Quantum Electronics **47** (3) 188–193 (2017)

https://doi.org/10.1070/QEL16305

Specific features of betatron oscillations and betatron emission in a hollow-channel plasma

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Abstract. **Transverse betatron oscillations and the corresponding betatron emission are investigated for electrons in a strongly nonlinear wake wave excited by a high-power laser pulse in a hollow-channel plasma. The expressions for the oscillation period and critical frequency in the synchrotron regime of betatron emission are derived. A two-stage scheme is considered, in which an electron bunch formed in a plasma possessing a certain set of parameters generates betatron emission in a plasma possessing another set of parameters. The emission spectrum at the second stage is calculated in the absence of bunch acceleration.**

Keywords: *laser-plasma acceleration, betatron oscillations, betatron emission.*

1. Introduction

Interaction of laser radiation with gas targets is of interest in problems of electron acceleration [1, 2] and generation of emission in difficult-to-access ranges, in particular, in the X-ray range [3–5]. In this interaction, the leading edge of a high-power laser pulse ionises a gas and the main part of the pulse excites a wake wave in the gas. The wave propagates in a plasma at a velocity close to the group velocity of the laser pulse and possesses a strong longitudinal electric field, which is used for particle acceleration [6]. In addition, the transverse fields of the wake wave result in oscillations of accelerated electrons. Such oscillations are called betatron oscillations and provide generation of betatron emission at a frequency, which for strongly relativistic electrons may be in the X-ray range. The betatron emission of laser-plasma sources was repeatedly observed experimentally [7–10], its characteristics (collimation, short pulse duration, good spatial coherence) proved to be suitable for studying object structures [11, 12].

At sufficiently high laser field intensities [$a_0 = eE_\mathrm{L}\times(mc\omega_\mathrm{L})^{-1} > 1$, where $e > 0$ and m are the absolute values of the electron charge and mass, respectively; c is the speed of light in vacuum; E_L and ω_L are the peak strength and frequency of the laser field] in a low-density plasma [with the concentration $n \ll n_\mathrm{cr} = m\omega_\mathrm{L}^2/(4\pi e^2)$], the wake wave is excited in the so-called strongly nonlinear regime (or regime of a plasma cavity) [13]. In this regime, the laser pulse completely ejects plasma electrons, which leads to the formation of an electron-free region behind the pulse. The plasma cavity has a shape close to spherical; in the literature, it is called a 'bubble'. The feature of the strongly nonlinear regime is the electron self-injection: plasma electrons are trapped in a plasma bubble and accelerated in it [14]. This substantially simplifies experiments because obtaining accelerated bunches and betatron emission does not need external electron bunches. A drawback of plasma acceleration methods is insufficient quality of accelerated bunches. A promising approach to solving the problem is the employment of a plasma with a deep channel produced in it, in which the plasma concentration along the axis of laser pulse propagation is almost zero [15]. The channel substantially affects the transverse dynamics of electrons and, hence, the characteristics of betatron emission, which attracts interest to studying these effects.

A phenomenological model of a bubble in the plasma allows one to describe its shape and the field distribution both for a homogeneous plasma [16] and for a plasma with a channel [17, 18], and to take into account the influence of electron bunches on the fields in the bubble [19, 20]. These results give a chance to study the dynamics of electrons in the fields of a plasma bubble analytically. In the present work we analyse betatron oscillations of electrons in a plasma with a channel (Section 2) and basing on this analysis, estimate the spectral characteristics of betatron emission (Section 3). Using the results obtained, in Section 4 we consider a two-stage scheme of betatron emission, in which an electron bunch formed at the first stage in the homogeneous plasma possessing a certain concentration is used for generating betatron radiation at the second stage in the homogeneous plasma possessing another concentration or in a plasma with a channel.

We use dimensionless variables, normalising charges to e, velocities V to c, momenta p to mc, energies to mc^2, time to ω_p^{-1}, coordinates to c/ω_p, concentrations n to n_0, electric (E) and magnetic (B) fields to $mc\omega_\mathrm{p}/e$. Here, $\omega_\mathrm{p} = (4\pi e^2 n_0/m)^{1/2}$ is the characteristic electron plasma frequency, and n_0 is the characteristic electron concentration in plasma (for example, outside the channel).

