Quantum Electronics **47** (3) 199–205 (2017)

https://doi.org/10.1070/QEL16304

Growth of emittance in laser-plasma electron acceleration in guiding structures

M.E. Veisman, S.V. Kuznetsov, N.E. Andreev

Abstract. **We study angular characteristics, such as emittance and fraction of electrons emitted into a prescribed solid angle, for electron bunches that are accelerated in wake fields generated in various guiding structures – capillary waveguides and plasma channels, in which intense laser pulses propagate. The investigation is performed at various injection energies, which simulate input conditions to different cascades of a multistage electron accelerator. It is shown that, in contrast to the limitations issuing from the requirement of trapping and accelerating a substantial part of bunch electrons, the condition of maintaining the low emittance of the bunch, needed for successful transfer of the accelerated bunch to the entry of the next acceleration cascade or other device, imposes much more strict limitations on the accuracy of focusing of laser radiation to the guiding structure.**

Keywords: *electron acceleration, wake fields, plasma channels, capillary waveguide, emittance.*

1. Introduction

The future of high-energy physics requires the development of accelerators capable of generating electron and positron bunches with an energy of ∼10 TeV [1], which is well above the energy available now in conventional accelerators. Existing microwave accelerators have a maximal rate of the electron energy increase of ∼100 MeV m^{-1}. With such an acceleration rate, the accelerator should have a length of 100 km in order to reach the electron energy of 10 TeV, which can hardly be realised in practice.

In many practical applications, cheaper and more compact accelerators are needed. For example, presently the application of accelerators in medicine for sterilising materials and producing isotopes is intensively expanded. The development of acceleration technique makes it possible to produce intense sources of hard radiation employed in biology, diagnosis of extreme states of matter, investigation and treatment of materials. All these possible applications necessitate the development of accelerators of new type that might be transferred from the category of unique to sufficiently cheap and compact devices, which, however, would be capable of generating electron beams possessing an energy from hundreds MeV to several GeV.

The laser-plasma method of electron acceleration that was first suggested in [2] opens prospects for solving these problems [3]. Laser-plasma accelerators, in which electrons are accelerated in the fields of a wake wave generated in a plasma by an intense laser pulse, have demonstrated the rate of the electron energy increase of above 100 GeV m^{-1} [4]. This gives a chance to obtain bunches of accelerated electrons possessing an energy of ∼1 GeV over an acceleration distance of ∼1 cm [5]. Up to now, the best results have been obtained at the Lawrence Berkeley National Laboratory, where the setup was created [6], on which the laser pulse with a peak energy of 300 TW (duration of 40 fs, wavelength of 0.815 μm) accelerates electrons in a gas-filled capillary to an energy of 4.2 GeV over a distance of 9 cm.

Further increase in a particle energy in laser-plasma accelerators requires an additional analysis of possible acceleration schemes. A single-stage electron acceleration is limited by the complexity of maintaining a stable accelerating field of high intensity in elongated plasma structures and by the acceleration length limitation due to the dephasing effect. For obtaining a higher energy of accelerated particles, one can use the scheme of multistage acceleration, where an electron bunch is multiply accelerated in similar laser-plasma sections. After each acceleration section, the bunch passes to an entry of the next stage for increasing the electron energy [7, 8].

The multistage acceleration scheme is principally capable of attaining the energy of accelerated particles exceeding multiply that obtained in a single section. However, there are the following difficulties.

First, the injection of an electron bunch to each successive acceleration cascade should match in time with the input of the laser radiation that generates the wake field. A sufficiently large increment of the electron bunch energy and the conservation of the number of electrons during the acceleration process require that electrons should be injected to a strictly specified domain of the wake wave phase, which possesses the corresponding focusing and accelerating characteristics. This implies a high synchronism between all the constituents of the multistage acceleration process.

Second, it is necessary to provide sufficiently good characteristics of the electron bunch after each stage, which, as a rule, become worse in the process of electron bunch acceleration in a wake field. Among these characteristics, the most important are the energy spread of the electron bunch, emittance, and bunch length. It is known [9, 10], that the influence of the bunch length on the energy spread is decisive in the acceleration process. In turn, at a relatively low average

M.E. Veisman, S.V. Kuznetsov Joint Institute for High Temperatures, Russian Academy of Sciences, Izhorskaya ul. 13, Bld. 2, 125412 Moscow, Russia; e-mail: bme@ihed.ras.ru;
N.E. Andreev Joint Institute for High Temperatures, Russian Academy of Sciences, Izhorskaya ul. 13, Bld. 2, 125412 Moscow, Russia; Moscow Institute of Physics and Technology (State University), Institutskii per. 9, 141701 Dolgoprudnyi, Moscow region, Russia

Received 6 February 2017
Kvantovaya Elektronika **47** (3) 199–205 (2017)
Translated by N.A. Raspopov

energy of bunch electrons, which is specific for first acceleration stages, the spread of the electron energy at the input to the next acceleration section will make the bunch length longer as well. The bunch characteristics are strongly affected by its charge, initial emittance, and nonlinear dynamics of the laser pulse [11–16]. Hence, in each acceleration section, the conditions of electron bunch injection should satisfy certain requirements, which provide an optimal balance between the rate of the electron energy increase and the maintaining of sufficiently good characteristics of the bunch.

