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Effect of atomic flux reversal in a fluctuating moving optical lattice

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Abstract. **The dynamics of an ensemble of noninteracting cold atoms is considered in the field of a moving optical lattice subjected to random amplitude and phase fluctuations. The effect of the reversal of an atomic ensemble under the action of lattice fluctuations is demonstrated, in which the atoms begin to move in the direction opposite to that of the lattice motion. A kinetic model is constructed that reproduces this effect and makes it possible to relate its emergence to the asymmetry of the amplitudes of the interlevel transitions in a momentum space.**

Keywords: *moving optical lattice, cold atoms, harmonic noise, atomic flux reversal.*

1. Introduction

Moving optical lattices formed by two counterpropagating, frequency-detuned laser beams are of interest from different points of view. For example, they are considered as a means of transferring macroscopic ensembles of atoms in entangled states into a given region during quantum computations [1, 2]. They can also be used to simulate various phenomena of solid state physics, for example, electron–phonon [3] and spin–orbit [4, 5] interactions. The somewhat unusual band structure of the energy spectrum of atoms in moving optical lattices allows the transport of atoms to be coherently controlled [6, 7]. Another interesting application of moving optical lattices is the investigation of the phase transition from a superfluid state to the state of a Mott insulator in Bose–Einstein condensates [8, 9].

Optical potentials are virtually always subjected to some extent to noise, resulting, for example, from fluctuations of laser radiation. As a rule, these noises are destructive in nature, causing heating and decoherence of atoms [10]. At the same time, in certain cases, the lattice noise can lead to quite interesting physical phenomena, for example, to noise-induced Landau–Zener transitions [11] and to a significant prolongation of the time of Zitterbewegung oscillations [12]. In a recent paper, Makarov and Kon'kov [13] considered the problem of the motion of atoms in an optical potential, which is a superposition of a random static potential and a moving lattice. It was found that in this case the fluctuations of the moving lattice not only can significantly enhance the transport of atoms, but also cause a spontaneous change in the direction of the atomic flux, i.e. after a certain time, the atoms begin to move against the direction of motion of the optical potential. In the present paper, we intend to investigate this effect in the absence of a random potential, i.e., in an optical potential that includes only a moving optical lattice.

2. Description of the model

Let us consider the case of a cigar-shaped optical trap inside which an optical lattice is formed. In the absence of the interatomic interaction, the behaviour of the atoms is described by the one-dimensional Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m_\mathrm{a}}\frac{\partial^2\Psi}{\partial X^2} + V(X,t)\,\Psi, \tag{1}$$

where m_a is the mass of the atom;

$$V(X,t) = V_0\cos(2k_\mathrm{L}X - \beta\omega_0 t);\ \beta = \mathrm{sgn}\,v;\ |v| = \frac{\omega_0}{2k_\mathrm{L}}; \tag{2}$$

v is the velocity of the lattice along the X axis; ω_0 is the frequency difference between the counterpropagating laser beams producing an optical lattice; and k_L is the wave number of these laser beams. We introduce the normalisation of the variables:

$$x = 2k_\mathrm{L}X,\ \tau = 8\omega_\mathrm{r}t,$$

where

$$\omega_\mathrm{r} = \frac{\hbar k_\mathrm{L}^2}{2m_\mathrm{a}}$$

is the recoil frequency. Then, equation (1) is transformed to the form

$$\mathrm{i}\frac{\partial\Psi}{\partial\tau} = -\frac{1}{2}\frac{\partial^2\Psi}{\partial x^2} + V(x,\tau),\ V(x,\tau) = A\cos(x - \beta\omega\tau), \tag{3}$$

where

$$A \equiv \frac{V_0}{8E_\mathrm{r}};\ \omega \equiv \frac{\omega_0}{8\omega_\mathrm{r}};$$

E_r is the recoil energy. Our next task is to simulate the random 'shaking' of the optical lattice. To this end, we use the method

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first used in paper [14] and then in papers [12, 13, 15]. We replace expression (2) for $V(x, \tau)$ by the expression:

$$V(x,\tau) = A[f(\tau)\cos x - \beta f(\tau + \tau_{\rm sh})\sin x)], \quad (4)$$

where $f(\tau)$ is some random signal with a finite bandwidth, and $f(\tau + \tau_{\rm sh})$ is a replica of this signal with a time shift $\tau_{\rm sh}$. In what follows, we shall consider signals whose spectral width is much narrower than the centre frequency ω or has the same order of magnitude. We are interested in the case when the magnitude of the time shift is given by formula

$$\tau_{\rm sh} = \frac{\pi}{2\omega}.$$

Then, if we set $f(\tau) \sim \cos\omega\tau$, then $V(x,\tau) \sim \cos(x - \beta\omega\tau)$.

