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Displacement and localisation of a transparent nanosphere by light-pressure forces in the field of a focused laser beam

A.A. Afanas'ev, D.V. Novitsky

Abstract. **We report the results of a numerical simulation of the Langevin equation describing the motion of a transparent nanosphere under the action of a resulting light-pressure force in the field of a continuous focused Gaussian laser beam. The conditions for the localisation of the nanosphere near the focal waist of the beam focused by the lens are determined. An analytical solution of the approximate (truncated) equation of motion is found, which almost exactly coincides with the results of the numerical simulation of the initial equation.**

Keywords: *nanosphere, laser beam focusing, light-pressure force, Langevin equation, nanosphere localisation.*

1. Introduction

A. Ashkin [1, 2] was the first to demonstrate the optical trapping and levitation of small plastic particles under the action of light-pressure forces using cw lasers. After his pioneering works, at the junction of optics, mechanics and laser physics there emerged and intensively developed a direction associated with the trapping of small particles by laser radiation. The effect of the action of light-pressure forces on small particles, leading to their manipulation, finds interesting practical applications in various technologies, in particular, in medicine and biology. The apparatus for trapping small particles and biological objects created by Ashkin et al. [3, 4] using light-pressure forces (so-called laser tweezers) is used in medicine and biology in the study of viruses and bacteria [5], DNA molecules [6], processes occurring in living cells [7], etc.

In addition to biomedical applications, light-pressure forces can be used to write concentration patterns in a liquid suspension of suspended plastic particles of small size [8]. Such suspensions have large values of the optical Kerr coefficient and can be employed as highly effective nonlinear media for four-wave mixing [9, 10] and stimulated concentration (diffusion) light scattering [11 – 13]. Light-pressure forces are also used to cool and localise atoms, collimate and control atomic beams, which can serve as an additional tool in experimental atomic physics (see, for example, [14]).

In experiments on manipulation of small particles with the help of light-pressure forces, focused laser beams are commonly applied to increase the radiation intensity [1, 2]. Afanas'ev et al. [15] theoretically investigated light-pressure forces acting on a transparent nanosphere located on the axis of a focused Gaussian beam and found an expression for the resultant force, which made it possible to predict the possibility of localising a nanosphere having a certain size and optical properties near the focal region of the lens.

In this paper, based on the Langevin equation, we study the motion of a nanosphere in the field of a continuous focused laser beam and theoretically demonstrate the effect of its localisation near the focal region of the lens.

A.A. Afanas'ev, D.V. Novitsky B.I. Stepanov Institute of Physics, National Academy of Sciences of Belarus, prosp. Nezavisimosti 68, 220072 Minsk, Belarus; e-mail: dvnovitsky@gmail.com

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2. Analysis of the resultant force

In a laser beam, a transparent nanosphere is subjected to the action of two light-pressure force components, i.e. the component $\boldsymbol{F}_{\rm scat}$ arising due to radiation scattering and acting in the beam propagation direction and the gradient component $\boldsymbol{F}_\nabla$ associated with the radiation nonuniformity ($\boldsymbol{F}_{\nabla\perp}$ is the force component acting across the beam and $\boldsymbol{F}_{\nabla z}$ is the force component acting along its z axis). In the case $\bar{m} = n_0/n > 1$ (n_0 and n are the refractive indices of the nanosphere material and the surrounding liquid, respectively), the transverse component of the gradient force $\boldsymbol{F}_{\nabla\perp}$ will contribute to the trapping of the nanosphere on the beam axis z.

Figure 1 shows a scheme for focusing a laser beam with a lens of focal length f.

In the case under study, the resultant force $\boldsymbol{F}_z = \boldsymbol{F}_{\rm scat} + \boldsymbol{F}_{\nabla z}$ acting on the nanosphere located on the z axis of the beam is determined by the expression [15]

$$\boldsymbol{F}_z = \boldsymbol{e}_z F_0 \frac{K[(1 - z/f)^2 + (z/z_{\rm d})^2] + 2(1 - z/z_{\rm w})}{f[(1 - z/f)^2 + (z/z_{\rm d})^2]^2}, \tag{1}$$

