}^{sum} = E_{cw}\exp(i\chi_{cw}) + \frac{2r_{ccw}}{\delta}E_{ccw}\exp[i(\chi_{ccw} + \varphi_{ccw})], \quad (12)$$

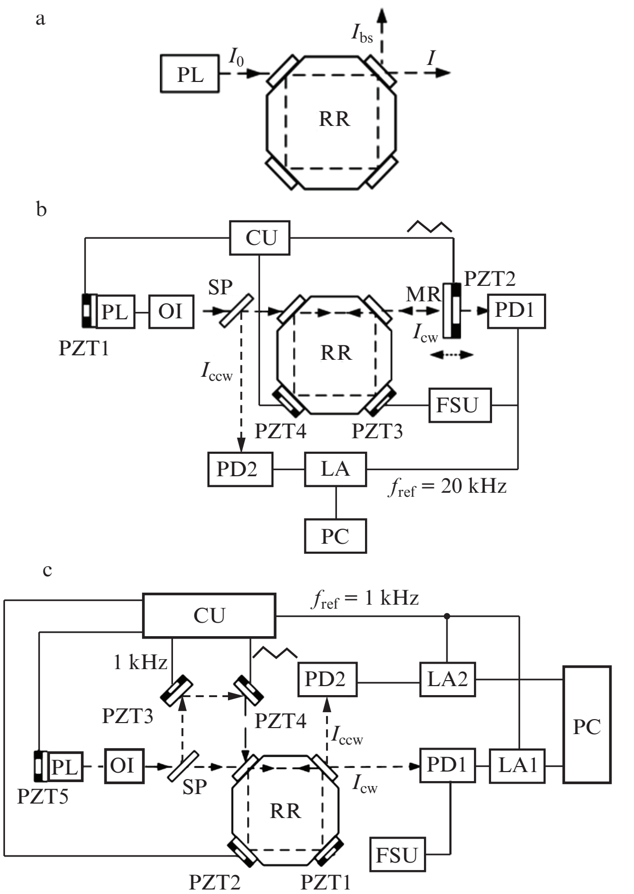

Figure 1. Schematics of measurement of complex CCs: (a) optical scheme for measuring the CC modulus for one of the counterpropagating waves, (b) scheme with a returnable mirror (RM) and (c) scheme with a mixer:
(PL) probe He–Ne laser with a radiation wavelength of 632.8 nm; (CU) control unit; (FSU) frequency stabilisation unit; (OI) optical isolator; (SP) semitransparent plate; (PD1) and (PD2) photodetectors; (PZT1 – PZT5) piezoelectric transducers; (LA1) and (LA2) lock-in amplifiers; (PC) personal computer; f_{ref} is the reference frequency of the signal applied to the input of the lock-in amplifier.

$$E_{ccw}^{sum} = E_{ccw}\exp(i\chi_{ccw}) + \frac{2r_{cw}}{\delta}E_{cw}\exp[i(\chi_{cw} + \varphi_{cw})]. \quad (13)$$

Because counterpropagating waves in the RR have the same frequency, the factor $\exp(i\omega t)$ (ω is the circular frequency) is omitted in these relations. The factor $2r/\delta$ appears as a result of the allowance for the coupling between the intensities of the forward and backscattered waves (11). Taking into account the smallness of the ratio $2r/\delta$ for the counterpropagating wave intensities, it is easy to obtain the expressions:

$$I_{cw} = I_{cw}^{(0)}\left[1 + \frac{4r_{ccw}}{\delta}\sqrt{\frac{I_{ccw}^{(0)}}{I_{cw}^{(0)}}}\cos(\chi + \varphi_{ccw})\right], \quad (14)$$

$$I_{cw} = I_{cw}^{(0)}\left[1 + \frac{4r_{ccw}}{\delta}\sqrt{\frac{I_{ccw}^{(0)}}{I_{cw}^{(0)}}}\cos(\chi + \varphi_{ccw})\right], \quad (15)$$

where $\chi = \chi_{ccw} - \chi_{cw}$ is the phase difference of the exciting oscillations; and $I_{cw}^{(0)}$ and $I_{ccw}^{(0)}$ are the mean values of the intensities of the counterpropagating waves (without allowance for the backscattering fields).

In moving the mirrors located outside the RR in question, the phase difference χ varies in the range $0-2\pi$, and there are also small changes in the intensities of counterpropagating waves from the fraction to one percent. The observed shift in the positions of the extrema of the interference patterns is due to the searched-for total phase shift φ. For example, at $\varphi = \pi$, the position of the intensity maximum of one of the waves will coincide with the position of the intensity minimum of a counterpropagating wave.