As the signal model $f(\tau)$, we use the so-called harmonic noise [16, 17], also known as a two-dimensional Ornstein–Uhlenbeck stochastic process. Harmonic noise is a solution of a system of stochastic differential equations in the form of the Langevin equation

$$\frac{df}{d\tau} = g, \quad \frac{dg}{d\tau} = -\Gamma g - \omega^2 f + \sqrt{2\varepsilon\Gamma}\,\xi(\tau), \quad (5)$$

where $\xi(\tau)$ is the Gaussian white noise; ε is a constant characterising the amplitude of the harmonic noise; and Γ is a positive constant characterising the degree of stochasticity. The first two moments of the harmonic noise are described by formulas

$$\langle f \rangle = 0, \quad \langle f^2 \rangle = \frac{\varepsilon}{\omega^2}.$$

In what follows, we set the constant ε so that the condition

$$\langle f^2 \rangle = \frac{1}{2} \quad (6)$$

is fulfilled. The spectral density of the harmonic noise is described by the formula

$$S(\Omega) = \frac{\Gamma}{\pi[\Omega^2\Gamma^2 + (\Omega^2 - \omega^2)]} \quad (7)$$

and has, for $\Gamma < 2\Omega$, a maximum at a frequency

$$\Omega = \omega_{\rm p} = \sqrt{\omega^2 - \frac{\Gamma^2}{2}}. \quad (8)$$

The width of the maximum is described by formula

$$\Delta\omega = \sqrt{\omega_{\rm p}^2 + \Gamma\omega'} - \sqrt{\omega_{\rm p}^2 - \Gamma\omega'}, \quad (9)$$

where

$$\omega' = \sqrt{\omega^2 - \frac{\Gamma^2}{4}}.$$

It follows from (6) and (7) that $\Delta\omega \to 0$ and $\omega_{\rm p} \to \omega$ at $\Gamma \to 0$. For small values of Γ, we have $\Delta\omega \approx \Gamma$.

3. Simulation of the dynamics of atoms in the momentum representation

One can achieve a significant simplification if we assume that in the course of the motion the atoms do not leave a certain bounded region in the space of the variable x

$$-\frac{L}{2} \leqslant x \leqslant \frac{L}{2},$$

whose size is described by the formula $L = 2\pi N$, where N is a sufficiently large integer. Setting periodic boundary conditions on the boundaries of this region, we can introduce a basic set of eigenstates of the momentum operator

$$|m\rangle = \frac{1}{\sqrt{L}}e^{ip_m x}, \quad p_m = \frac{2\pi m}{L},$$

where $m > 1$ is an integer. Then, the solution of the Schrödinger equation can be represented as the sum

$$\Psi(x,\tau) = \sum_{m=-\infty}^{\infty} a_m(\tau)e^{-iE_m\tau}|m\rangle, \quad E_m = \frac{p_m^2}{2}.$$

Substituting this expansion into (3), we obtain

$$\frac{da_m}{d\tau} = -i\sum_{n=-\infty}^{\infty} V_{m,n}(\tau)e^{i(E_m - E_n)\tau}a_n(\tau).$$

Taking into account (4), we find the expression for the matrix element $V_{m,n} \equiv \langle m|V|n\rangle$:

$$V_{m,n}(\tau) = \frac{A}{2}[f(\tau) + i\beta\,\mathrm{sgn}(m-n)f(\tau+\tau_{\rm sh})]\delta_{m,n\pm N}, \quad (10)$$

where δ_{ij} is the Kronecker symbol. It is convenient to represent nonzero matrix elements in the form:

$$V_{m,m-N} = \frac{A}{2}Y(\tau), \quad V_{m,m+N} = \frac{A}{2}Y^*(\tau),$$

where

$$Y(\tau) = f(\tau) + i\beta f(\tau + \tau_{\rm sh}). \quad (11)$$

Below we shall consider only the case $\beta = 1$. The equations of motion for the amplitudes a_m are transformed to the form