2. Betatron oscillations

The motion of electrons in the fields of a wake wave is determined by a Lorentz force F. Of most interest is to consider electrons trapped by the wave and accelerated or decelerated in it. Such electrons are relativistic and mainly move along the z axis, i.e., their parameter is $\beta_z = V_z/c \approx 1$. In this case, the expression for the Lorentz force can be written in the simplified form:

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Received 6 February 2017
Kvantovaya Elektronika **47** (3) 188–193 (2017)
Translated by N.A. Raspopov

$$F_z \approx -E_z, \quad F_r \approx -E_r + B_\varphi, \tag{1}$$

where z is the longitudinal coordinate (along which the laser pulse propagates); and r and φ are the radial and angular polar coordinates. A specific feature of a plasma bubble in a plasma possessing an arbitrary transverse concentration profile is that the longitudinal force acting on accelerated electrons is independent of the transverse coordinate, and the transverse force is independent of the longitudinal coordinate. In a homogeneous plasma these forces are linear [21]:

$$F_z \approx -\frac{\xi}{2}, \quad F_r \approx -\frac{r}{2}, \tag{2}$$

where $\xi = z - \beta_{\rm ph} t$ ($\xi = 0$ corresponds to the centre of the plasma bubble); $\beta_{\rm ph}$ is the phase velocity of the wake wave. Note that a structure of the wave is conserved as the latter propagates in the plasma; hence, one may approximately assume that the dependence of the wake wave on time and coordinate is only determined by the value of ξ (so called quasi-stationary approximation). From experimental data follows [22] that in the case of a linearly polarised laser pulse, the electron betatron oscillations have a selected direction in the plane of the laser pulse polarisation. For simplicity, we will consider the electron trajectories that only lie in the plane; for definiteness, let it be the yz plane.

First, consider the motion of electrons without a longitudinal force. In the case when particles are relativistic, with a small transverse momentum as compared to the longitudinal one (i.e., $|p_r| \ll p_z$), the Lorentz factor for the particle, $\gamma \approx p_z$, can be assumed independent of the transverse momentum, which allows one to describe a transverse motion by nonrelativistic Newton equations for an electron of mass γ. In this case, the focusing force (2) causes the transverse betatron oscillations of electrons at the betatron frequency $\omega_{\rm b0} = 1/\sqrt{2\gamma}$ (Fig. 1a).

If the electron undergoing betatron oscillations is accelerated or decelerated, this results in adiabatic variation of the amplitude of betatron oscillations by the law $r_{\max} \propto \gamma^{-1/4}$ ($p_{r\max} \propto \gamma^{1/4}$). Thus, the amplitude for the transverse momentum oscillations increases in the case of acceleration; however, the ratio $p_{r\max}/p_z \approx p_{r\max}/\gamma \propto \gamma^{-3/4}$ reduces.

In the case of a plasma with a vacuum channel and sharp walls, for which the concentration is $n(r) = \Theta(r - r_{\rm c})$, where $\Theta(X)$ is the Heaviside function, the transverse force can be found as a derivative of the wake potential Ψ: $F_r \approx \partial\Psi/\partial r$. In view of the general expression for the wake potential [17] this force can be expressed in the form:

$$F_r = -\Theta(r - r_{\rm c})\frac{r^2 - r_{\rm c}^2}{2r}. \tag{3}$$

Inside the channel plasma ions are absent and the transverse force does not affect electrons; hence, the particle moves along a straight line. At channel walls, the electron turns around (Fig. 1b). Consider the influence of the channel on the period of betatron oscillations $T_{\rm b}$. For an electron possessing the transverse momentum $p_{r\max}$ at the axis $r = 0$, the period can be written as

$$T_{\rm b} = 2\sqrt{2\gamma}\int_0^{r_{\max}} \frac{{\rm d}r'}{\sqrt{p_{r\max}^2/(2\gamma) - U(r')}}, \tag{4}$$

where the potential is

$$U(r) = \Theta(r - r_{\rm c})\left[\frac{r^2 - r_{\rm c}^2}{4} - \frac{r_{\rm c}^2}{2}\ln\left(\frac{r}{r_{\rm c}}\right)\right]; \tag{5}$$