Important parameters characterising the observed interference patterns are their contrasts (c_{cw} and c_{ccw}), defined as the ratio of the difference between the maximum and minimum intensities to their sum. In the case of a scheme with a returnable mirror (RM), for contrasts we have expressions

$$c_{cw} = \frac{I_{cw}^{max} - I_{cw}^{min}}{I_{cw}^{max} + I_{cw}^{min}} = \frac{8T_2\sqrt{\rho}}{\delta^2}r_{ccw}, \quad (16)$$

$$c_{ccw} = \frac{I_{ccw}^{max} - I_{ccw}^{min}}{I_{ccw}^{max} + I_{ccw}^{min}} = \frac{2}{T_2\sqrt{\rho}}r_{cw}, \quad (17)$$

(ρ is the RM reflection coefficient), and in the case of a scheme with a mixer, we have expressions

$$c_{cw,ccw} = \frac{4r_{ccw,cw}}{\delta}. \quad (18)$$

In deriving relation (18), we assumed that the intensities of intrinsic counterpropagating waves in the RR are equal. In practice, this is easily attainable, since the excitation of natural oscillations in opposite directions occurs through a single input mirror of the RR. The contrasts of the interference patterns measured in experiments were used to determine the CC moduli. The parameters δ, T_2 and ρ were measured additionally.

Optical schemes (Figs 1b and 1 c) have both advantages and disadvantages. The main advantage of the RM scheme is that the contrast of the interference pattern in the counterclockwise direction (17) can be increased in a controlled manner by reducing the RM reflection coefficient. The same effect is achieved when an attenuation filter is placed between the RR and the RM. Looking ahead, we note that in the course of our experiments there were no technical difficulties in measuring the CC moduli that are less than 0.01 ppm.

The main drawback of the RM scheme is the noticeable difference in contrasts for counterpropagating waves due to the asymmetry of the excitation of natural oscillations. This in some cases makes it difficult to measure the phase shift. The optical scheme with a mixer easily ensures the symmetry of excitation of counterpropagating waves. In turn, when using this scheme, it is necessary to proceed to the measurement of the variable components of the wave intensities emerging from the RR. Because of this, it becomes impossible to correctly measure the absolute values of the CC moduli, which is a significant drawback of the scheme with a mixer.

Our measurements were carried out in two stages, with the main advantages of these two schemes being used. First, the absolute values of the CC moduli were measured with the use of the RM scheme. Then, using the scheme with a mixer, the phase shift φ was measured.

Let us now describe the technical details of our measurement method. First of all, for complex CCs to be measured, it is necessary to 'lock-in' the PL frequency to the fundamental mode frequency of the RR under study. To this end, one of the PL mirrors was equipped with a piezoelectric transducer (PZT), which allows the lasing frequency to be controlled. The frequency was stabilised with respect to the amplitude resonances of the intensity of the radiation emerging from the RR. The resonance has the form of a Lorentz function. The frequency stabilisation unit (FSU) uses an error signal proportional to the first derivative of the given function with respect to time. To do this, a small amplitude (of the order of the width of the resonance) harmonic modulation with a frequency of about 10 kHz is introduced into the control signal of the PZT. The error signal is fed to the input of the PID (proportional-integral-differentiating) controller, whose output was connected to the PZT of the RR or PL. A detailed description of such stabilisation schemes can be found in [11].

With the optimal adjustment of the FSU, the average value of the intensity of radiation emerging from the RR was approximately 50% of its peak value. The time dependence of the intensity of radiation emerging from the RR with the operating FSU is shown in Fig. 2a. As a PL we used two types of monoblock He–Ne lasers made of glass ceramics: with a linear and ring resonators. In the first case, the power of single-mode lasing was about 1 mW, and in the second, it was 70 μW. The power of radiation emerging from the RR, as a rule, exceeded 1 μW. For the measurements we used silicon photodiodes with a photosensitive area diameter of 1 mm and a photocurrent–voltage conversion circuit on an LF411CN operational amplifier.

In using the RM scheme, the mean value of the intensity of radiation emerging from the RR was measured. To do this, the output of the photodetector (Fig. 1b) was connected to the input of a lock-in amplifier. The reference voltage was an alternating voltage with a frequency equal to twice the modulation frequency of the FSU (about 20 kHz). A triangular signal with a period of about 20 s and an amplitude corresponding to the longitudinal displacement of the RM for a distance greater than $\lambda/2$ was applied to the PZT RM. Depending on the magnitude of the measured CC modulus, a filter was selected, which makes it possible to obtain a contrast of the interference pattern at a level of 10%. A typical time dependence of the average intensity of radiation emerging from the RR in the counterclockwise direction when the RM is displaced is shown in Fig. 2b.