$$i\frac{da_m}{d\tau} = J_{m,m-N}(\tau)a_{m-N} + J_{m,m+N}(\tau)a_{m+N}, \quad (12)$$

where

$$J_{m,m-N} = \frac{A}{2}Y(\tau)e^{i(E_m - E_{m-N})\tau}; \quad (13)$$

$$J_{m,m+N} = \frac{A}{2}Y^*(\tau)e^{i(E_m - E_{m+N})\tau}. \quad (14)$$

We simulated the dynamics of atoms by numerically solving the system of equations (12) followed by averaging over 1000 harmonic noise realisations. The initial condition was chosen as follows:

$$a_m(\tau = 0) = (2\pi L^2\sigma_x^2)^{-1/4}\int_{x=0}^{L} e^{-\frac{(x-x_0)^2}{4\sigma_x^2} - ip_m x}dx, \quad (15)$$

where $\sigma_x = 10\pi$. With this initial condition, the quantum-mechanical mean value of the momentum operator at the initial instant of time is zero.

The results of the simulation are shown in Fig. 1. One can see that at $\Gamma = 0.1$ and 0.5, one can observe at the initial stage a fairly rapid acceleration of the atoms in the direction of positive x values, i.e., in the direction of motion of the optical lattice. After reaching a certain peak value of the average pulse, this process terminates, after which the atoms experience deceleration, and then accelerate in the opposite direction. The displacement of the atomic ensemble can be estimated from the formula

$$\langle x(\tau)\rangle = \langle x(\tau = 0)\rangle + \int_0^{\tau}\langle p(\tau')\rangle \mathrm{d}\tau',$$

where $\langle x(\tau = 0)\rangle$ is determined by the initial condition (15) and is equal to zero (Fig. 1b). One can see that, after changing the direction of motion, the atomic ensemble passes its initial position and continues to move further against the direction of motion of the optical lattice. Comparing the data obtained for different values of Γ, we can conclude that amplification of fluctuations of the moving lattice makes the flux reversal effect stronger. Thus, it is reasonable to assume that fluctuations play a very important role in the occurrence of this effect.

The effect of atomic flux reversal can be explained using the kinetic approach. The simplest form of the kinetic equations for populations $\rho_m \equiv |a_m|^2$ looks like this [18]:

$$\frac{\mathrm{d}\rho_m}{\mathrm{d}\tau} = \sum_n \Gamma_{m,n}(\rho_n - \rho_m), \quad \Gamma_{m,n} = \langle |J_{m,n}|^2\rangle. \qquad (16)$$

We can neglect interference effects, which decay shortly. This makes it possible to average the coefficients $J_{m,n}$ over a sufficiently large time interval, subsequently setting the length of this interval to infinity. As a result, the expressions for nonzero coefficients are as follows:

$$\bar{J}_{m,m-N} = \frac{\mathrm{i}\pi A}{2}[F(\Omega = \omega_{m-N,m}) + \mathrm{i}F'(\Omega = \omega_{m-N,m})],$$

$$\bar{J}_{m,m+N} = \frac{\mathrm{i}\pi A}{2}[F(\Omega = \omega_{m+N,m}) - \mathrm{i}F'(\Omega = \omega_{m+N,m})],$$

where $\omega_{m\pm N,m} = E_{m\pm N} - E_m$, and F and F' are the Fourier transforms of the functions $f(\tau)$ and $f(\tau + \tau_{\mathrm{sh}})$, respectively. Assuming that the correlation time of the harmonic noise is sufficiently large, we can use the approximation

$$F'(\Omega) \approx \mathrm{e}^{\mathrm{i}\Omega\tau_{\mathrm{sh}}}F(\Omega). \qquad (17)$$

Taking into account (6), we have $|F(\Omega)|^2 = S(\Omega)/2$. Thus, we obtain an expression for the amplitude of the transitions

$$\Gamma_{m,m\pm N} = \frac{\pi^2 A^2 S(\omega_{m,m\pm N})}{4}(1 \pm \sin\omega_{m,m\pm N}\tau_{\mathrm{sh}}). \qquad (18)$$

This formula does not work if $\omega \approx \omega_{m,m\pm N}$ and $\omega_{m,m\pm N}\tau_{\mathrm{sh}} = \pm\pi/2$. In this case, approximation (17) becomes inapplicable. Formula (18) assumes that the amplitudes of transitions between states with positive momenta are larger than the amplitudes of transitions between states with negative momenta. It follows that the states with negative momenta, which correspond to the motion that is antidirectional to that of the optical lattice, are 'dark' and are able to accumulate the population with time due to noise-induced transitions. As a result, it is possible to reverse the atoms in the opposite direction.