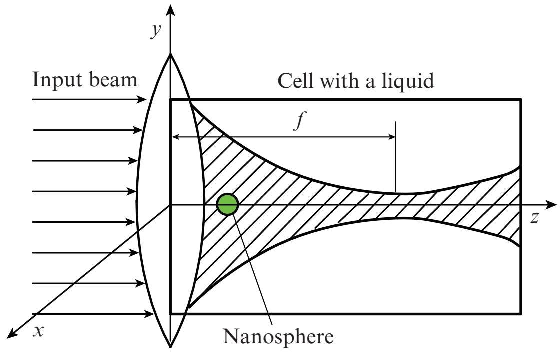

Figure 1. Scheme for focusing a Gaussian beam with a lens of focal length f in a cell with a liquid into which the nanosphere is immersed.

where $F_0 = 2\pi(n/c)\alpha I_0$; $K = 4k^4\alpha f/3$; $z_d = kr_0^2$ is the diffraction length of the beam with an input radius r_0; k is the wave number; I_0 is the intensity of the beam;

$$\alpha = \frac{\bar{m}^2 - 1}{\bar{m}^2 + 2} R^3 \equiv \alpha_0 R^3$$

is the polarisability of a nanosphere of radius R; and

$$z_w = \frac{f}{1 + (f/z_d)^2}$$

is the coordinate of the beam-waist point.

Using the results of [15], we will present some characteristic features of the resultant force $\boldsymbol{F}_z$, which will be required in what follows.

It follows from (1) that at $\alpha_0 > 0$ the longitudinal component of the gradient force $F_{\nabla z} \propto 1 - z/z_w$ in the region $z < z_w$ is co-directed with the scattering force $F_{scat} \propto K$. At the waist point $z = z_w$, it is zero, and in the region $z > z_w$ it reverses the direction. It was shown in [15] that under the condition $\Gamma = Kz_w/z_d \leqslant 1$ in the region

$$z_w + \frac{f}{K}(1 - \sqrt{1 - \Gamma^2}) \leqslant z \leqslant z_w + \frac{f}{K}(1 + \sqrt{1 - \Gamma^2}) \qquad (2)$$

the resultant force is $F_z \leqslant 0$. From the relation $\Gamma = 1$ we find a critical radius of the nanosphere

$$R_f = \sqrt[3]{\frac{3z_d}{4k^4 z_w f \alpha_0}}, \qquad (3)$$

where $F_z = 0$ at point $z = z_w + f/K$. Thus, nanospheres with $R \leqslant R_f$ can be localised in region (2). For example, for a latex nanosphere in water ($n_0 = 1.58$, $n = 1.33$) at $f = 2$ cm, $r_0 = 0.1$ cm and $k = 10^5$ cm^{-1}, we have $R_f \approx 25$ nm. Figure 2 shows the dependence of F_z on the dimensionless coordinate $x = z/f$ for different values of R and the parameters given above.

In the absence of beam focusing (for $f \to \infty$), from (1) follows the expression

$$\boldsymbol{F}_z = \boldsymbol{e}_z \frac{2F_0}{[1 + (z/z_d)^2]^2} \left\{ \frac{2}{3} k^4 \alpha [1 + (z/z_d)^2] - \frac{z}{z_d^2} \right\}, \qquad (4)$$

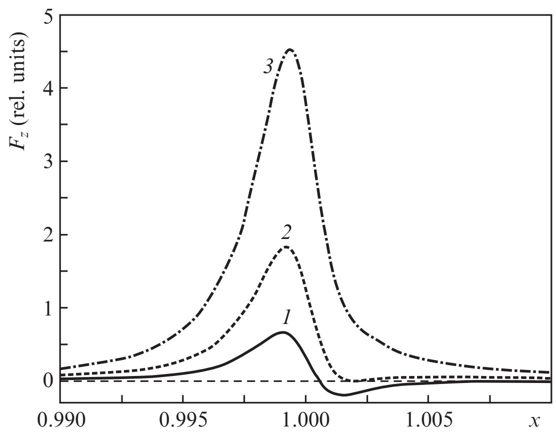

Figure 2. Dependences of the resultant force F_z on the normalised coordinate $x = z/f$ for R = (1) 20, (2) 25 and (3) 30 nm.

which can also be obtained from the corresponding formulas of paper [16]. Obviously, in this case, because of the diffraction spreading of the beam at $\alpha > 0$, the longitudinal component of the gradient force $F_{\nabla z} \propto z/z_d^2$ and the scattering force $F_{scat} \propto \alpha$ are always the opposite. The critical radius of the nanosphere is found from the relation

$$R_\infty = \sqrt[3]{\frac{3}{4k^4 z_d \alpha_0}}. \qquad (5)$$

For the above parameters

$$\frac{R_f}{R_\infty} = \sqrt[3]{1 + (z_d/f)^2} \approx 10^2.$$

Thus, in the absence of focusing, the condition $F_z \leqslant 0$ can be reached for very small nanospheres.