$r_{\max}$ is determined from the condition of turning denominator (4) to zero. The oscillation period can be presented as a sum of two times: $T_{\rm b} = T_{\rm b}^{\rm c} + T_{\rm b}^{\rm w}$, where $T_{\rm b}^{\rm c} = 4r_{\rm c}\gamma/p_{r\max}$ is the time of electron motion in the channel (at $r < r_{\rm c}$), and $T_{\rm b}^{\rm w}$ is the time of electron motion inside the walls of the channel (at $r > r_{\rm c}$). The curves determined by expression (4) are shown in Fig. 2.

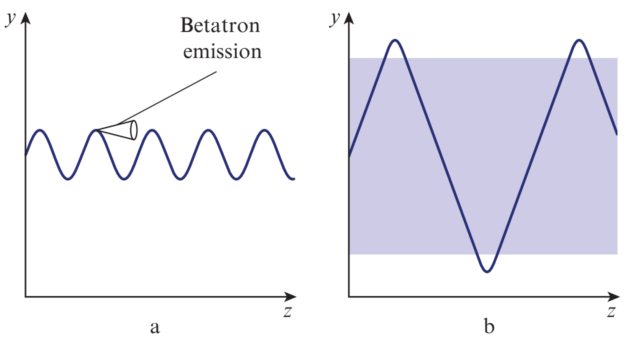

Figure 1. Schematic of the trajectory of the betatron oscillation of a particle (a) in the homogeneous plasma and (b) in the plasma with a channel (the channel is shown by grey colour).

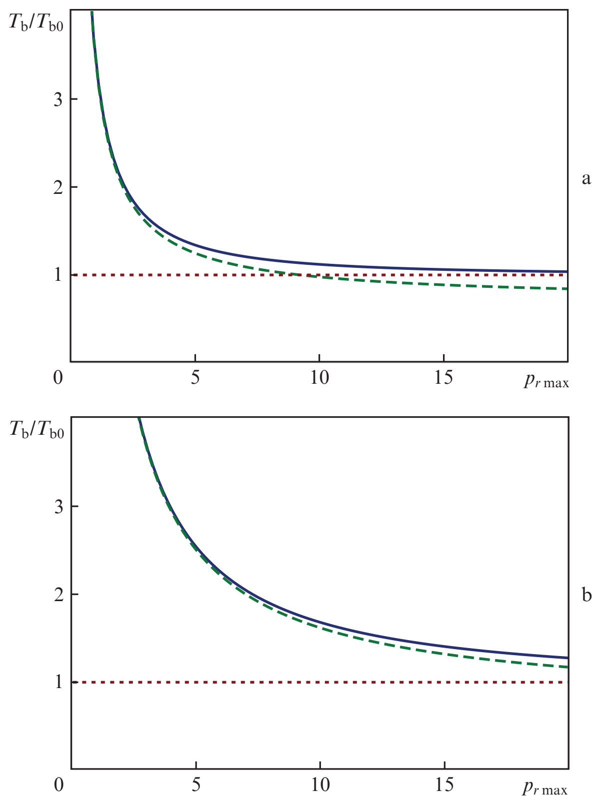

Figure 2. Dependences of the ratio of the period of betatron oscillations in the plasma with a channel, $T_{\rm b}$, to the period of oscillations in the homogeneous plasma, $T_{\rm b0}$, on the transverse momentum of an electron for the channel radius $r_{\rm c}$ = (a) 0.3 and (b) 1. Dashed curves refer to the calculation by formula (7) in the approximation of a small momentum; dotted lines refer to the solution for the homogeneous plasma. The Lorentz factor of the electron is $\gamma = 400$.

