Transverse electron acceleration in the field of terahertz radiation. Terahertz synchrotron

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Abstract. **We study transverse acceleration of an electron introduced to a terahertz pulse along the direction of the electromagnetic field wave vector in the presence of an external permanent magnetic field. We estimate the possible increment of the electron energy as well as the acceleration length and turn angle of the electron leaving the pulse. The developed acceleration scheme may be employed (in addition to the electron accelerator itself) in a terahertz synchrotron, possible parameters of which are estimated.**

Keywords: transverse electron acceleration, terahertz pulse, terahertz synchrotron.

1. Introduction

Electron acceleration by pulses of terahertz (THz) radiation is an attractive 'linear' acceleration method in contrast to nonlinear methods realised with wake fields of a laser pulse [1] or a bubble regime [2]. Presently, this method has been used for accelerating electrons to the energies of a fraction of keV [3], an electron beam being introduced normally to the wave vector of THz radiation. The acceleration time has been actually limited by a single half-period of THz radiation, which is definitely longer than the characteristic period of optical-range waves.

If the accelerated electron is initially relativistic, then the acceleration time can be noticeably increased by introducing an electron (or an electron bunch) along the wave vector of the electromagnetic radiation. In this case, the energy increment will correspondingly rise [4]. Such a scheme of electron acceleration was previously considered in the problem on laser pulse output onto an optical surface of the 'vacuum–transparent medium' interface [4–6]. Because of the complexity of this scheme for experimental implementation, it has not been realised yet. However, the scheme, as compared to the transverse acceleration in a THz field, has one important advantage: the phase velocity of an inhomogeneous electromagnetic wave on such a surface can be controlled in certain limits, up to nonrelativistic velocities [4]. This may substantially increase the duration and, hence, value of acceleration. Such a scheme cannot be yet realised in the THz range because of the lack of necessary dielectrics. However, the idea seems promising because the transverse

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inhomogeneity of the electromagnetic wave in this case has a noticeably greater scale since the THz wavelength is much longer than the optical wavelength. Potentially, this circumstance removes the most of experimental difficulties.

There is an obvious possibility of realising the scheme suggested above for accelerating relativistic electrons by a THz pulse in open vacuum. In these conditions, the phase velocity of radiation cannot be less than the speed of light in vacuum. The electron can be accelerated by a terahertz pulse in a strongly dispersive waveguide or cavity. In this case, the propagation velocity of THz radiation can be less than the speed of light at a certain (small) part of the waveguide. This scheme was realised in experiments [7], where the energy increment of 5 keV was obtained for electrons with the initial energy of 60 keV.

The scheme suggested in the present work provides substantial increments of energy without adjusting the phase velocity of THz radiation in open vacuum, because an accelerated particle is longer kept in the accelerating half-wave of radiation simply due to its greater length. Quantitative estimates of such acceleration are given below, and a scheme of THz synchrotron is considered.

2. Motion of a charged particle in a transverse electromagnetic field in the presence of an external permanent magnetic field

We will quantitatively mathematically estimate the effect by using the model of charged particle acceleration in crossed permanent electric and magnetic fields, not assuming the equivalence of their absolute values. This is experimentally attained by introducing an additional magnetic field along the axis of the magnetic field of THz radiation; the latter is assumed linearly polarised. The system of equation for particles in the laboratory coordinate system is written in the form:

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = -eE_x + \frac{e}{c}v_z H_y,$$