The characteristics of an electron bunch are also strongly affected by a drift gap between the acceleration sections. Characteristics of the electron bunch should not worsen while it passes the drift gap. Note again that the bunch is elongated in the drift gap caused by repulsion of particles due to their charge and energy spread [17]. Also important is the angular divergence of the electron beam, determined by its emittance at the output from an acceleration section.

Finally, the process of multistage acceleration is greatly affected by imperfect focusing of laser radiation into each successive acceleration section [18, 19] and also by mutual misalignment of acceleration sections over their entire set. This problem is being yet out of focus of researchers.

The present work is aimed at studying the angular characteristics (the emittance and angular divergence) of an electron beam accelerated in a wake waves behind the laser pulses, which propagate in various guiding structures – capillary waveguides and plasma channels.

In multistage acceleration, the electron bunch obtained at the output from the preceding acceleration cascade passes to the entry of the next acceleration cascade. The angular characteristics of the bunch depend on those at the output from the preceding cascade and on the dynamics of the electron bunch between the cascades. However, in the first approximation, one may consider acceleration of the electron bunches that have initially zero emittance at the input to each cascade, assuming that in the process of acceleration the electron bunch acquires the emittance far exceeding its initial value. In this approach, the electron bunch with the zero emittance at the input to each cascade plays a role of a probe instrument, which determines how this acceleration cascade with a particular initial injection energy will affect the characteristics of accelerated electron bunches. This approach is used in the present work.

We also estimate the part of electrons that can be transferred from the exit of a preceding cascade to the entry of the next acceleration cascade (at a prescribed input aperture of the latter) with taking no special measures such as plasma lenses [20, 21]. The shorter the drift gap between the cascades and the greater the input aperture of the next cascade, the greater this part of electrons. From the results obtained it follows that the angular characteristics of accelerated bunches critically depend on the accuracy of laser radiation focusing to the guiding structure.

2. Investigation technique

Acceleration of electron bunches in the wake fields, generated by laser pulses in parabolic plasma channels and capillary waveguides was simulated by using the previously developed model [19, 22]. This model utilises obtained analytical expressions for the angular and radial modes of laser radiation in these guiding structures, expressions for the angular harmonics of the wake field, and equations of motion for bunch electrons, which were solved numerically by the Runge–Kutta method. This model allows one to determine the characteristics of electron bunches on large spatial and temporal scales. It also takes into account an arbitrary asymmetry of the laser pulses and asymmetric injection of the latter into the guiding structure under the condition that the inequality $(k_0R)^{-1} \ll 1$ (for waveguides) [23, 24] or $(k_0r_{ch})^{-1} \ll 1$ (for plasma channels) [22] holds. Here, R is the internal radius of the waveguide, r_{ch} is the characteristic radius of the plasma channel [see formula (1) below], $k_0 = \omega_0/c$ is the absolute value of the vacuum wave vector of the laser radiation, ω_0 is the laser radiation frequency, and c is the speed of light.

For revealing the role of the main geometrical factors related to the focusing and propagation of the laser radiation, which generates the accelerating wake fields, we will neglect nonlinear processes in the interaction between plasma and the laser radiation. We also take the maximal power P_0 of the laser radiation much less than the critical power P_{cr} for the relativistic self-focusing: $P_0 \ll P_{cr}$, where $P_{cr} = 0.017\gamma_{ph}^2$ TW; $\gamma_{ph} = \omega_0/\omega_p = \sqrt{n_{cr}/n_{e0}}$ is the relativistic gamma-factor of the plasma wave; $\omega_p = \sqrt{4\pi n_{e0}e^2/m}$ is the plasma frequency; e and m are the electron charge and mass, respectively ($e < 0$); n_{e0} is the concentration of the background electrons in plasma; and $n_{cr} = m\omega_0^2/(4\pi e^2)$ is the critical concentration.

We will assume that the plasma density in capillary waveguides is homogeneous and the dielectric function of the plasma inside the capillaries is constant, $\varepsilon = 1 - n_{e0}/n_{cr} = \text{const}$; in addition, $|1 - \varepsilon| \ll 1$ (see [19] for details).

We assume that the profile of the electron concentration $n(r)$ in plasma channels in the transverse direction is parabolic and does not vary in the longitudinal direction:

$$n(r) = n_{e0}[1 + (r/r_{ch})^2]. \quad (1)$$

For a channel radius matched with the radius of the laser beam possessing a Gaussian transverse profile of the electric field in the linear approximation we have $r_{ch} = R_{fit}$, where

$$R_{fit} = k_p r_0^2/2; \quad (2)$$

$k_p = \omega_p/c$ is the absolute value of the wave vector of the plasma wave; and r_0 is the radius of a laser spot corresponding to the field $E \propto \exp(-r/r_0^2)$. Under the condition $r_{ch} = R_{fit}$, the intensity of the electric field of the laser pulse does not change as it propagates in the channel [22].

