Quantum Electronics **47** (5) 421–425 (2017)

https://doi.org/10.1070/QEL16351

A new generation of cryogenic high-*Q* Fabry–Perot resonators for ultrastable lasers

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Abstract. **We have performed a comparative analysis of the characteristics of laser resonators fabricated from various materials. The greatest attention is paid to the problem of Brownian noise leading to fluctuations in the resonator length and limitation of the frequency stability of a laser actively stabilised by the transmission peak of a high-*Q* Fabry–Perot resonator. A possibility of designing new-generation ultrastable laser systems based on cryogenic crystalline resonators is shown.**

Keywords: *high-Q silicon resonator, ultrastable laser system, thermal noise.*

1. Introduction

Highly stable laser systems have been in demand in many areas of modern fundamental and applied physics. They find application in the problems of precision spectroscopy, metrology of time and frequency, navigation and telecommunication. Laser systems with a relative frequency instability at a level of 10^{-15}–10^{-16} and an averaging time of 1 s represent a key element of modern optical clocks [1, 2]. Furthermore, ultrastable laser systems can be used as independent sources of stable optical frequency signals. Inferior in stability for long averaging times (over 100 s) compared to optical atomic clocks with an instability level of 10^{-18}, laser systems stabilised by macroscopic resonators have better mass-dimensional characteristics and higher reliability, which makes them attractive for the development of transportable and compact devices. Such devices may find application in systems of satellite navigation, radiolocation, radio astronomy, and in highly stable communication lines.

Most modern laser frequency locking systems make use of ultrastable optical Fabry–Perot resonators. The laser radiation frequency can be locked to the resonator transmission peak using, for example, the Pound–Drever–Hall ltechnique [3]. As a result, the laser radiation frequency stability is determined by the frequency stability of the resonator eigenmodes and the signal-to-noise ratio in the error signal. The mode frequency, in turn, is directly related to the distance between the resonator mirrors.

Random changes in the distance between the resonator mirrors can be caused by external vibrations, changes in the linear dimensions of the resonator body due to temperature fluctuations, and also thermal Brownian noise, which is inevitably present in the resonator components: resonator body, mirror substrates and coatings. To reduce the effect of temperature fluctuations, the resonator body is made of a material having a 'zero' temperature expansion point. At this point, the coefficient of linear thermal expansion of a material vanishes. In addition, the resonator is placed in a thermally insulated high-vacuum chamber. In paper [4], a possibility of temperature stabilisation near the room temperature to an accuracy of 100 μK was demonstrated.

The correct choice of the shape of a resonator body and the way of its suspension as well as application of vibration compensation can significantly reduce the impact of vibrations. One of the possible approaches is the use of biconical shape (Fig. 1) and vertical resonator location with a suspension in the plane of its gravity centre [4].

Currently, ultrastable laser systems can provide a relative frequency instability $\delta\nu/\nu$ at a level of 10^{-16} with an averaging time of 1–1000 s [5, 6], limited mainly by thermal Brownian noise. In paper [7], two laser systems stabilised with respect to independent cryogenic silicon resonators are compared, and it is shown that the spectral width of laser radiation is about

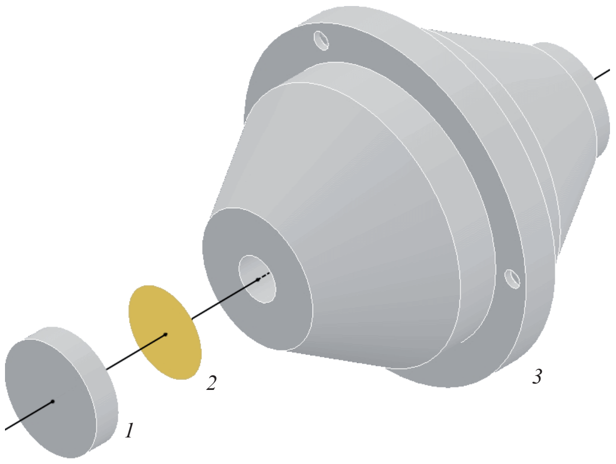

Figure 1. Resonator components: (*1*) mirror substrate; (*2*) reflective coating; (*3*) resonator body.

