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Scaling relations for a neutron yield in a plasma with inertial electrostatic confinement*

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Abstract. **We discuss the possibilities of producing a high-power source of thermonuclear neutrons under inertial electrostatic confinement of a plasma in the process of periodic oscillations of hydrogen isotope nuclei in the field of a virtual cathode of an electrostatic trap. The investigations are performed using analytical scaling relations, which explicitly give the dependence of a neutron yield on the electrostatic trap parameters under various operating conditions.**

Keywords: *inertial electrostatic confinement, oscillating plasma spheres, source of thermonuclear neutrons.*

1. Introduction

Inertial electrostatic confinement (IEC), the idea of which was first proposed and formulated in [1–5], underlies a number of modern projects aimed at creating high-power neutron sources [6–9] (see also monograph [10]). To date, one of the most promising schemes for the IEC application is the concept of the periodically oscillating plasma sphere (POPS) [6, 7]. In the POPS scheme, nuclei of hydrogen isotopes are periodically localised near the centre of the spherical axis or the axis of a cylindrical electrostatic trap due to multiple oscillations of these nuclei in the virtual cathode field formed by a flux of injected fast electrons. The pulsating regime of plasma formation was demonstrated both in experiments performed using a classical electrostatic trap [11] and in experiments with a nanosecond vacuum discharge [12].

In the classical POPS scheme, every oscillation of the ion flux requires constant energy expenditure to maintain the required magnitude of the electrostatic field. Therefore, at really achievable values of the final concentration of the hydrogen isotope nuclei of $10^{19}-10^{20}$ cm^{-3}, this scheme is ineffective for energy applications. At the same time, this scheme can form the basis of a relatively simple and effective method for producing a source of high-power thermonuclear neutrons with a particle flux exceeding 10^{12} neutrons per second [6]. The source's power of 10^9-10^{10} neutrons per second are already available in the experiments with an IEC-based neutron source [10].

Proposals aimed at achieving a positive energy yield of a thermonuclear reaction using IEC are based on the use of an external magnetic field in combination with an electrostatic trap [9, 13, 14]. In this case, an operating mode of a neutron source device can be achieved when the supplied energy is spent to maintain the necessary electric field during the period of the first or several first oscillations of hydrogen isotope nuclei, and subsequent oscillations will occur in the field of electrons magnetised in the potential well. According to various estimates, this requires a field with a strength of about 10 kT. To date, several proposals for the implementation of experiments of this kind are known, but publications on this subject are still absent.

The present work is devoted to a theoretical study of the features of a source of thermonuclear neutrons based on inertial electrostatic confinement of a plasma. The issues of energy reproduction using IEC are not discussed. It should be noted that theoretical research in the field of IEC is based on the results of complex numerical calculations, which are performed with the use of electrodynamic and kinetic codes, as a rule, for the given parameters of installations (see, for example, [10]). However, a useful approach to investigating the capabilities of IEC relies on the use of fairly simple models that allow an analysis in a wide range of variation of the problem parameters. In this paper we perform this analysis using analytical scaling relations that explicitly describe the dependence of the neutron yield on the parameters of a spherical electrostatic trap under various operating conditions. We consider isotopes of hydrogen of one type, i.e. deuterium, which simplifies the problem from the point of view of synchronising the periods of oscillations of ions of different types in an electrostatic field.

2. Scaling relations for the parameters of a convergent ion flux in the POPS scheme

We assume that a flux of deuterium nuclei converges from a sphere with an initial radius r_0 of the inner cathode to a sphere with a minimum radius r when the concentration of nuclei changes from the initial value n_0 to a maximum value n. In the case of the spherical trap geometry, the initial ion concentration, according to the Poisson distribution, is determined by the formula

$$n_0 = f\frac{3\Phi}{2\pi e r_0^2}, \tag{1}$$

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where e is the electron charge; Φ is the height of the potential well (potential of the electric field at the centre of the system); and f is the ratio of the concentration of deuterium nuclei to the concentration of electrons injected into the system, which we will henceforth assume to be unity. The oscillation frequency ν is determined by the time of flight of the deuterium nucleus inside the electrostatic trap:

$$\nu \equiv \frac{u_a}{2r_0} \approx 8\times 10^6 \frac{\Phi^{1/2}}{r_0}, \tag{2}$$

where $u_a = u/2 = [\Phi/(4m)]^{1/2}$ and u is the average and maximum values of the velocity of deuterium nuclei; m is the proton mass; Φ and r_0 are measured in kV and cm, respectively; and the frequency ν is measured in Hz.