In our experiments we used four-mirror RRs with perimeters of 16 or 28 cm. The total RR losses varied from 400 to 1800 ppm. Multilayer dielectric mirrors with TiO_2–SiO_2 and Ta_2O_5–SiO_2 layers deposited by ion deposition were used. The total integral scattering (TIS) of mirrors did not exceed 30 ppm.

As already noted above, when measuring the phase shift φ, it is necessary to register the variable components of the intensities of the waves emerging from the RR (Fig. 1c). This is due to the fact that contrasts of interference patterns in the scheme with a mixer, as a rule, amount to several tenths of a percent. Registration of such small changes in intensity against the background of other amplitude noises seems to be an unsolvable task in this situation. In measuring variable intensity components, harmonic modulation of the measured parameter is introduced, which makes it possible to significantly increase the signal-to-noise ratio of the measuring scheme.

For this reason, two folding mirrors (Fig. 1c) are installed on two PZTs. One of them is fed with a harmonic signal with a frequency of about 1 kHz and a deviation amplitude corresponding to a change in the phase shift of counterpropagating

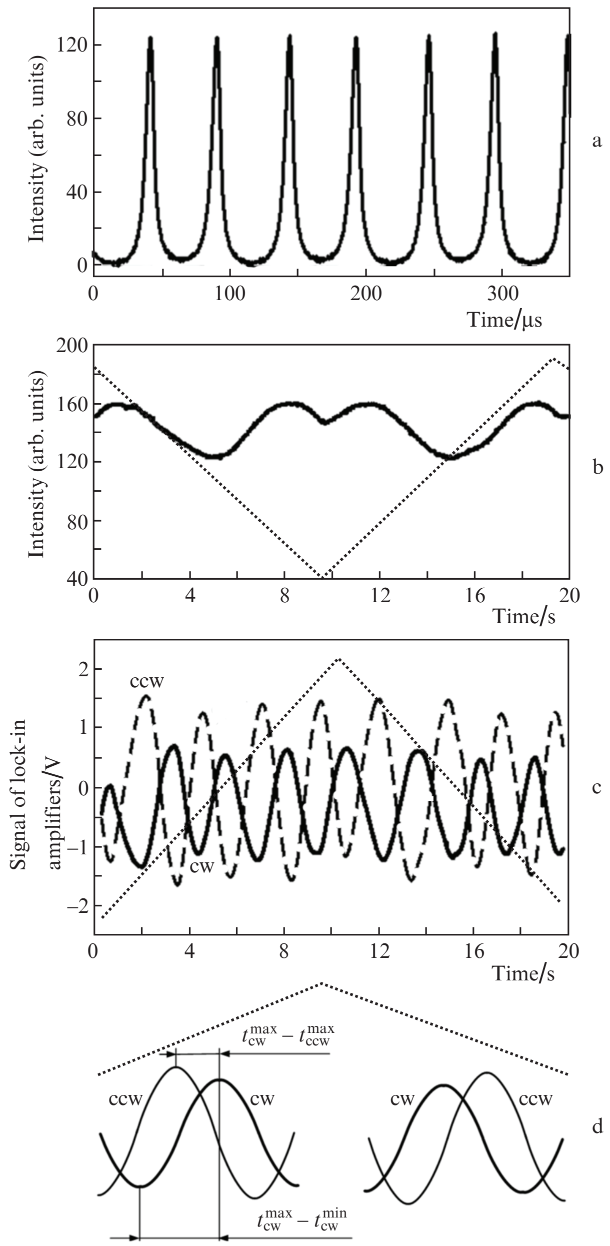

Figure 2. (a) Time dependence of the intensity of radiation emerging from the RR during the operation of the FSU, (b) time dependence of the average intensity of radiation emerging from the RR with the moving RM (solid curve) and the waveform applied to the PZT of the RM (dashed line) (the measured value of the CC modulus is 2.5 ± 0.3 ppm), (c) time dependences of the signals of lock-in amplifiers recording the variable components of the intensities of counterpropagating waves emerging from the RR (curves) and the waveform applied to the PZT of the RM (dashed line) and (d) illustration of the procedure of phase shift measurements (see the text).

waves of approximately π. A triangular signal with a period of 20 s and an amplitude corresponding to a change in the phase shift of counterpropagating waves by more than 2π is fed to the other PZT. In this case, the longitudinal displacement of the distance between the fronts of the counterpropagating waves entering into the RR under study is greater than λ. The time dependences of the variable intensity components are recorded by two lock-in amplifiers whose control inputs are fed with a harmonic modulation signal. A typical time dependence of signals from the outputs of lock-in amplifiers is shown in Fig. 2c. In contrast to the RM scheme, where small (∼10%) changes in the intensity of radiation emerging from the RR are recorded, in the scheme with a mixer the output voltages of the lock-in amplifiers change the sign when the PZT moves. This is due to the fact that the mean intensity value is subtracted from the interference patterns.