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Received 8 March 2017
Kvantovaya Elektronika **47** (5) 421–425 (2017)
Translated by M.A. Monastyrskiy

10 mHz at an averaging time of 1 – 1000 s. This opens up new possibilities for improving the accuracy of optical clocks and precision spectroscopy.

In the present work, a comparative analysis of the thermal mechanical noises of Fabry – Perot interferometers fabricated from various materials is performed, and a possibility of the development of ultrastable new-generation laser systems based on crystal resonators is examined.

2. Thermal noise

Thermal noise is one of the main problems in physics of precision measurements. Random thermal jitter of a resonator, especially of the mirrors, causes fluctuations in its length, and consequently, in the eigenfrequencies [8]. These micro-movements are produced by any body having a nonzero temperature.

To calculate thermal fluctuations, a fluctuation-dissipation theorem (FDT) is used, which relates the thermal noise spectrum and dissipative characteristics of a system. The use of FDT enables estimation of extremely weak thermal noises by investigating more measurement-accessible effects of energy dissipation in a system. According to FDT, the spectral power density of thermal noises in a mechanical system is proportional to the imaginary part of the transfer function of resonant oscillations for the case of mechanical perturbation of a given frequency. Traditionally, to calculate the response function, the eigenmode expansion method is used; however, this method is cumbersome since it requires a total calculation of the system resonances with regard to their effect on the resonator length. A simpler method was proposed in [9], where the thermal noise spectrum and the power dissipated by a system were calculated under the action of a periodic force applied to the resonator surface at those places that directly determine the resonator length, i.e. in the region of laser radiation reflection. In this case, the thermal noise spectrum and general damping of mechanical oscillations can be represented as a sum of the contributions from the individual elements of a resonator in accordance with the energy filling fractions and damping parameters [8, 10]. This method yields phenomenological estimation formulas that allow one to calculate the contributions to the spectrum of the frequencies f of the Fabry – Perot resonator thermal noise using the values characterising its shape and dimensions, and also elastic constants and loss factors of the materials from which the resonator is manufactured. In particular, the resonator body contribution to the spectral power density of thermal oscillations is

$$S_{\rm sp}(f) = \frac{2k_{\rm B}T}{\pi f}\frac{L}{3\pi R^2 E_{\rm sp}}\varphi_{\rm sp}, \tag{1}$$

where T is the resonator body temperature; L is the resonator length; R is the resonator body radius; and $E_{\rm sp}$ and $\varphi_{\rm sp}$ are the Young modulus and the coefficient of mechanical losses of the resonator body material, respectively. The thermal noise contribution of the mirror substrates is

$$S_{\rm sub}(f) = \frac{2k_{\rm B}T}{\pi f}\frac{1-\sigma^2}{\sqrt{\pi}\,w_0 E_{\rm sub}}\varphi_{\rm sub}, \tag{2}$$

where w_0 is the resonator mode radius on the mirror; and $E_{\rm sub}$, σ and $\varphi_{\rm sub}$ are, respectively, the Young modulus, the Poisson coefficient and the coefficients of mechanical losses of the mirror substrate material. For the contribution of the mirror coatings to the thermal noise spectrum, we have the expression

$$S_{\rm coat}(f) = \frac{4k_{\rm B}T}{\pi^2 f}\frac{d}{E_{\rm coat}w_0^2}(1+\sigma)(1-2\sigma)\varphi_{\rm coat}, \tag{3}$$

where d is the thickness of the reflective coating; and $E_{\rm coat}$ and $\varphi_{\rm coat}$ are the Young modulus and the coefficient of mechanical losses in the mirror coatings.