The potential Φ is naturally chosen from the condition that the energy of the nuclei accelerated at the centre of the system should be close to the energy corresponding to the maximum of the fusion reaction cross section. Thus, for deuterium nuclei, this value should be close to 100 kV. At a potential Φ = 100 kV and a radius r_0 = 8 cm, the oscillation frequency is 10 MHz. For a given Φ, an increase in the oscillation frequency requires a decrease in the radius of the system, r_0. At Φ = 100 kV, in order to reach a frequency of 100 MHz, the radius of the system should be ~0.8 cm. As the oscillation frequency increases, the initial concentration n_0 of the nuclei in the trap grows. According to (1) and (2)

$$n_0 = \frac{6\Phi\nu^2}{\pi e u_a^2} \approx 5\times 10^{-5}\nu^2, \tag{3}$$

where n_0 is taken in cm^{-3}. The initial concentrations of deuterium nuclei are $\sim 5\times 10^9$ and $\sim 5\times 10^{11}$ cm^{-3} at a frequency of 10 and 100 MHz, respectively. According to (2) and (3), the total number of nuclei accelerated in a spherical system during one period is determined by the expression

$$N_i = \frac{4\pi}{3} r_0^3 n_0 \approx 8.3\times 10^{17} \frac{\Phi^{3/2}}{\nu}. \tag{4}$$

To estimate the degree of convergence of the flux of nuclei along the radius to the centre of the system, we use the 'ballistic' approximation, within which the degree of convergence is determined from the condition of adiabatic compression of the plasma. This approach does not take into account such important effects as the formation of a volume charge and two-beam instability, but it makes it possible to determine the magnitude of the limiting degree of convergence and to compare the results for the two main methods of the formation of a deuterium nucleus flux, i.e. direct injection of nuclei and ionisation of a deuterium gas by an electron beam. The compression ratio θ in this approximation is expressed in terms of the final and initial (T_0) temperatures. Under the conditions of the problem in question, when use is made of the value of the adiabatic exponent for an ideal gas, it has the form

$$\theta \equiv \frac{r_0}{r} = \left(\frac{\Phi}{T_0}\right)^{1/2}. \tag{5}$$

To estimate the degree of convergence of the nucleus flux in a system with direct injection, the initial temperature is usually taken equal to room temperature, T_0 = 0.025 eV. In this case, according to (5), for the compression ratio and the final concentration of the nuclei in the spherical system, we have expressions

$$\theta \approx 2\times 10^2 \Phi^{1/2}, \tag{6}$$

$$n \equiv n_0\theta^3 \approx 4\times 10^2 \Phi^{3/2}\nu^2, \tag{7}$$

where n is taken in cm^{-3}. Estimate (6) at a potential of 100 kV gives for the compression ratio a very high value, i.e. 2×10^3. If the formation of nuclei occurs as a result of ionisation of the deuterium gas by an electron beam, then the initial temperature should be close to the electron energy responsible for ionisation. Using T_0 = 2 eV in (5), we obtain an order of magnitude less value of the compression ratio and three orders of magnitude less value of the final concentration:

$$\theta \approx 22\Phi^{1/2}, \tag{8}$$

$$n \approx 0.53\Phi^{3/2}\nu^2. \tag{9}$$

At Φ = 100 kV and ν = 10 MHz, the final concentration of deuterium nuclei is about 4×10^{19} cm^{-3} in the case of a system with direct injection of the nuclei and about 4×10^{16} cm^{-3} in the case of a system with ionisation by an electron beam. At ν = 100 MHz, these values are, respectively, 4×10^{21} and 4×10^{18} cm^{-3}.