The procedure for measuring the phase shift φ is explained in Fig. 2d. The absolute value of the phase shift is found by processing the time dependences of the variable intensity components of counterpropagating waves. In this case, the position of each of the extrema of dependences on the time axis is determined by parabolic approximation of its shape. The calibration value is the time shift between the positions of the minimum and maximum of one of the waves, which is taken as the phase shift equal to π. The shift φ is calculated by the formula

$$\varphi = \pi \frac{t_{\mathrm{cw}}^{\max} - t_{\mathrm{ccw}}^{\max}}{t_{\mathrm{cw}}^{\max} - t_{\mathrm{cw}}^{\min}}. \quad (19)$$

Here, $t_{\mathrm{cw}}^{\max}$ and $t_{\mathrm{cw}}^{\min}$ are the positions of the maximum and minimum of the variable intensity components of the wave directed clockwise; and $t_{\mathrm{ccw}}^{\max}$ is the position of the maximum of the wave directed counterclockwise.

When processing interference patterns of the variable intensity components of counterpropagating waves, it is necessary to answer the question: Is the total phase shift due to backscattering higher or smaller than π? The answer allows one to take into account the fact that the left and right fronts of the slow-scan voltage pulse of the PZT of the optical mixer are distinguished by the sign of the change in the phase difference of counterpropagating waves. On one of the fronts the difference grows, and on the other it decreases. Hence it follows that the measured value of φ could both exceed π, and be less than it.

3. Results of measurements

We measured complex CCs in about thirty RRs. The CC moduli lied in the range of tenths to a few ppm. The relative uncertainty of these measurements did not exceed 10%. In a number of cases, there was a significant (by 2–3 times) difference in the CC moduli of counterpropagating waves.

As for the phase shift φ, its value in the resonators in question varied over a wide range: from 2 to 4 rad. The uncertainty of these measurements can be estimated as ∼0.05 rad.

Of course, the goals of our research were not limited to the development of a method that would allow us to measure complex CCs in the RR. During the experiments and analysis of their results, many questions arise that require clear physical explanations. For example, why can CC moduli in the RR with mirrors that have approximately the same TIS values differ by more than an order of magnitude? Or, why can CC moduli differ by 2–3 times for counterpropagating waves? Even more incomprehensible is the large spread in the magnitude of the total phase shift.

The unusual behaviour of complex CCs in the RR can be attributed to the peculiarities of the backscattering field interference of the entire set of sources located on the surface of RR mirrors. We tried to simulate the interference processes using the antiphase movement of two PZTs installed on the mirrors of the RR under study. Let us briefly describe it.

With the antiphase movement of PZTs, the RR perimeter remains the same and the observed changes in complex CCs are determined by the interference of waves of two backscattering sources (Fig. 3). For simplicity, the complex plane has CCs for one of the counterpropagating waves. Each of the mirrors is characterised by its partial complex CC. The total complex CCs are the vector sums of their partial parts. In the antiphase movement of two PZTs, the total vector of the CCs of mirrors 1 and 2 (denoted by $\boldsymbol{r}_1$) remains fixed in the complex plane, and the total vector of the CCs of mirrors 3 and 4 (denoted by $\boldsymbol{r}_2$) makes a circular motion. It is easy to see that for an antiphase movement of the PZTs, we have periodically alternating maximum and minimum values of the modulus of the total CC:

$$r^{\max} = |\boldsymbol{r}_1| + |\boldsymbol{r}_2|, \quad (20)$$

$$r^{\min} = ||\boldsymbol{r}_1| - |\boldsymbol{r}_2||. \quad (21)$$

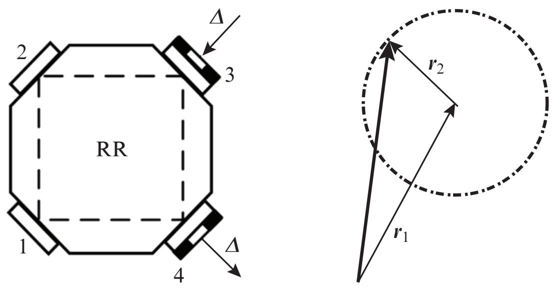

Figure 3. Illustration of the regime of antiphase movement of two PZTs: optical scheme of a RR with two PZTs and the motion of the vectors of complex CCs in the complex plane (see the text) (Δ is the mirror displacement).