$$\frac{\mathrm{d}p_z}{\mathrm{d}t} = -\frac{e}{c}v_x H_y. \tag{1}$$

This system is simpler than that considered in [4] because it has no component of the magnetic field along the *z* axis. Motion occurs in the *xz* plane. Here, p_x and p_z are the components of an electron momentum along the *x* and *z* axes, respectively; v_x and v_z are the components of an electron velocity along the *x* and *z* axes, respectively; *e* is the electron charge; and *c* is the speed of light in vacuum. The electric field
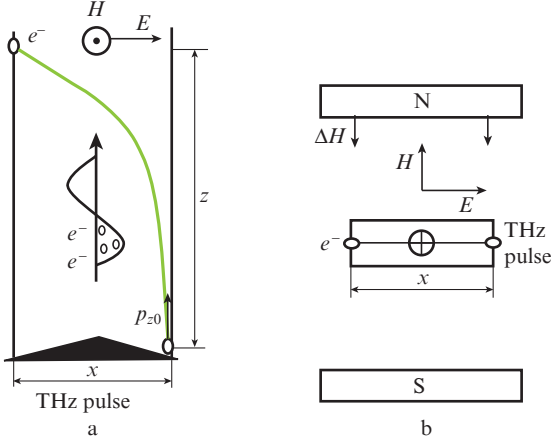
Figure 1. Scheme of a possible experiment:
(a) top view, the field H is directed toward a reader, a trajectory of an accelerated electron is shown by the grey curve, an approximate shape of the THz pulse is shown to be comprised of two half-periods, accelerated electrons are concentrated in the second half-period where $E > H$; (b) view following an outgoing THz pulse, the permanent magnets and the direction of additional permanent magnetic field are shown.

vector E_x is directed along the x axis, whereas the magnetic field vector H_y is directed along the y axis (Fig. 1). Without loss of generality, for the initial conditions one may take $p_x = 0$, $p_z = p_{z0}$, $x = 0$, $y = 0$ and $z = 0$ at $t = 0$.

If $E = E_x$ and $H = H_y$ are functions of coordinates and time as in the case of electron acceleration in a THz pulse in an external magnetic field, then system (1) can only be investigated numerically; such investigation will be performed elsewhere. Now we limit ourselves to the case of constants E and H, stipulating for the analysis of possible experimental realisation of the acceleration in the scheme suggested. We will analyse the case $|E| \geqslant |H|$; in the contrary case, the acceleration is absent.

The system has the integral of motion that can be written in the form

$$\gamma = \gamma_0 - \frac{eEx}{mc^2}. \tag{2}$$

Here, $\gamma = 1/\sqrt{1 - (v_x^2 + v_z^2)/c^2}$ is the relativistic energy factor; γ_0 is its initial value; x is a current value of the transverse coordinate; and m is the mass of the electron. The z-component of the momentum is $p_{z0} = mc\sqrt{\gamma_0^2 - 1}$. In addition, (2) implies that the motion occurs in the negative x direction; otherwise, the particle is decelerated.

The initial conditions for (1) are formulated in such a way that the electron in a conditional experiment is introduced along the z axis in parallel with the wave vector of the THz pulse. Principally, system (1) can be solved for a nonzero value of p_{x0} as well, i.e., for the case of nonparallel introduction of an electron. However, in this case, the effective phase velocity of the electromagnetic wave along the z axis will be greater than c, and the time of electron trapping in the accelerating half-wave will be shorter than in the case of the parallel introduction of the electron.

System (1) has the solutions that can be written implicitly for x and z:

$$ct\sqrt{1 - \frac{H^2}{E^2}} = \sqrt{x^2 - 2xx_2} + (x_1 - x_2) \times$$

$$\times \ln\left(1 - \frac{x}{x_2} + \sqrt{\frac{x^2 - 2xx_2}{x_2^2}}\right), \tag{3}$$

$$ct - \frac{H}{E}z = \sqrt{1 - \frac{H^2}{E^2}}\sqrt{x^2 - 2xx_2}.$$

Here we introduced the notations

$$x_1 = \frac{\gamma_0 mc^2}{eE}, \quad x_2 = x_1\left(1 - \sqrt{1 - \gamma_0^{-2}}\frac{H}{E}\right)\left(1 - \frac{H^2}{E^2}\right)^{-1}.$$

In the degenerate case $H = 0$, the solution for z is obtained in the form:

$$z = x_1\sqrt{1 - \gamma_0^{-2}}\ln\left[\frac{ct}{x_1} + \sqrt{1 + \left(\frac{ct}{x_1}\right)^2}\right]. \tag{3a}$$

In the case of transverse acceleration of an electron, its longitudinal velocity along the z axis reduces and it leaves the accelerating half-wave of the THz radiation. Similarly to [4], one can write out the condition of electron trapping in the accelerating half-wave of a length Λ in the form

$$ct - z \leqslant \Lambda. \tag{4}$$

The solution of three equations (3), (4) or (3), (3a), (4) will be just the sought acceleration γ [in this case, the equal sign is used in (4), see below].