The equations of motion for electrons in wake fields and expressions for the wake fields are presented in [19]. It is shown in this work that in the case of asymmetry of any type (it is always the case in real experiments), the wake potential in the plasma contains both zeroth and higher-order angular harmonics (dependent on the azimuth angle φ in the cylindrical system of coordinates as $\exp(\mathrm{i}l\varphi)$ for the l-th harmonics). The source of the l-th harmonic of the wake potential in this case is the lth angular harmonic of the square absolute value of the dimensionless transverse (lying in the plane normal to the propagation axis) component of the electric field $|\boldsymbol{a}_\perp|_l^2$. Here, $\boldsymbol{a}_\perp = e\boldsymbol{E}_\perp/(m\omega_0 c)$, and $\boldsymbol{E}_\perp$ is the transverse component of the electric field of the laser pulse.

From results of works [18, 23–25] and [22] follows that near the axis of a guiding structure, the vector structure of electromagnetic fields of the laser radiation can be approximately described by a single scalar function within the accuracy of first-order terms of the expansion over the small parameter $(k_0R)^{-1} \ll 1$ or $(k_0r_{ch})^{-1} \ll 1$. This function is determined by the second-order wave equation with two boundary conditions. One condition is specified at the input to the guiding structure at $z = 0$ (where z is the axis of the guiding structure). The other condition is specified at $r = R$ (the condition of the continuity of tangential components of the electric and magnetic fields on the inner wall of the capillary waveguide) or at $r \to \infty$ (the condition of vanishing electromagnetic fields at $r \to \infty$) [22].

In this case, the value $|\boldsymbol{a}_\perp|^2_l$ can be expressed as [22],

$$|\boldsymbol{a}_\perp|^2_l = a_0^2 \sum_p (S_{rp} S^*_{rp-l} + S_{\varphi p} S^*_{\varphi p-l}),$$

$$S_{rp} = \sum_{\sigma=\pm1} \sum_n \mathfrak{E}_{p\sigma n}(\xi,\zeta) D_n^{(p)}(r), \tag{3}$$

$$S_{\varphi p} = \sum_{\sigma=\pm1} \sum_n \sigma \mathfrak{E}_{p\sigma n}(\xi,\zeta) D_n^{(p)}(r),$$

where

$$\mathfrak{E}_{l\sigma n}(\xi,\zeta) = C_{l\sigma n} F_\parallel(\xi + \Phi_{l\sigma n}(\zeta)) \exp[-\mathrm{i}\Phi_{l\sigma n}(\zeta)] \tag{4}$$

are the mode coefficients with the constant factors $C_{l\sigma n}$, which depend on the dimensionless coordinates related to the laser pulse $\xi = k_0(z - ct)$ and $\zeta = k_0 z$, which are determined by the boundary conditions at the input to the guiding structure and have a different form for capillary waveguides and for plasma channels; $a_0 = eE_{\max}/(m\omega_0 c)$; $E_{\max}$ is the maximal electric field intensity of the laser pulse prior to entering the guiding structure; $F_\parallel$ is the laser pulse longitudinal profile prior to entering the guiding structure; $\Phi_{l\sigma n}$ are the phases; and $D_n^{(p)}$ are the radial functions, expressions for which are determined by the boundary conditions specified above and are different for capillary waveguides and plasma channels.

For the capillary waveguides filled with the plasma possessing a constant dielectric function $\varepsilon(r < R) = \varepsilon_c$, the following relationships are valid:

$$D_n^{(l)} = J_l(u_{ln} r/R), \tag{5}$$

$$\Phi_{l\sigma n} = (\zeta/2)(k^2_{\perp l\sigma n} + 1 - \varepsilon_c), \tag{6}$$

$$k_{\perp l\sigma n} = \frac{u_{l-\sigma n}}{\mathcal{R}}\Big(1 - \mathrm{i}\frac{\mu_+}{\mathcal{R}}\Big) \text{ for } l \neq 0,$$
$$k_{\perp 0-1n} = \frac{u_{1n}}{\mathcal{R}}\Big(1 - \mathrm{i}\frac{\mu_B}{\mathcal{R}}\Big), \;\; k_{\perp 01n} = \frac{u_{1n}}{\mathcal{R}}\Big(1 - \mathrm{i}\frac{\mu_E}{\mathcal{R}}\Big), \tag{7}$$

where J_l are the lth-order Bessel functions of the first kind; u_{ln} are the corresponding nth roots; $k_{\perp l\sigma n}$ are the transverse wave vectors of the modes of the electromagnetic field; $\mathcal{R} = k_0 R$;

$$\mu_B = \frac{1}{\sqrt{\varepsilon_w - 1}}, \;\; \mu_E = \varepsilon_w \mu_B, \;\; \mu_+ = \mu_B + \mu_E \tag{8}$$

are the factors dependent on the properties of the capillary walls; and ε_w is the permittivity of the capillary wall.