From the results of measurements of these values, it is possible to determine the moduli $|\boldsymbol{r}_1|$ and $|\boldsymbol{r}_2|$, equal to the half-sum and half-difference of the maximum and minimum values, respectively. The period of alternation of these extrema is equal to λ (in terms of the change in the RR perimeter when one PZT is displaced). The only difficulty in interpreting the results of such measurements is that it is impossible to determine to which particular pair of mirrors the found values of the moduli belong.

In measuring complex CCs in counterpropagating directions, we obtain three periodic dependences: for the CC moduli of counterpropagating waves and the total phase shift. Figure 4 shows some of these dependences.

It should be noted that each of the studied RRs is characterised by a unique set of these three dependences. This greatly complicates the analysis and generalisation of the results of measurements of complex CCs. Of course, we can limit ourselves to the fact that based on the results of these measurements one can provide the LG developers with the predicted values of the lock-in threshold and the nonlinear scale factor correction. However, in order to understand the physics of the formation of the backscattering fields in the RR, it is necessary to develop a model that adequately describes the results of our experiments. This description is presented in the second part of this work.

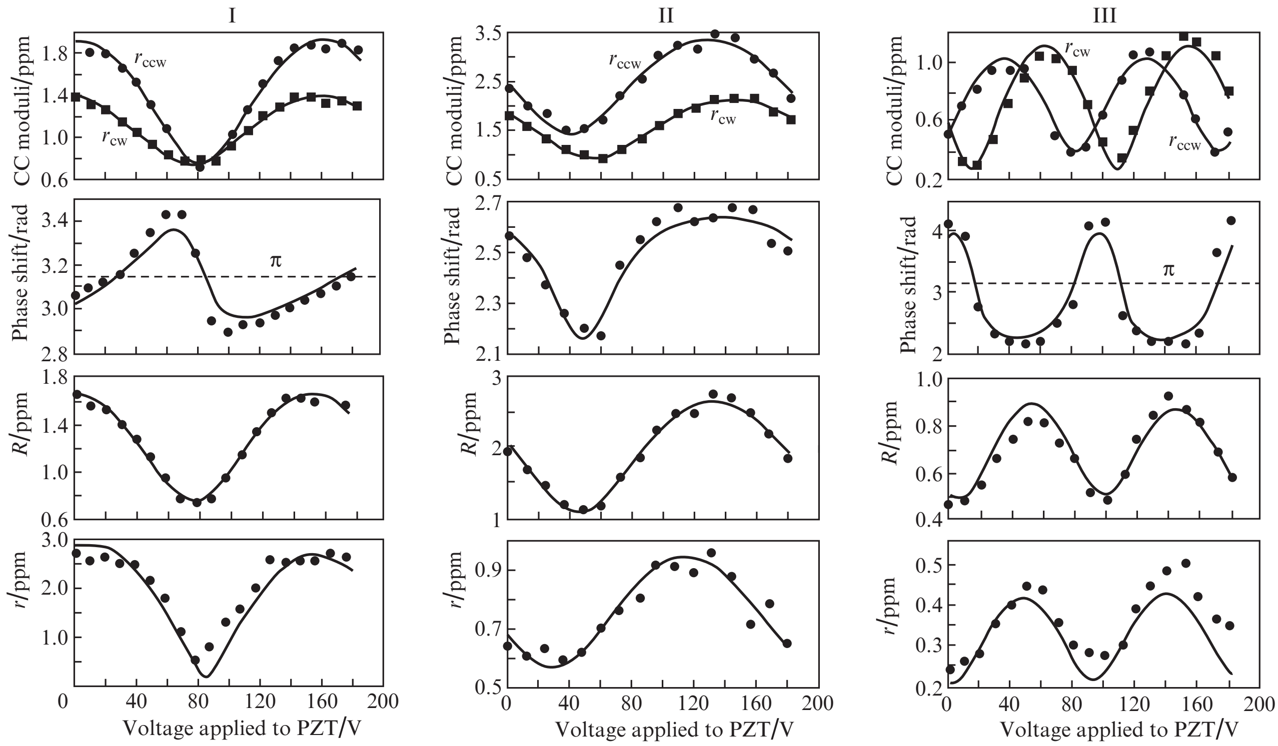

Figure 4. Dependences of the CC moduli of counterpropagating waves, phase shift and conservative and dissipative CC components in the anti-phase movement of two PZTs for three (I–III) resonators on the voltage applied to the PZT. The solid curves are approximations, and the points are the experiment.