There are two limiting cases, in which period (4) can be calculated approximately. In the case when an electron deeply penetrates the channel walls (i.e., $r_{\max} \gg r_c$), the oscillation period T_b will be close to that of betatron oscillations in a homogeneous plasma: $T_b \approx T_{b0} = 2\pi\sqrt{2\gamma}$. This limiting case is realised if the condition $p_{r\max} \gg r_c\sqrt{\gamma}$ holds (dotted lines in Fig. 2). Since in our consideration the transverse momentum is limited by the condition $p_{r\max} \ll \gamma$, this approximation can only be valid for sufficiently narrow channels.

One more limiting case corresponds to shallow penetration of an electron into the channel walls, i.e., $|r_{\max} - r_c| \ll r_c$. Then, the expression for potential (5) can be written in the approximate form

$$U(r) \approx \Theta(r - r_c)\frac{(r - r_c)^2}{2}, \qquad (6)$$

and the corresponding force proves to be linear: $F_r(r) \approx -(r - r_c)$ at $r > r_c$. The part of the electron trajectory in the channel walls is described by the harmonic oscillator equation with the focusing force twice that in a homogeneous plasma, which, respectively, results in the time of electron turn in the channel wall to be by a factor of $\sqrt{2}$ smaller than in the case of the homogeneous plasma: $T_b^w = 2\pi\sqrt{\gamma}$. Thus, in this case, the total oscillation period is

$$T_b = 2\pi\sqrt{\gamma} + \frac{4r_c\gamma}{p_{r\max}}. \qquad (7)$$

This approximation is valid at $p_{r\max} \ll \sqrt{\gamma r_c}$ (dashed curves in Fig. 2).

In adiabatic variation of the Lorentz factor of the particle, its transverse momentum cannot vary faster than $\gamma^{1/4}$ (in the case of the homogeneous plasma). Hence, in accelerating particles in a plasma channel the parameter $p_{r\max}/\sqrt{\gamma}$ at a sufficient channel width should become far less than r_c, so that the second approximation will be valid.

3. Betatron emission

In Section 2, we have considered the betatron oscillations both in a homogeneous plasma and in a plasma with a channel. Here we study the emission of particles resulted from these oscillations, which is also called 'betatron' emission. An instantaneous emission of a relativistic particle is concentrated within a very narrow angle ($1/\gamma$) in the direction of particle motion (Fig. 1a). In the case when a relativistic particle undergoes betatron oscillations in the plasma, the angle of trajectory inclination varies within the limits from 0 to $p_{r\max}/p_z$. If $1 \ll p_{r\max} \ll p_z$, then $p_{r\max}/p_z \gg 1/\gamma$, i.e., this angle is much greater than the angle of the directional characteristic of the particle instantaneous emission. This means that the betatron emission will be observed mainly in the z-axis direction at angles less than $p_{r\max}/\gamma$, and for a prescribed direction the emission will come from a very small part of the particle trajectory being presented in the form of short pulses (by two pulses per oscillation period). In this case, the part of the trajectory from which the emission passes in the discussed direction can be approximated by a circular arc. Hence, the spectrum of the betatron emission of a particle in this direction will be determined only by the particle energy and the local curvature of the corresponding trajectory arc [23].

The power of spontaneous emission is proportional to the trajectory curvature squared; hence, the particle emission is most intensive from the trajectory point that is mostly distant from the z axis. Emission at this point is directed along the z axis and its total spectrum is given by the function S [3, 4]:

$$\frac{dW}{d\omega} \propto S\left(\frac{\omega}{\omega_c}\right), \quad S(X) = X\int_X^{\infty} K_{5/3}(Y)\,dY, \qquad (8)$$

where W is the energy emitted; $K_{5/3}$ is the 5/3-order Macdonald function; $\omega_c = 3\gamma^3/(2\rho)$ is the critical frequency; and ρ is the radius of curvature for the electron trajectory at the point $r = r_{\max}$.