Solutions of these algebraic equations can only be obtained numerically. Those are presented in Fig. 2 for the two values of the initial particle energy $\gamma_0 = 5, 30$ and for the three values of the electric field intensity $E = 10^6$, 10^7 and 10^8 V cm^{-1}.

The length z over which an electron is accelerated varies from several x_1 in the left part of the figures to the value of $2\gamma_0^2\Lambda$ at $(H/E)^2 = 1 - \gamma_0^{-2}$, which is much greater than x_1. As the ratio H/E further approaches unity, the possible acceleration still increases; however, this is attained at very large actually unrealisable transverse values of x (see below).

If $(H/E)^2 = 1 - \gamma_0^{-2}$, then system (3), (4) has a simple explicit solution. In this case, $x_1 = x_2$ and electron moves along the z axis uniformly at a velocity of $c\sqrt{1 - \gamma_0^{-2}}$. The growth of energy is

$$\gamma = \gamma_0\sqrt{1 + \left(\frac{2eE\Lambda}{mc^2}\right)^2}. \tag{5}$$

In turn, now the length of acceleration is $2\gamma_0^2\Lambda$, and the transverse deviation [which just determines the value of $2\gamma_0^2\Lambda$ (5)] is

$$x = x_1 - \sqrt{x_1^2 + (2\Lambda\gamma_0)^2}. \tag{6}$$

The ratio of (6) to (5) determines the value of the turn angle: at large (as compared to x_1) values of $\gamma_0\Lambda$ the angle is $1/\gamma_0$.

Expressions (5) and (6) are valuable because conventionally they are responsible for the right part of Fig. 2 (near unity on the abscissa axis). Since the curves of the $\Delta\gamma$ dependence on H/E in these figures are smooth, one may conclude that for neighbouring values of H/E, the values of $\Delta\gamma$ will also be close. These considerations will be needed for estimating a potential experiment.
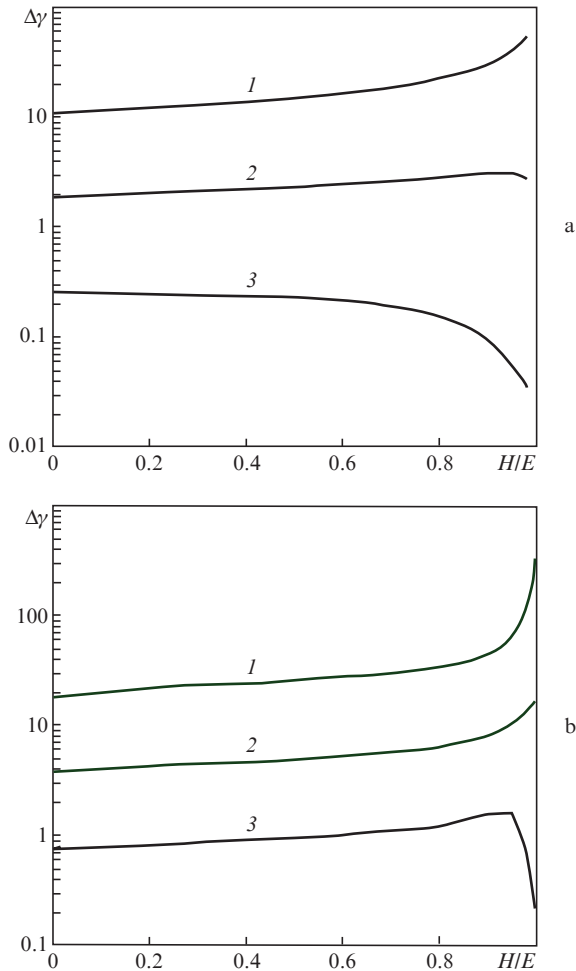
Figure 2. Dependences of electron energy variation $\Delta\gamma = \gamma - \gamma_0$ at $\gamma_0 =$ (a) 5 and (b) 30 for $E = (1)$ 10^8, (2) 10^7 and (3) 10^6 V cm^{-1}.