4. Model of backscattering field interference for conservative and dissipative sources

Let us start with the simplest case. Suppose that in the RR there are only two point backscattering sources whose size is much smaller than the wavelength λ. Each of these sources is characterised by a partial CC modulus, a phase shift arising due to backscattering and a longitudinal coordinate on the optical axis of the RR. We note that in the case of a point source, the CC moduli of counterpropagating waves are equal. The total complex CCs of this system can be expressed as:

$$\tilde{r}^{\,\mathrm{sum}}_{\mathrm{cw,ccw}} = R\exp(\mathrm{i}\varphi_R) + r\exp(\mathrm{i}\varphi_r \pm 2\mathrm{i}kl). \qquad (22)$$

Here, $k = 2\pi/\lambda$ is the wave number; R and r are the CC moduli for two sources: φ_R and φ_r are the phase shifts arising due to backscattering; and l is the distance between the point sources, measured along the optical axis of the RR. The reference point coincides with the position of the source with the CC modulus equal to R. The '–' sign in the second term refers to the complex CC of a wave scattered in the counterclockwise direction.

The physical meaning of the sign '–' in (22) is that a wave scattered from a point source in the counterclockwise direction falls into the starting point after a complete round trip of the RR perimeter (a multiple of an integer number of wavelengths) minus the distance l. The factor 2 in front of the wave number in the exponent takes into account the fact that in the case of backscattering, the phase shift is formed from two equal-sized terms: the first is due to the arrival of the main wave to the source of the scattering, and the second is due to the return of the scattered field to the initial point.

It is easy to see that if point sources introduce the same phase shifts due to backscattering (Fig. 5a), then we are dealing with symmetric backscattering. When the distance between the sources is changed, the CC moduli remain the same for counterpropagating waves. Depending on the distance l, they vary from a maximum value equal to $R + r$ to a minimum value equal to $R - r$ (we assume that $R > r$). The total phase shift when the sources are displaced does not change and is equal to 2φ ($\varphi = \varphi_R = \varphi_r$).

The situation changes qualitatively when point sources introduce unequal phase shifts. When the longitudinal distance between the sources changes, a difference appears in the CC moduli of counterpropagating waves, described by the relation

$$r^2_{\mathrm{cw}} - r^2_{\mathrm{ccw}} = 4rR\sin(2kl). \qquad (23)$$

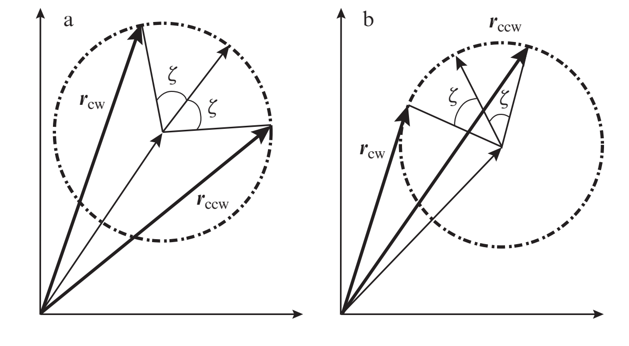

Figure 5. Vector diagrams of addition of two complex CCs of point backscattering sources at (a) $\varphi_R = \varphi_r$ and (b) $\varphi_R \neq \varphi_r$; $\zeta = 2kl$ is the phase shift.

The total phase shift also does not remain constant and, when moving sources, can vary over a very wide range (depending on the ratios between R, r, φ_R and φ_r).

We will also consider that there are two types of backscattering sources – conservative and dissipative. These types of sources are considered in theoretical works [2, 12], where the influence of the backscattering effects on the RL characteristics is analysed. The first of them introduces a phase shift $\varphi_R = \pi/2$, and the second one – $\varphi_r = \pi$. In this case, dissipative backscattering sources determine the parameter S_+ (or the lock-in threshold), while conservative sources determine the parameter S_-.

Despite all the seemingly primitive nature of the model in question, it describes well the main results of our measurements of complex CCs. The model can be complicated by presenting each of the resonator mirrors in the form of a set of dissipative and conservative point backscattering sources randomly 'scattered' over its surface. However, even in this case, complex CCs will be formed in the process of interference of the fields of two effective conservative and dissipative point sources displaced relative to each other by a distance l.