For parabolic plasma channels with the concentration profile (1), the expressions for the radial functions $D_n^{(l)}$ and phase factors $\Phi_{l\sigma n}$ have the form

$$D_n^{(l)} = L_n^{|l|}(\varkappa) \exp(-\varkappa/2) \varkappa^{|l|/2}, \;\; \varkappa \equiv k_p r^2/r_{ch}, \tag{9}$$

where $L_n^{|l|}$ are the generalised Laguerre polynomials; and

$$\Phi_{l\sigma n} = \frac{\zeta}{2}\frac{n_{e0}}{n_{cr}}\Big(1 + \frac{2n + l + 1}{k_p r_{ch}}\Big). \tag{10}$$

From the boundary conditions related to the continuity of electromagnetic fields at the entry to the guiding structure (at $z = 0$) and from expressions (5) and (9) for the radial modes of the electromagnetic field inside the capillary waveguides and plasma channels, respectively, we obtain the expressions for the constant factors $C_{l\sigma n}$ of mode coefficients (4):

$$C_{0\sigma n} = -N_{1n}^{-1} \sum_{k=\pm1} \frac{1 + k\eta}{2\sqrt{1 + \eta^2}}\Big(k\frac{1-\sigma}{2} + \frac{1+\sigma}{2}\Big)\mathcal{Z}_{-kn},$$

$$C_{l\sigma n} = N_{|l|-\sigma n}^{-1} \frac{1 + \sigma\eta}{2\sqrt{1 + \eta^2}} \mathcal{Z}_{|l|-\sigma n} \;\text{ for } l \neq 0,$$

$$N_{kn} = \int_0^1 y J_k^2(u_{kn} y)\,\mathrm{d}y, \tag{11}$$

$$\mathcal{Z}_{kn} = \int_0^1 y F_k(y) J_k(u_{kn} y)\,\mathrm{d}y,$$

$$F_k(y) = \frac{1}{2\pi}\int_0^{2\pi} \exp(-\mathrm{i}k\varphi) F_\perp(yR,\varphi)\,\mathrm{d}\varphi$$

for the capillary waveguides [where $F_\perp(r,\varphi)$ is the transverse profile of the laser field at $z = 0$, and η is the degree of laser pulse polarisation, $\eta = 0$ stands for linear polarisation, and $\eta \neq 0$ stands for elliptical polarisation] and

$$C_{l\sigma n} = N_{|l|-\sigma n}^{-1} \frac{1 + \sigma\eta}{2\sqrt{1 + \eta^2}} \mathcal{Z}_{|l|-\sigma n},$$

$$N_{kn} = \int_0^\infty \exp(-y) y^{|k|} ([L_n^{|k|}(y)]^2\,\mathrm{d}y, \tag{12}$$

$$\mathcal{Z}_{kn} = 2^{|k|/2+1} \int_0^\infty F_k\big(\sqrt{2y/(k_p r_{ch})}\big) \exp(-y) y^{|k|/2} L_n^{|k|}(2y)\,\mathrm{d}y$$

for the plasma channels.

The values $|\boldsymbol{a}_\perp|^2_l$ and the corresponding expressions for wake field harmonics [19] were used for calculating the longitudinal (accelerating) and transverse (focusing or defocusing) forces acting on the accelerated electrons. These forces were employed for solving the equations of motion numerically and determining the electron trajectories for a particular length z of laser pulse propagation in the guiding structure.

Note that formulae (3)–(12) extend the model of electron acceleration in capillary waveguides considered in [19] to the case of plasma channels.

After determining the position of accelerated bunch electrons in the phase space, the normalised transverse emittance ϵ_n was calculated, which is related to the angular emittance ϵ_* by the relationship $\epsilon_n = \gamma_b \beta_b \epsilon_*$ [26]:

$$\epsilon_n = 4\sqrt{\overline{R^2}\,\overline{\alpha^2} - (\overline{R\alpha})^2}\,\gamma_b \beta_b,$$

$$\overline{R^2} = N_b^{-1} \sum_i R_i^2, \;\; \overline{\alpha^2} = N_b^{-1} \sum_i \alpha_i^2,$$
$$\overline{R\alpha} = N_b^{-1} \sum_i R_i \alpha_i, \;\; \alpha_i = \arctan(\tilde{P}_{ri}/P_{zi}), \tag{13}$$
$$\tilde{P}_{ri} = \frac{x_i P_{xi} + y_i P_{yi}}{R_i}, \;\; R_i = \sqrt{x_i^2 + y_i^2},$$

where P_{xi}, P_{yi}, P_{zi} are the transverse and longitudinal momenta of the ith electron in the bunch, expressed in terms of mc; x_i, y_i are the transverse coordinates of the ith electron in the bunch; N_b is the number of electrons in the bunch; γ_b is the bunch gamma-factor, determined as the maximal value of

$$\sqrt{1 + P_{xi}^2 + P_{yi}^2 + P_{zi}^2};$$

and $\beta_b = \sqrt{1 - \gamma_b^{-2}}$ is the bunch beta-function.

In addition to the emittance, in the present work we have calculated the distribution of the electron density over a solid angle $\Omega(\Theta)$ with the apex angle Θ relative to the z axis of the guiding structure, which was normalised to unity:

$$S(\Omega) = N_\Sigma^{-1} \mathrm{d}N(\Omega)/\mathrm{d}\Omega, \;\; \mathrm{d}\Omega = 2\pi \sin\Theta \mathrm{d}\Theta, \tag{14}$$

where $\mathrm{d}N(\Omega) = N(\Omega, \Omega + \mathrm{d}\Omega)$ is the number of bunch electrons within the element of solid angle $\mathrm{d}\Omega$ with the apex angle $\mathrm{d}\Theta$; and N_Σ is the total number of electrons in the bunch. By using the density $S(\Omega)$ (14), we calculate the fraction of all electrons $P(\Theta)$ which moves inside the solid angle $\Omega(\Theta)$ with the apex angle Θ:

$$P(\Theta) = \int_0^{\Omega(\Theta)} S(\Omega')\mathrm{d}\Omega'. \tag{15}$$

Once $P(\Theta)$ at the exit from the accelerating cascade and the input aperture of the next cascade are known, one can determine the fraction of electrons entering the next accelerating cascade for the case when electrons freely propagate between the cascades.