Graphics of the function $S(X)$ is shown in Fig. 3. The spectrum width is determined by the critical frequency ω_c, which depends on the radius of the trajectory curvature at the point most distant from the axis. The curvature, in turn, is related to the transverse force acting on the electron $\rho = \gamma/F_r(r_{\max})$. The critical frequency in this case is

$$\omega_c = \frac{3}{2}\gamma^2 F_r(r_{\max}). \qquad (9)$$

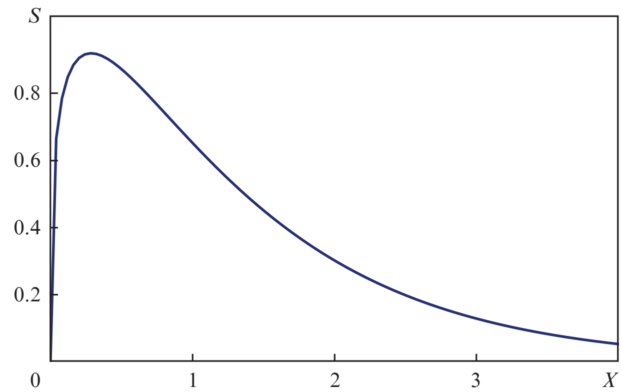

Figure 3. Graphic of function $S(X)$.

Consider the ratio between the critical frequencies in the homogeneous plasma and in the plasma with a channel. In view of (2) the critical frequency for the homogeneous plasma is written as

$$\omega_{c0} = \frac{3}{4}\gamma^2 r_{\max} = 3\left(\frac{\gamma}{2}\right)^{3/2} p_{r\max}. \qquad (10)$$

The frequency linearly increases with increasing electron transverse momentum.

For a plasma with a channel, the critical frequency can be found from expressions (3) and (5). In the limiting cases, analytical expressions can be obtained. In the case when an electron deeply penetrates into the channel walls ($r_{\max} \gg r_c$), the critical frequency will be close to that in the homogeneous plasma ω_{c0}. In the contrary case, where $|r_{\max} - r_c| \ll r_c$, by using the quadratic approximation for potential (6), we obtain $\omega_c \approx \sqrt{2}\,\omega_{c0}$. Making allowance for the next infinitesimal order yields

$$\omega_c \approx \sqrt{2}\,\omega_{c0}\left(1 - \frac{p_{r\max}}{3r_c\sqrt{\gamma}}\right). \qquad (11)$$

Dependences of the ratio of the critical frequencies on the electron momentum for the plasma with a channel and the homogeneous plasma are presented in Fig. 4. In the regime of electron moderate penetration into the channel walls, which is realised when the electrons are accelerated in a sufficiently wide channel, the critical frequency in the plasma with a channel is greater than in the homogeneous plasma by a factor of $\sqrt{2}$, which corresponds to twice increased plasma concentration.

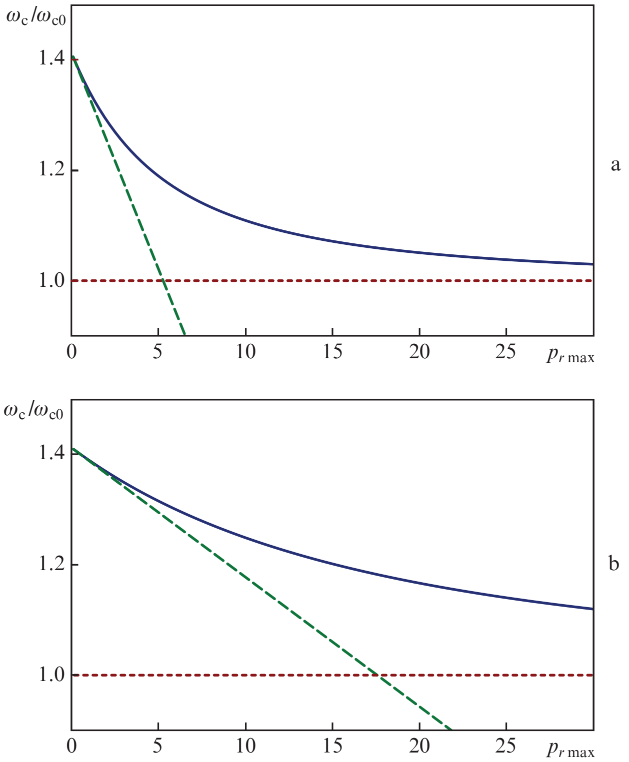

Figure 4. Dependences of the ratio of the critical frequency in the plasma with a channel to the critical frequency in the homogeneous plasma on the transverse momentum of an electron for the channel with radius r_c = (a) 0.3 and (b) 1. Dashed curves refer to the calculation by formula (11) in the small-momentum approximation; dotted curves refer to the solution for the homogeneous plasma. The Lorentz factor of electrons is $\gamma = 400$.