Within the framework of this model, it is possible to reduce the result of any measurement of complex CCs to the determination of three parameters: conservative (R) and dissipative (r) CC components and phase shift $\zeta = 2kl$. These parameters are determined by the relations:

$$R = \sqrt{r_{\rm cw}^2 + r_{\rm ccw}^2 - 2r_{\rm cw}r_{\rm ccw}\cos\varphi}/2, \qquad (24)$$

$$r = \sqrt{r_{\rm cw}^2 + r_{\rm ccw}^2 + 2r_{\rm cw}r_{\rm ccw}\cos\varphi}/2, \qquad (25)$$

$$\zeta = \arcsin\left[\frac{r_{\rm cw}^2 - r_{\rm ccw}^2}{\sqrt{(r_{\rm cw}^2 - r_{\rm ccw}^2) - 4r_{\rm cw}^2 r_{\rm ccw}^2\cos^2\varphi}}\right]. \qquad (26)$$

From the presented relations it is seen that the conservative component of the CC modulus for the mirror is $R = ½S_-$, and the dissipative component is $r = ½S_+$.

Thus, the physical process of backscattering in a RL can most clearly be characterised by three parameters: R, r and ζ. The regime of antiphase movement of two PZTs in the RR is described in the framework of the backscattering field interference model of two pairs of conservative and dissipative sources:

$$\tilde{r}^{\rm \,sum}_{\rm cw,ccw} = R_1\exp({\rm i}\pi/2) + r_1\exp({\rm i}\pi \pm {\rm i}\zeta_1)$$

$$+ [R_2\exp({\rm i}\pi/2) + r_2\exp({\rm i}\pi \pm {\rm i}\zeta_2)]\exp(\pm 2{\rm i}k\Delta l). \qquad (27)$$

Each of the pairs of the mirrors (1–2 and 3–4 in Fig. 3) has its own set of parameters: R_1, r_1, ζ_1 or R_2, r_2, ζ_2. The movement of PZTs is accompanied by a change in Δl of the longitudinal distance between the pairs of the mirrors. As in the case of relation (22), the sign '–' refers to a wave propagating in the counterclockwise direction.

Equation (27) was used to solve the inverse problem associated with the determination of six parameters characterising the antiphase movement of PZTs: R_1, r_1, ζ_1 and R_2, r_2, ζ_2. Their values were obtained by approximating the dependences of complex CCs on the displacement of PZTs. In the framework of this paper, we cannot describe in detail the algorithm for solving the inverse problem. We only note that the initial experimental material including three dependences [$r_{\rm cw,ccw}(U)$ and $\varphi(U)$, where U is the voltage applied to one of the PZTs] is sufficient (mathematically) to determine the six parameters.

Of course, we cannot determine to which specific pair of the mirrors these parameters refer (this has already been mentioned above). However, it is possible to 'inter-relate' the values of the conservative and dissipative components of the CCs for the pairs of the mirrors. This allows one to answer an important question: Is there a correlation between the conservative and dissipative backscattering components? The experiments carried out by us indicate the absence of such a correlation.

The measured dependences of the CC moduli (Fig. 4) were approximated by relation (27). The results of this approximation are shown in Fig. 4 by solid curves. Figure 4 also presents the calculated dependences of the conservative and dissipative CC components. One can see that the model of backscattering field interference of conservative and dissipative sources describes well the results of our experiments. Within the framework of this model, it is possible to explain the noticeable difference in the CC moduli ($r_{\rm cw} \neq r_{\rm ccw}$) observed in a number of experiments, as well as the behaviour of the total phase shift in the antiphase movement of PZTs.

The effect of speckle structures on the formation of the backscattering fields should be specially mentioned. A mirror of a modern LG is a slightly rough surface with a height of irregularities of the order of 1 Å and a correlation length not exceeding a fraction of a micrometer. As a result, the backscattering field of a wave in the RR is formed as a result of the interference of the fields of a very large number of sources with uncorrelated phases. A similar problem was first considered by Lord Rayleigh in 1880 [13] in describing the statistics of the resultant field A of a large number of harmonic vibrators having random phases Ψ_p:

$$A = \alpha\left|\sum_p \exp({\rm i}\Psi_p)\right|. \qquad (28)$$

In this case, A is also a random variable whose probability density is described by the well-known Rayleigh distribution:

$$f(A) = \frac{A}{\sigma^2}\exp\left(-\frac{A^2}{2\sigma^2}\right), \qquad (29)$$

where σ is the scale factor.

With such statistics, the spread in the measured values of the CC moduli can be comparable with their average value. Note that this distribution is also used in describing the statistics of speckle structures of scattered-light fields [14].