3. Motion of a nanosphere and effects of its localisation

The Langevin equation for a nanosphere under the action of force (1) has the form

$$m\frac{d^2 z}{dt^2} + 6\pi R\eta \frac{dz}{dt} = F_z, \qquad (6)$$

where $m = 4\pi R^3 \rho/3$ is the mass of a nanosphere made of a material with a density ρ; and η is the dynamic coefficient of viscosity of the resting liquid surrounding the nanosphere. For the analysis, taking into account expression (1), it is convenient to write this equation in the form

$$\frac{d^2 x}{dt^2} + \delta \frac{dx}{dt} = G_0 \frac{K[(1-x)^2 + (x/x_d)^2] + 2(1 - x/x_w)}{[(1-x)^2 + (x/x_d)^2]^2}, \qquad (7)$$

where $\delta = 9\eta/(2\rho R^2)$ is the coefficient of friction; $G_0 = n\alpha_0 I_0/(2c\rho f^2)$; $x_d = z_d/f$; and $x_w = (1 + 1/x_d^2)^{-1}$. Below, equation (7) is solved in the general form by numerical methods, and in the case of its simplification, analytically.

3.1. Discussion of the results of the numerical solution

Figure 3 shows the results of a numerical solution of equation (7) with zero initial conditions $x(t = 0) = 0$ and $dx/dt|_{t=0} = 0$. One can see that there occurs a localisation of nanospheres with radii $R \leqslant R_f = 25$ nm [curves (1) and (2)]. In this case, the time needed to reach the localisation point of a nanosphere with $R = 20$ nm is about three times greater than that for a nanosphere with $R = 25$ nm. Although the coordinates of the localisation points depend on R [see (2)], for the given parameters in the scale used in Fig. 3, they almost coincide and are close to the coordinate of the lens focus point $x \approx 1$. In this case, a nanosphere with $R = 30$ nm $> R_f$ [curve (3)] after passing through this point continues to move at a much slower velocity due to a decrease in the intensity of the divergent beam and a change in the direction of the longitudinal component of the gradient force.

It is seen from Fig. 3a that the velocity of motion of nanospheres approaching the localisation points increases. This obviously does not correspond to a 'rapid' decrease in the resultant force in the region $x \approx 1$ (see Fig. 2). This apparent discrepancy is due to the relatively 'rough' scale of Fig. 3a. The velocity of the nanosphere as it approaches the localisa-

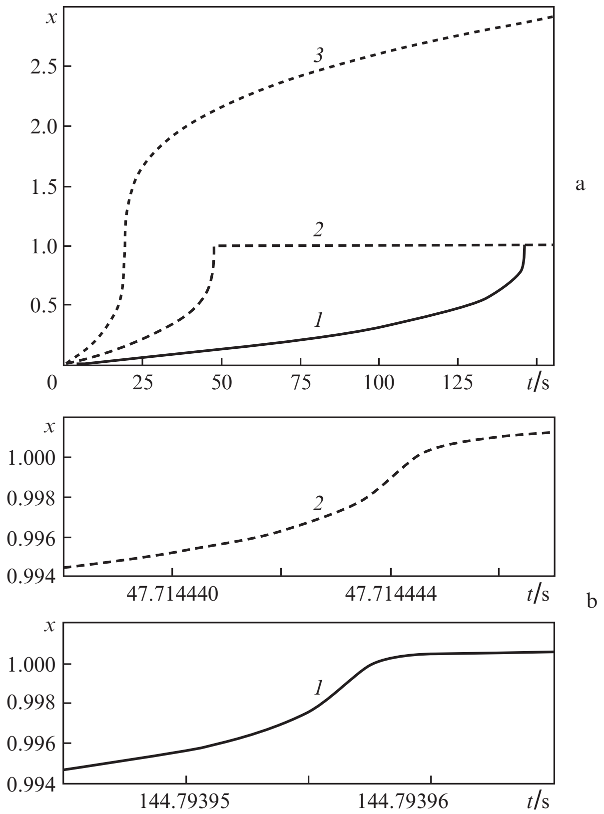

Figure 3. Time dependences of the nanosphere coordinates at $G_0 = 10^5$ for R = (*1*) 20, (*2*) 25 and (*3*) 30 nm.