3. Calculation results

In the calculations, the laser pulse had a dimensionless amplitude $a_0 = eE_{max}/(m\omega_0 c) = 0.5$ and the maximal power P_0/P_{cr} = 0.14, the relativistic gamma-factor was $\gamma_{ph} = 80$. The profile of the laser pulse was Gaussian with the radius $r_0 = 50$ μm in the transverse direction and the FWHM duration $\tau_L = 56$ fs, the laser radiation wavelength was $\lambda_0 = 0.8$ μm. The calculations were performed for the two guiding structures: a silicon capillary waveguide with an inner radius $R = 82$ μm and a plasma channel with the electron concentration $n(r)$ (1) and matched radius R_{fit} (2). In the calculations, we took $r_{ch} = R_{fit} = 122$ μm.

Electron bunches were injected along the axis of the guiding structure to the first period of the plasma wave behind the laser pulse. Under the injection phase ξ_{inj} we will imply the distance along the longitudinal coordinate ξ, related to the laser pulse, from the electron bunch centre to the point that corresponds to the maximal longitudinal acceleration force. In order to provide fitting for the maximal part of the most of bunch electrons into the focusing phase of the wake field, the injection phase was chosen in such a way that the dimensionless value $k_p\xi_{inj}/k_0$ be equal to 0.3 (for the capillary waveguides and low injection energies $E_{inj} = 50$ MeV) or 0.1 (in the rest cases), where $k_p = \omega_p/c$.

At input to the capillary waveguide, the laser and, hence, the wake fields are irregular (see, for example, [24, 25]). Thus, in modelling the acceleration of electron bunches in capillary waveguides, electrons were injected not to the capillary face ($z = 0$), but to a distance z_{inj} deep into the capillary (z_{inj} = $800/k_p$). This injection can be technically realised if the first part of the capillary will be empty rather than filled with plasma. An alternative solution of the problem of laser field irregularity at the capillary entry is to employ a specifically profiled (conical) input [25].

The investigations performed showed that the normalised transverse emittance of an accelerated electron bunch in the totally symmetric geometry of the problem (that is, under the exact focusing of the laser radiation into the guiding structure) is mostly affected by the longitudinal and transverse bunch sizes. In all calculations, the electron bunches had Gaussian concentration distributions over the longitudinal (along the propagation z axis) and transverse directions. The initial length $k_p\sigma_z$ and the transverse size $k_p\sigma_r$ of the electron bunch were, respectively, 0.05 and 0.15.

In Fig. 1, one can see the emittance of electron bunches as a function of the length z along the guiding structure, normalised to the dephasing length $L_{ph} = \lambda_0\gamma_{ph}^3 = 41$ cm, at various injection energies E_{inj}. The injection phase in this case is $\xi_{inj} = 0.1k_0/k_p$ for all the curves, except for those pertaining to the capillary waveguides and the injection energy E_{inj} = 50 MeV, for which $\xi_{inj} = 0.3k_0/k_p$. The various cases of focusing laser radiation onto the face of the guiding structure are considered: the exact focusing without deviations of the laser axis from that of the guiding structure (the angle θ between the axes is zero) and the case of inaccurate focusing, where $\theta = 0.06$ mrad for the plasma channels and 0.03 mrad for the capillary waveguides.

From Fig. 1 follows that the angles $\theta = 0.06$ mrad for the plasma channels and $\theta = 0.03$ for the capillary waveguides are the limiting angles, at which the low-emittance (no more than several mm mrad) accelerated electron bunches can be obtained at the acceleration distance $z \approx 0.5L_{ph}$ corresponding to the maximal energy $\Delta E_e = E_e - E_{inj}$ acquired by the electron bunch*. Such a limitation in the angles θ is, at least, by an order of magnitude stronger than the limitation that follows from the consideration of the energy characteristics of the electron bunch [19]. For the first acceleration cascade with the electron injection energy E_{inj} = 50 MeV, at the chosen parameters the electron bunch with low emittance ($\epsilon_n \approx 1$ mm mrad) can only be obtained with plasma channels.

Results of similar calculations are presented in Fig. 2 for one injection energy E_{inj} = 0.5 GeV, various displacements dx of the focusing point of laser radiation relative to the axis of the guiding structure, various degrees of the laser beam asymmetry modelled by a distinct from unity ratio r_y/r_x of the semi-axes of the transverse profile ellipse of the laser pulse (given in the form $E(x,y) = E_0\exp(-x^2/r_x^2 - y^2/r_y^2)$, where $r_x^2 + r_y^2 = 2r_0^2$, at $r_0 = 50$ μm), and for the exact (symmetric) focusing.

From Fig. 2 follows that the requirements for deviation of a laser pulse spot shape from cylindrically symmetric, which issue from the condition of maintaining the low emittance are not very strict (those are similar to the requirements issuing from the condition of maintaining the large acquired energy ΔE_e and large part of trapped and accelerated particles N_{tr} [18, 19]). However, the requirements for the deviation dx of a laser radiation focusing point from the axis of the guiding structure and requirements for the angle θ between the laser beam and guiding structure axis are stronger by approximately an order of magnitude than those issuing from the conditions of maintaining the large values of N_{tr} and ΔE_e.