3. Possible experiment

A scheme of a potential experiment is shown in Fig. 1. A group of relativistic electrons is introduced into a 'bipolar' pulse of THz radiation with a box-type spatial cross section. The external magnetic field should be directed in such a way as to make the magnetic field in the first half-wave greater than the electric field. In this case, electrons in the first half-wave are more efficiently decelerated and pass to the second accelerating half-wave. Seemingly, one should rely on the right part of curves in Fig. 2, where the value of the external magnetic field is not large [$H \sim E(1 - \sqrt{1 - \gamma_0^{-2}}) \approx E/(2\gamma_0^2)$]. For $\Lambda/c \sim 1$ ps, we have $2eE\Lambda/(mc^2) = 1$ at $E = 8.5 \times 10^6$ V cm^{-1}. Hence, at the fields $E \sim 10^8$ V cm^{-1} reached presently and $\gamma_0 = 5$, the change of energy is $\Delta\gamma \approx 55$. Electrons with such initial energies γ_0 can be obtained from standard beta-radioactive sources (for example, ^{90}Sr + ^{90}Y, the energy is $\gamma_0 = 5.5$, or ^{60}Co, the energy is $\gamma_0 = 4$). In this case, the acceleration length is $z = 1.5$ cm and the deviation is $x = 0.3$ cm. To prevent the THz radiation from divergence over such a distance it is sufficient to have a pulse with the transverse size of 0.3×0.2 cm, total duration of 2 ps and energy of 1.5 J. The external magnetic field should be ~ 0.6 T, which can be provided by permanent magnets.

For accelerating electrons possessing the initial energy $\gamma_0 = 30$ the transverse size of the pulse should be 2.0×2.5 cm

at the field intensity and pulse duration mentioned above. The required pulse energy of 90 J and acceleration $\Delta\gamma \approx 330$ will be attained at a distance of 45 cm (here, the external magnetic field is rather small, ~ 0.025 T). Thus, in possible experiments the acceleration per unit length of more than several GeV m^{-1} is reached already at $E \sim 10^8$ V cm^{-1}; however, it falls as γ_0 increases. At stronger fields of THz radiation, the rate of acceleration is higher.

Electrons possessing $\gamma_0 = 5$ can be detected by the turn angle of an accelerated electron bunch – it will be $\sim 12°$, that is, the accelerated bunch will depart from the initial electron beam at a distance of several centimetres, or at a distance of two–three acceleration lengths. Unaccelerated electrons in this direction are absent; hence, bunch detection is simple. At $\gamma_0 = 30$, the turn angle will be $\sim 2°$, and the bunch will deviate from the main beam already at a distance of ~ 1 m. In this case, seemingly, one can use a conventional scheme of electron energy measurements.

In the estimates given above, we used the maximal values of side deviation x, at which electron is still in the accelerating half-wave. However, the transverse size of a THz pulse (denoted by x_m) can be less than this deviation. In this case, $\Delta\gamma$ is determined from (2) by the simple formula: $\Delta\gamma = eE|x_m| \times (mc^2)^{-1}$ [here, the sign '<' is used in (4)]. In addition, the acceleration can be reduced by using a stronger permanent magnetic field (the left part of curves in Fig. 2). However, the acceleration length is substantially shorter in this case, and a possible experimental setup can be rather compact. Hence, the scheme of the transverse electron acceleration by a THz pulse gives a chance to substantially vary experimental parameters, which is not so in [4–6].

4. Terahertz synchrotron

Acceleration of electrons considered above is accompanied by their emission. By realising several successive acceleration cycles, with the acceleration direction in each of the cycles being perpendicular to the input electron velocity, the electron (or electron bunch) will finally turn by 360°, which allows one to 'loop' the process. Such an experimental scheme presents a synchrotron, which is a terahertz synchrotron by the acceleration method. Let us estimate the maximal possible frequency of synchrotron emission and the possibility of principal realisation of the device.

The energy losses due to emission per single acceleration cycle are given by the formula from [8] (§73). For electron motion described by expression (3), the problem is solved exactly. By using the second equation (3) we obtain the expression for the energy losses per single cycle:

$$\varepsilon = \frac{2e^4E^2}{3m^2c^3}\left(1 - \frac{H}{E}\sqrt{1 - \gamma_0^{-2}}\right)^2 \gamma_0^2 t. \qquad (7)$$

Here, $t = L/c$; and L is the acceleration length.