tion point tends to zero at small distances and small time intervals, which remains practically invisible on the scale of Fig. 3a. The process of a monotonous decrease in the velocity of nanospheres is illustrated in Fig. 3b, on which curves (*1*) and (*2*) are plotted on a scale with a substantially larger spatiotemporal resolution.

A criterion for the stable localisation of a nanosphere near the point $x \approx 1$ is a significant excess of the kinetic energy of its thermal (Brownian) motion by the potential of the gradient force F_∇ [3, 16]. In our case, this criterion is determined by the condition

$$\frac{2\pi n}{c}\alpha_0 R^3 \frac{P}{\pi r^2} \geqslant 10 k_{\rm B} T, \qquad (8)$$

where P is the radiation power; $r = f/(kr_0)$ is the focal spot radius; $k_{\rm B}$ is the Boltzmann constant; and T is the temperature of the surrounding liquid. Using the experimental data of [3], we estimate the minimum value of the nanosphere radius $R_{\rm min}$ at which equality (8) is satisfied. In the experiment, Ashkin et al. [3] used polystyrene nanospheres in water ($\bar{m} = 1.24$ and $\alpha_0^{\rm exp} = 0.15$) and an argon laser ($\lambda = 514.5$ nm) with a beam radius $r^{\rm exp} = 0.29 \times 10^{-4}$ cm. For the conditions of this experiment, the minimum radius $R_{\rm min}^{\rm exp}$ of the nanosphere was estimated by Harada and Asakura [16] and found equal to 9.45 nm. Combining equality (8) with an analogous equality for the experimental parameters [3], we can obtain an expression

$$R_{\rm min} = R_{\rm min}^{\rm exp} \sqrt[3]{\frac{\alpha_0^{\rm exp}}{\alpha_0}\left(\frac{r}{r^{\rm exp}}\right)^2}. \qquad (9)$$

For $k = 2\pi n/\lambda_0 = 1.62 \times 10^5$ cm^{-1}, $r_0 = 0.1$ cm and $f = 1$ cm, we find from (9) that $R_{\rm min} \approx 16.9$ nm and, thus, $R_{\rm f} = 24.5$ nm. The minimum radius $R_{\rm min}$ of the nanosphere obtained by us was 1.8 times larger than $R_{\rm min}^{\rm exp}$ [16], which is due to the difference in the polarisabilities of the nanospheres (α_0 and $\alpha_0^{\rm exp}$) and the radii of the focal spots of the beam (r and $r^{\rm exp}$). Thus, for a given pump power $W^{\rm exp}$ [3], the localisation of nanospheres in our case (in a single-beam trap) is possible if their radii satisfy the condition 16.9 nm $\leqslant R \leqslant$ 24.5 nm.

3.2. Analysis of the analytical solution of the truncated equation

It follows from Fig. 3 that for the given parameters the average velocity $\langle v \rangle$ of motion of nanospheres in the region $x \leqslant 1$ is estimated as 10^{-1}–10^{-2} cm s^{-1}. The process of establishing the velocity of motion of nanospheres occurs in a time $\tau \approx 1/\delta$, which in our case at $\eta = 10^{-2}$ P and $\rho = 1$ g cm^{-3} is very small: $\tau = 10^{-10}$ s. During this time, the nanospheres travel a distance $\Delta z \leqslant 10^{-11}$ cm. At the same time, the motion velocity of the nanosphere 'quasi-statically tracks' the changes in the resultant force, and the second derivative in (7) can be neglected. The resulting so-called truncated equation admits separation of variables, and, therefore, at $x(t = 0) = 0$ its solution can be represented as the integral:

$$\frac{\delta}{G_0}\int_0^x \frac{[(1-x')^2 + (x'/x_{\rm d})^2]^2 {\rm d}x'}{K[(1-x')^2 + (x'/x_{\rm d})^2] + 2(1 - x'/x_{\rm w})} = t. \qquad (10)$$