* In all the calculations performed, $\Delta E_{e\,max} \approx 0.5$ GeV for acceleration in plasma channels and ~0.3 GeV for acceleration in capillary waveguides.

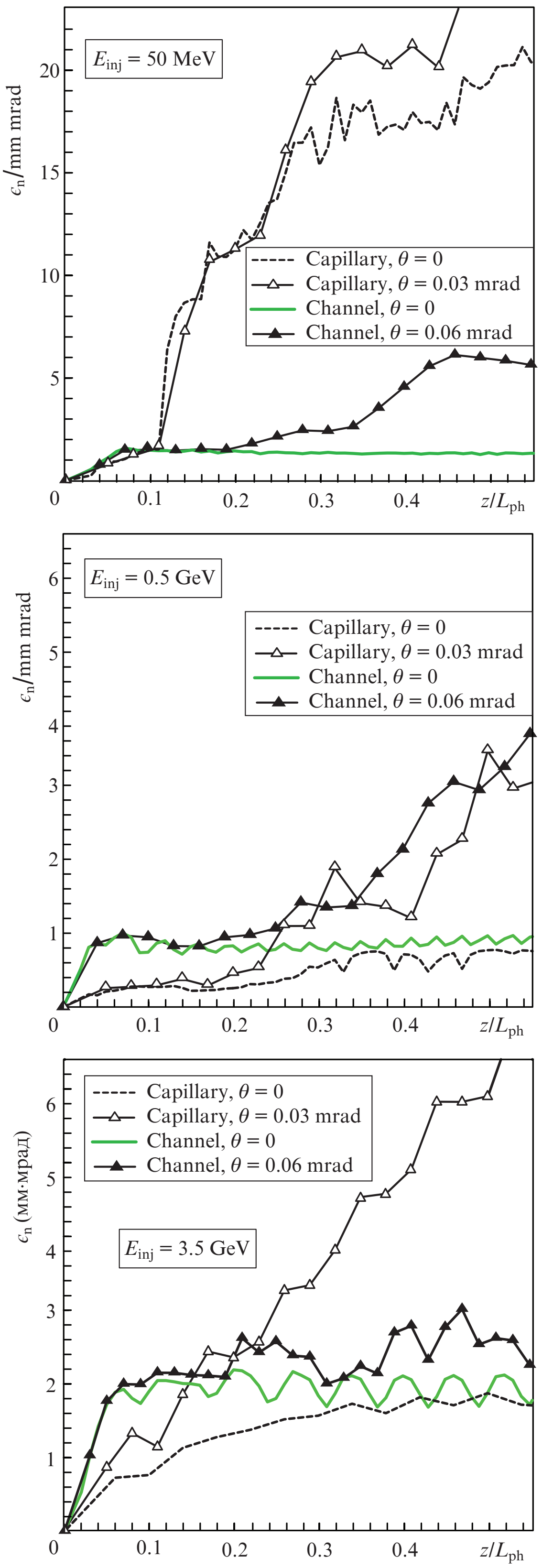

Figure 1. Emittance of electron bunches vs. the dimensionless length of the guiding structure for the parameters specified in the text and figures, at various injection energies E_{inj}.

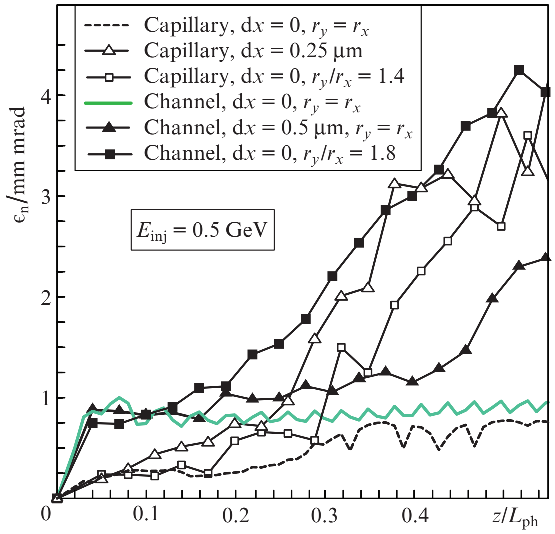

Figure 2. Emittance of electron bunches vs. the dimensionless length of the guiding structure at various displacements dx, various ratios r_y/r_x, and at the exact (symmetric) focusing. The other parameters are the same as in Fig. 1.