3. Power of a neutron source

In a separate oscillation, the neutrons are produced within a time during which the deuterium nuclei fly through the central region of the compressed plasma. In the simplest formulation of the problem, considering the spectrum of the nuclei to be monoenergetic and taking into account that the neutron is produced only in one of the two equiprobable channels of the DD reaction, the rate of DD neutron generation can be given in the form

$$\dot{N} = \frac{1}{4} n^2 \sigma(\varepsilon) u \Omega, \tag{10}$$

where σ is the cross section for the synthesis reaction; ε is the energy of the nuclei; and $\Omega = 4\pi r^3/3$ is the volume corresponding to the maximum concentration of the nuclei.

The time of flight of the deuterium nucleus through the region of a compressed plasma is related to the frequency of the oscillations by the expression

$$t_g = \frac{2r}{u} = \frac{1}{2\nu\theta}, \tag{11}$$

in which the oscillation frequency is given by formula (2).

Taking into account the periodic operating mode of IEC systems, the effective neutron generation rate, calculated from the period of one oscillation, is usually considered as a characteristic of the neutron source power. Taking into account (11), the effective generation rate is

$$\dot{N}_{\text{eff}} = \frac{\dot{N}}{2\theta}. \tag{12}$$

The use of the effective generation rate in the case of a periodic operation of the device makes it possible to calculate the total number of neutrons produced by the time t:

$$N = \dot{N}_{\rm eff}\, t.$$

Taking into account formulas (10), (12), (2), (6) and (8) for the effective rate of neutron generation in systems with direct injection of nuclei and ionisation of the gas by an electron beam, we respectively obtain

$$\dot{N}_{\rm eff} \approx 10^{24} \Phi^3 \nu\sigma, \qquad (13)$$

$$\dot{N}_{\rm eff} \approx 10^{22} \Phi^3 \nu\sigma. \qquad (14)$$

The effective rate of neutron generation increases linearly with increasing frequency of oscillations. For a system with direct injection of the nuclei, at a potential $\Phi = 100$ kV ($\sigma = 0.1$ barn), this velocity is $\sim 10^{12}$ neutrons per second for an oscillation frequency $\nu = 10$ MHz and $\sim 10^{13}$ neutrons per second for $\nu = 100$ MHz. For a system with ionisation by an electron beam, the effective generation rate is approximately two orders of magnitude lower. These results, which follow from simple scaling relations, are in good agreement with numerical calculations that are discussed in detail in [10]. We note the strong dependence of the power of the neutron source on the potential Φ of the electrostatic field. In the energy range of the deuterium nuclei from 100 to 1000 keV, the DD reaction cross section is virtually independent of the particle energy and is approximately 0.15 barn; hence, we can assume that the power of the neutron source in this case increases in proportion to the cubic potential of the field. This means that in a system with direct injection of the nuclei, at an oscillation frequency of 100 MHz and a potential of $\Phi = 200$ kV, we can expect the production of a neutron flux exceeding 10^{14} neutrons per second.

4. Conclusions

We have obtained analytical dependences of the power of a neutron source on the basis of an electrostatic trap operating in the POPS regime on the field potential and oscillation frequency of the ion flux. The results of the calculation by the analytical model are in satisfactory agreement with the results of numerical calculations. The rates of DD-neutron generation exceeding 10^{12} neutrons per second, which are in demand for modern projects aimed at creating a high-power source of thermonuclear neutrons, correspond to an oscillation frequency $\nu = 10$ MHz. The scaling coefficients found show that the power of the neutron source increases monotonically with increasing frequency of the oscillations of the flux and even stronger (according to a law close to cubic) with increasing field potential. This gives reason to believe that the flux of DD neutrons can exceed 10^{14} neutrons per second at an oscillation frequency of 10–100 MHz and a potential of about 200 kV.

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