Since the choice of experimental parameters for a scheme of a THz accelerator is wider than for the scheme from [4–6], various regimes of THz synchrotron operation can be calculated. However, for example, under the condition $(H/E)^2 = 1 - \gamma_0^{-2}$ the radius of the synchrotron will be $2\gamma_0^3\Lambda$. Its main frequency will be $3\Lambda/c$; hence, it is just the same THz oscillator, but with unacceptable large dimensions of the synchrotron. In addition, the energy losses due to emission per single acceleration cycle will be extremely small. Thus, the range of small additional permanent magnetic fields is not suitable for

realising a THz synchrotron; the operation range of fields in the synchrotron should correspond to the left part of curves in Fig. 2, that is, a sufficiently large external magnetic field is needed. As mentioned, the acceleration length at the fields corresponding to the left part of the curves, determined by Eqns (3a) and (4), is $L \sim (2-4)x_1$.

For definiteness, we take $H = 0$. In this case, the acceleration per single cycle is $\gamma = \sqrt{\gamma_0^2 + [eEL/(mc^2)]^2}$. The problem formulation implies that the second term under the radical sign is greater than the first term by a factor of $2-4$ (for definiteness, we set $L = 3x_1$); hence, $\Delta\gamma \approx 2.16\gamma_0$. By taking $mc^2\Delta\gamma$ equal to (7) at $H = 0$, we obtain the critical value of the energy γ_0, at which the losses due to emission are compensated for by the energy increment in a single cycle; this will be just the theoretical limit for particle acceleration in the synchrotron:

$$\gamma_{0cr} \approx \frac{1.08mc^2}{\sqrt{e^3 E}}. \qquad (8)$$

At $E = 10^6$ V cm^{-1}, we have $\gamma_{0cr} = 4.5 \times 10^6$, and at $E = 10^8$ V m^{-1}, γ_{0cr} is less by an order of magnitude. Correspondingly, the experimental setup admits less values of γ_0. The turn angle α ($\tan\alpha = x/z$) per single acceleration cycle will be $\sim 40°$, that is, nine acceleration cycles are necessary for closing a loop. Such a synchrotron will be small even at substantial energies γ_0: its diameter D will be $27x_1/\pi \sim 4.8\gamma_0$ (in cm) already at $E = 10^6$ V cm^{-1} (that is, approximately 5 m for $\gamma_0 = 10^4$). The main synchrotron frequency in this case will be $3c\gamma_0^3/D \sim 0.66 \times 10^{20}$ s^{-1}, which corresponds to the 42-keV energy of a quantum.

5. Conclusions

The scheme suggested for the transverse electron acceleration (and kinetic energy acquisition) in the field of a coaxial THz pulse with a permanent magnetic field seems simpler than the scheme considered previously for electron acceleration by a laser pulse when the electron approaches the optical surface of the 'vacuum–transparent medium' interface. The parameters of a THz pulse needed for reaching a sufficiently large acceleration are ordinarily realised in modern experiments [9]. At fields available presently, the rate of energy accumulation by an electron in a THz pulse will be above GeV m^{-1}, and at higher fields, it will be well above this limit of nonlinear acceleration methods [1, 2]. The general charge of particles accelerated simultaneously far exceeds the value of tens of picocoulombs discussed in [1, 2]. This problem should be studied separately taking into account random magnetic fields [10] (or currents [11]), generated by accelerated particles.

Estimates show that it is preferable to realise a THz accelerator at comparatively small external magnetic fields produced by permanent magnets. In this case, the dimensions of a single acceleration section are sufficiently large already at $\gamma_0 \geqslant 100$. Correspondingly, the required THz pulses should have large transverse sizes, that is, a substantially high energy. On the contrary, it is preferable to realise a THz synchrotron with strong permanent magnetic fields having the intensity comparable to that of the field in the pulse. Then the device will be sufficiently smaller (table-top).

Due to the considerable spatial dimensions and rather high density [12] of an accelerated electron bunch, the latter has various interesting applications, such as a relativistic mirror, ultrashort-wavelength (gamma) laser [13, 14] and so on.

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