4. Two-stage method for obtaining betatron emission

A two-stage scheme is interesting for obtaining betatron emission. In this scheme, an electron bunch is trapped and accelerated in a plasma bubble possessing one set of parameters, whereas the main emission occurs in a plasma with another parameter set. Because of the different plasma parameter sets, in this section we will use dimensional parameters, so that the critical frequency (10) in the homogeneous plasma will be written in the form

$$\omega_c = \frac{3}{4}\frac{\gamma^2\omega_p^2 r_{max}}{c}. \qquad (12)$$

A bunch formed in the homogeneous plasma with a concentration n has the electron distribution over the transverse coordinates and momenta, in which the bunch radius r_0 and the maximal transverse momentum p_{r0} are related by the expression

$$p_{r0} = \sqrt{\frac{\gamma}{2}}\, m\omega_p r_0. \qquad (13)$$

Here, γ corresponds to the Lorentz factor averaged over bunch electrons under the assumption about the quasi-monoenergetic character of the bunch. Experiments show that laser-plasma accelerators allow one to obtain quasi-monoenergetic bunches with the electron energy spread on the order of several percent [24] that will not substantially affect the spectrum of betatron emission.

Since the emission power is proportional to the oscillation amplitude, the most intensive emission will be from electrons possessing the maximal oscillation amplitude; hence, the emission spectrum will approximately correspond to the spectrum of oscillations with the amplitude $r_{max} = r_0$. In addition, since the power of the synchrotron emission is proportional to γ^4/ρ (where ρ varies not faster than $\gamma^{3/2}$) and, thus, strongly depends on the Lorentz factor of electrons, the greatest contribution will be made by the emission from the trajectory part where electrons have the highest energy (in the case of acceleration it is the final part, in the case of deceleration it is the first part), and the critical frequency will be determined by the Lorentz factor γ at this trajectory part. Also, we assume that electron acceleration at the second stage can be neglected.

The transfer between the two stages possessing different parameters is assumed sufficiently short so that the parameters of the electron bunch remain almost constant. Since the electron bunch is ultrarelativistic, the forces of Coulomb repulsion for electrons at this part can be neglected, because their influence reduces inversely to γ^2. The nonstationary character of the wake fields along such a transfer, observed at the distances on the order of several plasma wavelengths (that is, much less than the length of betatron oscillations), cannot substantially affect the bunch parameters as well. Hence, the bunch radius will vary due to the bunch angular divergence $\theta \approx p_{r0}/(mc\gamma)$. Trajectories of electrons near the axis $r = 0$ have the greatest divergence; hence, the admissible length l of the transfer between the two stages can be approximately estimated from the inequality $l\theta < r_0$, which, in view of (13), results in the relationship $k_p l < \sqrt{2\gamma}$, where k_p is the wave number of plasma oscillations at the first stage. If this condition does not hold, an external action is needed for maintaining the bunch parameters.

Consider how the critical frequency in the spectrum of the betatron emission of this bunch will change if at the second stage the bunch passes to the homogeneous plasma with the concentration n'. If $n' > n$ then the amplitude of the bunch oscillations will not change in such a transition, that is, $r'_0 = r_0$. Then, according to formula (12) the critical frequency at the second stage is

$$\omega'_c = \omega_c\left(\frac{\omega'_p}{\omega_p}\right)^2 = \omega_c\frac{n'}{n}. \qquad (14)$$