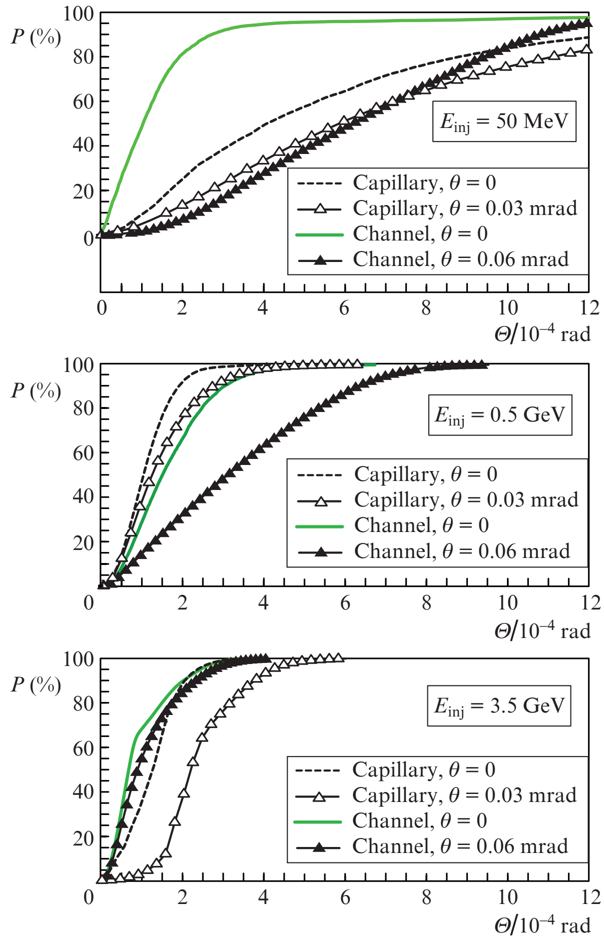

Figure 3. Fraction of electrons $P(\Theta)$, the velocity vectors of which are within a solid angle with the apex angle Θ for the point $z = 0.5L_{ph}$, which corresponds to the maximal energy ΔE_e acquired by bunch electrons. Calculations are performed for various guiding structures, various injection energies E_{inj}, and various focusing conditions, corresponding to different angles θ. The other parameters are similar to those in Fig. 1.

Part of electrons outgoing within the solid angle with the apex angle Θ is shown in Fig. 3 as a function of the angle Θ for the acceleration distance $z = 0.5L_{ph}$ and various injection energies E_{inj} and focusing conditions. One can see that even the small inaccuracy of laser radiation focusing revealed in a small angle θ (0.03 and 0.06 mrad), may markedly widen the angular spectrum and reduce the part of electrons outgoing within a small solid angle near the axis of the guiding structure. This effect is especially pronounced in plasma channels and at small injection energies, which is specific to first acceleration cascades.

In this case, the part of electrons reduces, which, having passed a distance X, may enter the input aperture of radius R of the next acceleration cascade, is reduced: for example, at R = 82 μm and X = 0.4, we obtain $\Theta = 2 \times 10^{-4}$. In this case, according to Fig. 3, if the injection energy of the first acceleration cascade is 50 MeV and the laser radiation is exactly focused ($\theta = 0$), about ~80% of electrons enter the plasma channel of the second acceleration cascade, whereas in the case of inaccurate focusing (θ = 0.06 mrad) only 8% of electrons do. As shown above, at comparatively low injection energies, the emittance of the electron bunch accelerated in the capillary waveguides is much greater than that in the case of acceleration in plasma channels even under the exact focusing. This is revealed in that (Fig. 3), regardless of the focusing conditions, part of electrons capable of fitting the entry of the second acceleration cascade does not exceed 20% at the parameters R = 82 μm and X = 0.4 m in the case of the capillary waveguide acceleration.

Calculations for high injection energies, which give an estimate for subsequent acceleration cascades, show that in the considered example (R = 82 μm, X = 0.4 m), about 80%–90% of electrons get into the entry of the next acceleration cascade (at Θ = 0.2 mrad) in the case of exact focusing and ~30% of electrons in the case of inaccurate focusing at θ = 0.06 mrad for the plasma channel and injection energy E_{inj} = 0.5 GeV or at θ = 0.03 mrad for the capillary waveguide and E_{inj} = 3.5 GeV.

Stronger than for plasma channel requirements to the accuracy of laser radiation focusing in the case of the capillary waveguide are explained by that the laser fields in a plasma channel (and, hence, the generated wake fields) are more regular. This is related to softer conditions of laser radiation reflection from walls of the plasma channel possessing a smoothly varying profile of electron concentration.

The emittance was also calculated for wider laser beams at r_0 = 100 μm. In this case, the matched internal radius of the capillary waveguide was $R = 1.64 r_0$ = 164 μm [23, 24], and the matched radius of the plasma channel according to (2) was R_{fit} = 490.9 μm. The power of laser radiation in this case is four times greater as compared to the initial calculations; however, it is still less than the critical power, $P_0/P_{cr} = 0.56$. According to [18], the limiting admissible angle θ_{max} between the propagation direction of the laser beam and the axis of the guiding structure, estimated from the energy characteristics and from the number of trapped particles, can be determined as $\theta_{max} \simeq 2R/L_{ph} = 2R/(\lambda_0 \gamma_{ph}^3)$. If the radius R increases twice, then this angle becomes also twice greater. However, according to calculations, the angle determined from the condition of maintaining the low emittance does not increase in this case.

4. Conclusions

For practical applications and for possible multistage electron acceleration to high energies, it is important to provide the minimal angular divergence of the accelerated electron bunch, that is, the minimal emittance and the maximal part of electrons outgoing within a small solid angle near the axis of the guiding structure of the accelerator.