Calculating integral (10) and performing simple algebraic transformations, we find the expression

$$\frac{\delta}{G_0}[H(x) - H(0)] = t, \qquad (11)$$

where

$$H(x) = A_0 W(x) + B_0 L(x) + U(x);$$

$$A_0 = \frac{4}{K^2}\left[1 - \frac{1}{x_{\rm w}}\left(1 - \frac{2}{K^2 x_{\rm w}}\right)\right]; \quad B_0 = \frac{4}{K^4 x_{\rm w}};$$

$$L(x) = (2 + K) - 2\left(K + \frac{1}{x_{\rm w}}\right)x + \frac{K}{x_{\rm w}}x^2;$$

$$U(x) = \frac{1}{K}\left[\left(1 - \frac{2}{K} + \frac{4}{K^2 x_{\rm w}}\right)x - \left(1 - \frac{1}{K x_{\rm w}}\right)x^2 + \frac{1}{3x_{\rm w}}x^3\right].$$

The function $W(x)$, depending on the parameter Γ (or R), is determined by the relations [17]:

$$W(x) = x_{\rm w} \times \begin{cases} -\dfrac{1}{\sqrt{1-\Gamma^2}}{\rm artanh}\dfrac{K(x - x_{\rm w}) - 1}{\sqrt{1-\Gamma^2}} & \text{at } \Gamma < 1\,(R < R_{\rm f}), \\ -\dfrac{1}{K(x - x_{\rm w}) - 1} & \text{at } \Gamma = 1\,(R = R_{\rm f}), \\ \dfrac{1}{\sqrt{\Gamma^2 - 1}}\arctan\dfrac{K(x - x_{\rm w}) - 1}{\sqrt{\Gamma^2 - 1}} & \text{at } \Gamma > 1\,(R > R_{\rm f}). \end{cases}$$

Figure 4 shows the dependences $x(t)$, calculated by formula (11) for the same parameters as in Fig. 3. The comparison of the curves in Figs 3 and 4 shows that they completely coincide. Thus, the obtained truncated equation admitting an analytic solution is equivalent to the initial equation (7). Consequently, the proposed method of simplifying the initial equation can be used in a wide range of R values to study analytically transport and localisation of transparent nanospheres by light-pressure forces.

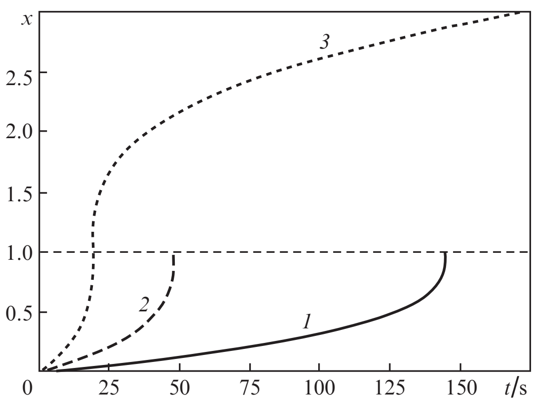

Figure 4. Time dependences of the nanosphere coordinates calculated from the approximate formula (11) with the same parameters as in Fig. 3.

4. Conclusions

Based on the Langevin equation, we have studied displacement and localisation of a transparent nanosphere by the resultant light-pressure force in the field of a focused Gaussian laser beam. Using the numerical solution of this equation, we have obtained the time dependence of the nanosphere coordinate for various values of its radius. It is shown that for the radius of the nanosphere, $R \leqslant R_{\rm f}$, its localisation at point $z \approx f$ occurs due to the compensation of the scattering force of the longitudinal component of the gradient force ($\boldsymbol{F}_{\rm scat} + \boldsymbol{F}_{\nabla z} = 0$). A criterion for the stable localisation of nanospheres in a focused beam of an argon laser is presented and the necessary values of their minimum radii $R_{\rm min}$ are given.

The use of the truncated Langevin equation (for $d^2z/dt^2 = 0$) is justified, the validity of which is due to the 'quasi-static tracking' of the change in the resulting force $F_z(z)$ by the nanosphere velocity as a function of the coordinate. The exact analytical solution of the truncated equation of motion is obtained and it is shown that it completely coincides with the result of the numerical solution of the initial equation.

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