The results of our experiments indicate (although indirectly!) the effect of the speckle structure of the backscattering fields on the formation of complex CCs. We investigated twelve RRs assembled from the mirrors having approximately the same quality. The value of the TIS of the mirrors with layers of TiO_2–SiO_2 did not exceed 20 ppm. As a result of the measurements, twenty-four values of the conservative and dissipative CC components were obtained. Their average values (standard deviations are given in parentheses) are as follows: $\langle R\rangle = 1.4$ ppm ($s_R = 0.82$ ppm) for the conservative component and $\langle r\rangle = 0.24$ ppm ($s_r = 0.24$ ppm) for the dissipative component.

We also note that we did not find a correlation between conservative and dissipative components for the pairs of the mirrors. This means that in the case of a 'large' conservative component, a 'large' dissipative component is not always observed.

As for the mean values of $\langle R\rangle$ and $\langle r\rangle$, the measured values are not surprising. In particular, the numerical estimate of $\langle R\rangle$ can be carried out under the assumption that the light is isotropically scattered by the mirror into a solid angle 4π sr. Let us make use of the relations from [10]:

$$\frac{\Phi}{4\pi} = \left(\frac{w_0}{4L}\right)^2, \tag{30}$$

$$R = \sqrt{\frac{S\Phi}{4\pi}}. \tag{31}$$

Here, w_0 is the waist radius of the fundamental RR mode; S is the TIS of the mirror; and Φ is the solid angle of the fundamental mode into which the radiation scattered by the mirror is incident. The estimate of R in our case gives 1.5 ppm, which in order of magnitude coincides with the value of $\langle R\rangle$ measured by us.

Note that the model of an isotropically scattering mirror does not allow us to explain such a large spread in the values of the measured parameters. For the data of this experiment, the ratio of the maximum and minimum values of R was $\sim$15. Taking into account the fact that the intensity of the scattered radiation is proportional to the square of the CC modulus, it will be necessary to assume that the TIS of the mirrors differ by more than 200 times (!). To explain such a significant spread in the values by the presence of individual dust-like particles (not seen at the stage of assembly and alignment!) on mirrors seems to us extremely doubtful. The only reasonable explanation for such a huge spread in the values of the CC moduli may be the presence of a speckle structure in the backscattering fields.

This feature of the behaviour of the measured complex CCs should be taken into account when analysing the results of measurements by averaging over a sufficiently large array of the RRs studied.

5. Conclusions

First of all, we should mention the main result of our work. Complex CCs in the RR in the absence of a working gas mixture have been directly measured for the first time. Thanks to this, we have been able to study more deeply the physical aspects of the backscattering field interference processes in the RR. Based on the results of our experiments, we have proposed a physical model of backscattering field interference of conservative and dissipative sources. This model makes it possible to explain a number of features of the formation of complex CCs. These include a significant difference in the CC moduli of counterpropagating waves and a variation in the total phase shift over a wide range. It is easy to show that the method proposed by us allows (in principle!) the complex CCs of individual RR mirrors to be measured.

Of course, all this is of undoubted practical value. To date, during assembly and alignment of LG RRs, the following parameters are controlled: the polishing quality of mirror substrates, coefficients of transmission, total integral scattering and phase anisotropy of mirrors, as well as losses for the main types of vibrations of the assembled RRs. The backscattering parameter is not controlled. Ring resonators are assembled and aligned 'blindly'. As a result, a noticeable number of produced RRs are rejected because of the entry of large dust-like particles into the working area of the mirrors.

The use of the proposed method for measuring complex CCs in the metrological system for monitoring the RR parameters will make it possible to fill this gap and help increase the percentage of output of high-quality LGs. Using the results of measurements of complex CCs, it is possible to formulate a criterion for rejecting assembled RRs, based, for example, on the predicted magnitude of the LG lock-in threshold [15, 16].

It also seems extremely interesting to use the method for measuring complex CCs when developing an alignment system that allows one to minimise the partial values of the dissipative and conservative CC components for individual mirrors. Attempts to design similar alignment systems were made in the 1980s – 1990s. The main idea of the approach of the authors of patents [8, 9] is based on the fact that when turning a mirror around its axis, it is possible to direct the dark part of the speckle into the counterpropagating mode of the RR being adjusted.

It was assumed *a priori* that there is a direct correlation between the backscattering intensity into the counterpropagating RR mode and the lock-in threshold in the RL. The results of our experiments have shown that such a correlation is absent. However, the main idea of the approach to the problem of minimising the LG lock-in threshold proposed by the authors of the patents mentioned above seems to us promising. It is only necessary to equip such devices with means of measuring complex CCs and displacement of the aligned mirror.

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