Figure 2 shows the time dependence of the average momentum, obtained by numerically solving kinetic equations (16). We can note a fairly good qualitative agreement between this result and the curves presented in Fig. 1a. Some quantitative discrepancy may be due to the influence of inter-

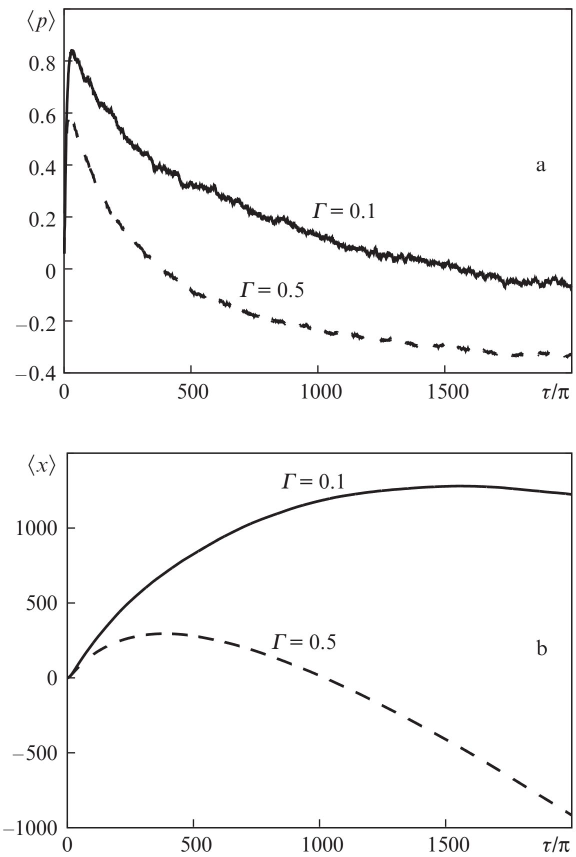

Figure 1. Time dependences of (a) the average momentum and (b) average displacement of atoms, obtained by solving equations (12), $A = 0.25$, $\omega = 1$.

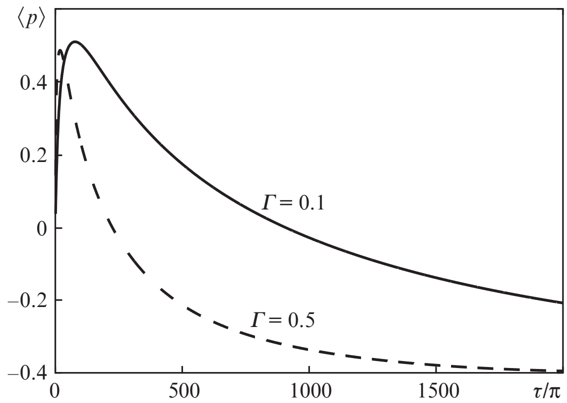

Figure 2. Time dependence of the average momentum of atoms, obtained by solving the system of kinetic equations; the values of the parameters are the same as in Fig. 1.

ference terms, which can enhance the initial acceleration of atoms in the direction of motion of the optical lattice. Thus, we can conclude that our kinetic model reproduces quite well the main features of the physical system under consideration.

4. Conclusions

We have considered the dynamics of cold atoms in the field of a moving optical lattice subjected to random amplitude and phase fluctuations. It has been shown that fluctuations lead to a spontaneous reversal of the atomic flux, as a result of which they begin to move in a direction that is opposite to the motion of the optical lattice. A kinetic model has been constructed that describes the dynamics of atoms in a momentum space. The analysis carried out with its help allows us to conclude that the reversal of the atomic flux is due to noise-induced pumping of the momentum states corresponding to motion in the direction opposite to that of the optical lattice.

At the same time, there are a number of questions, the answers to which could help both to discover new properties of the flux reversal effect and to assess the prospects for its experimental implementation. First, it is interesting how the atomic dynamics will change when the internal degrees of freedom are taken into account? Secondly, the question remains: is the effect of atomic flux reversal possible if the moving lattice is subjected to deterministic modulation instead of random fluctuations? The answers to these questions are expected to be found in further work.

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