For revealing the factors affecting the angular divergence of the accelerated electron bunch we have investigated the emittance and the part of particles outgoing within a prescribed solid angle for the electron bunches accelerated in the wake fields generated in various guiding structures, capillary waveguides and plasma channels, in which the intense laser pulses propagate.

It was shown that the angular divergence of the electron bunch is determined by the initial energy of electron beam injection into the accelerating cascade and, to the highest degree, by the geometry of accelerating wake fields, which, in turn, is determined by the geometry of the laser electromagnetic field that generates the wake fields. The latter geometry is related to the accuracy of laser radiation focusing to the guiding structure.

It was established that, in contrast to the limitations related to the requirement of trapping and accelerating a substantial part of bunch electrons, the condition of maintaining the low bunch emittance, needed for the efficient transfer of the accelerated bunch to the input of the next acceleration cascade or other device, imposes the constraints on the accuracy of laser radiation focusing into the guiding structure, which are stricter by an order of magnitude. In particular, the characteristic tolerance for the deviation angle of the laser radiation propagation direction from the axis of the guiding structure is at most 0.1 mrad for the parameters considered (see Fig. 3). The characteristic tolerance for the relative shift of the laser radiation focusing point, calculated as the ratio of the shift of the focusing point from the axis of the guiding system to the characteristic transversal dimension of the guiding structure does not exceed 0.5%.

Acknowledgements. The work was partially supported by the Presidium of the Russian Academy of Sciences (Programme ‘Extreme laser radiation: physics and fundamental applications’).

References

1. Zisman M.S. https://accelconf.web.cern.ch/accelconf/BIW2008/papers/moiotio01.pdf.
2. Tajima T., Dawson J.M. *Phys. Rev. Lett.*, **43**, 267 (1979).
3. Katsouleas T. *Plasma Phys. Controlled Fusion*, **46**, B575 (2004).
4. Malka V., Fritzler S., Lefebvre E., et al. *Science*, **298** (5598), 1596 (2002).
5. Kostykov I.Yu., Pukhov A.M. *Usp. Fiz. Nauk*, **185**, 89 (2015) [*Phys. Usp*., **58**, 81 (2015)].
6. Leemans W.P., Gonsalves A.J., Mao H.-S., et al. *Phys. Rev. Lett.*, **113**, 245002 (2014).
7. Cheshkov S., Tajima T., Horton W., Yokoya K. *Phys. Rev. ST Accel. Beams*, **3**, 071301 (2000).
8. Kimura W.D., van Steenbergen A., Babzien M., et al. *Phys. Rev. Lett.*, **86**, 4041 (2001).
9. Andreev N.E., Kuznetsov S.V., Pogorelsky I.V. *Phys. Rev. ST Accel. Beams*, **3**, 021301 (2000); https://doi.org/10.1103/PhysRevSTAB.3.021301.
10. Andreev N.E., Kuznetsov S.V. *Plasma Phys. Controlled Fusion*, **45** (12A), A39 (2003).
11. Andreev N.E., Kuznetsov S.V. *IEEE Trans. Plasma Sci.*, **36** (4), 1765 (2008).
12. Andreev N.E., Baranov V.E., Cros B., et al. *Nucl. Instrum. Methods Phys. Res., Sect. A*, **653** (1), 66 (2011); www.sciencedirect.com/science/article/pii/S0168900211001975.

13. Andreev N.E., Kuznetsov S.V., Cros B., et al. *Plasma Phys. Controlled Fusion*, **53** (1), 014001 (2011); https://doi.org/10.1088/0741-3335/53/1/014001.
14. Kuznetsov S.V. *Zh. Eksp. Tekhn. Fiz*., **142**, 190 (2012).
15. Andreev N.E., Baranov V.E., Cros B., et al. *J. Plasma Phys.*, **79**, 143 (2013).
16. Kuznetsov S.V. *Zh. Eksp. Tekhn. Fiz*., **150**, 195 (2016).
17. Steinhauer L.C., Kimura W.D. *Phys. Rev. ST Accel. Beams*, **2**, 081301 (1999).
18. Andreev N.E., Kuznetsov S.V., Veysman M.E. *Nucl. Instrum. Methods Phys. Res., Sect. A*, **740**, 273 (2014).
19. Veisman M.E., Kuznetsov S.V., Andreev N.E. *Quantum Electron*., **46**, 288 (2016) [*Kvantovaya Elektron.*, **46**, 288 (2016)].
20. Su J.J., Katsouleas T., Dawson J.M., Fedele R. *Phys. Rev. A*, **41**, 3321 (1990).
21. Steinke S., van Tilborg J., Benedetti C., et al. *Phys. Plasmas*, **23** (5), 056705 (2016).
22. Veysman M.E., Andreev N.E. *J. Phys. Conf. Ser*., **744**, 012109 (2016).
23. Veysman M.E., Cros B., Andreev N.E., Maynard G. *Phys. Plasmas*, **13** (5), 053114 (2006).
24. Veysman M.E., Andreev N.E., Cassou K., et al. *J. Opt. Soc. Am. B*, **27** (7), 1400 (2010).
25. Veysman M.E., Andreev N.E., Maynard G., Cros B. *Phys. Rev. E*, **86**, 066411 (2012).
26. Edwards D.A., Syphers M.J. *An Introduction to the Physics of High Energy Accelerators* (New York: Wiley, 1993).