If $n' < n$ then the bunch radius at the second stage will be determined by the amplitude of oscillations of electrons possessing the greatest transverse momentum $p'_{r0} = p_{r0}$. In this case, according to (13) the amplitude of oscillations is $r'_0 = \sqrt{n/n'}\, r_0$, which leads to the following relationship for the critical frequency at the second stage:

$$\omega'_c = \omega_c\sqrt{\frac{n'}{n}}. \qquad (15)$$

Now consider the case when the electron bunch of radius r_0 formed in the homogeneous plasma with a concentration n passes to the plasma with a channel of radius $r_c > r_0$ and the concentration outside the channel n'. As shown in Section 3, in the case when electrons do not deeply penetrate into the channel walls, that is, $\max(r - r_c) \ll r_c$, the critical frequency for the same transverse momentum of an electron will be greater by a factor of $\sqrt{2}$ than the critical frequency in the homogeneous plasma with the same concentration. In view of this fact, from relationships (12) and (13) one obtains

$$\omega_c' = \omega_c \sqrt{\frac{2n'}{n}}. \quad (16)$$

Dependences of the critical frequency at the second stage on the plasma concentration are shown in Fig. 5 for the cases of the homogeneous plasma and the plasma with a wide channel. Thus, we have calculated the emission spectrum for an electron bunch at the second stage under the assumption that the bunch is not accelerated.

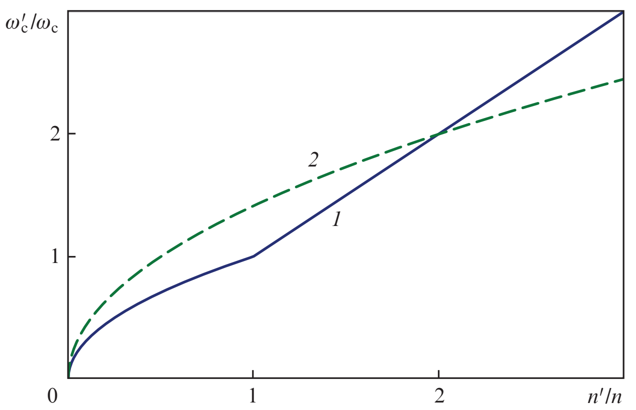

Figure 5. Dependences of the ratio of the second-stage critical frequency to the first-stage critical frequency on the plasma concentration for (*1*) the homogeneous plasma and (*2*) plasma with a channel at the second stage.

5. Conclusions

We have considered the transverse betatron oscillations of electrons and the corresponding betatron emission in a strongly nonlinear wake wave excited by a laser pulse in the plasma for the cases of a homogeneous plasma and a plasma with a cylindrical channel having sharp walls. It is shown that for strongly relativistic particles ($|p_r| \ll p_z$) the period of betatron oscillations in the plasma with the channel depends on the transverse momentum of electrons. For greater transverse momenta, this period corresponds to that of oscillations in the homogeneous plasma, and for small momenta, it grows in inverse proportion to the momentum. If electrons are accelerated in a sufficiently wide channel, the regime of a small transverse momentum is realised, which substantially increases the oscillation period as compared to the case of the homogeneous plasma.

The influence of the channel on the betatron emission has been analysed in the synchrotron regime ($p_{r\,\max} \gg 1$). The expression for the critical emission frequency as a function of the transverse electron momentum has been derived. It has been shown, that at large momenta, the critical frequency coincides with that of emission in the homogeneous plasma, and at small momenta it turns to be greater by a factor of approximately $\sqrt{2}$.

A two-stage scheme of betatron emission is considered in which an electron bunch formed at the first stage in the homogeneous plasma possessing a certain concentration is used for generating the betatron emission at the second stage in the plasma possessing another concentration. The cases of the homogeneous plasma and the plasma with a wide channel at the second stage are separately considered. It has been shown how the critical frequency of the emission spectrum depends on the concentration in these two cases in the absence of acceleration.

Acknowledgements. I.Yu. Kostykov acknowledges the support of the Russian Science Foundation (Grant No. 16-12-10383) to study the dynamics of betatron oscillations and A.A. Golovanov acknowledges the support of the Government of the Russian Federation (Grant No. 14.B25.31.0008) and the Presidium of the Russian Academy of Sciences (Programme 'Extreme laser radiation: physics and fundamental applications') to study betatron emission.

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