Equations (1) – (3) allow the low-frequency wing of the thermal noise spectrum to be estimated. Indeed, the lowest frequency resonance of mechanical oscillations of an interferometer represents the fundamental oscillation of the Fabry – Perot resonator body with a frequency of $\nu \sim 10$ kHz, while, at averaging times of 1 – 1000 s, only relevant is the spectral noise density in the sub-Hz frequency range. Although the formulas used are not exact, they are capable, with an accuracy better than 10 %, of describing experimental achievements of recent years in the area of employing the resonators for laser radiation frequency stabilisation.

Formulas (1) – (3) contain a generalised phenomenological parameter, i.e. the coefficient of mechanical losses φ equal to an inverse Q-factor ($Q = 1/\varphi$) of mechanical oscillations of the interferometer elements. It is assumed that Q does not depend on the oscillation frequency (at least, in the low-frequency region), and this is due to the nature of a dominant damping mechanism of mechanical oscillations in the material of constructional elements – the so-called structural damping. Of special interest is the problem of measuring the Q parameter for different materials, especially if it reaches large values (above 10^4). Such experiments require high accuracy and skill [11].

The choice of the resonator material has a decisive effect on its potential level of stability. First and foremost, the thermal noise amplitude is influenced by the Q-factor and temperature; the value of the latter is determined by the zero point position.

Currently, ULE glass is a widely spread material for high-Q resonators used in systems for laser frequency stabilisation [12]. Several systems based on resonators made of ULE glass have been designed and are used by our group at Lebedev Physical Institute (LPI) [13, 14]. The zero point of the ULE glass thermal expansion coefficient is usually located in the range of room temperatures, which makes this glass convenient for applications. However, the recrystallisation process that constantly occurs in the glass results in a linear frequency drift of the resonator eigenmodes at a level of hundreds millihertz per second. Nevertheless, an increase in the linear dimensions of the resonator made it possible to attain a relative instability of laser radiation frequency of 8×10^{-17} at averaging times up to 1000 s [6].

One of the promising materials for the fabrication of resonators is monocrystalline silicon [5]. This material is not susceptible to recrystallisation, has excellent elastic properties and also possesses a zero point at a temperature of $T_0 = 124$ K and a Q-factor of $\sim 10^8$, which is at least an order of magnitude higher than that for all known alternative materials.

Table 1 shows the results of calculations performed for five realisations of resonators using formulas (1) – (3). Numerical estimates have been carried out for various combinations of the resonator body materials, mirror substrates and their reflective coatings (the material parameters are presented in Table 2). Geometric parameters of the resonators are the same and correspond to those we have employed in

Table 1. The calculated limit of the relative frequency instability for resonators from different materials (the dimensions of resonators correspond to those described in [4]).

Substrate/body material	Coating	Substrate noise contribution (%)	Coating noise contribution (%)	Frequency instability for 1 s (10^{-16})
ULE/ULE	SiO_2/Ta_2O_5	89	11	2.879
ULE/SiO_2	SiO_2/Ta_2O_5	25	75	1.539
Si/Si	SiO_2/Ta_2O_5	2	98	0.200
Si/Si	GaAs/AlGaAs	9	91	0.084
GaAs/GaAs	GaAs/AlGaAs	34	66	0.133

Table 2. Material parameters.

Material	E/Pa	T_0/K	σ	φ
ULE	6.8×10^{10}	300	0.18	1.6×10^{-5}
Si	18.8×10^{10}	124	0.266	10^{-8}
GaAs	8.6×10^{10}	56	0.31	10^{-7}
SiO_2/Ta_2O_5	–	–	–	4×10^{-4}
GaAs/AlGaAs	–	–	–	2.5×10^{-5}

the first-generation systems [4, 13]. It is remarkable that the reflective coating largely contributes to the thermal noise amplitude. For rough estimates of the contribution ratio of the resonator elements to the total noise, we may use the proportion

$$S_{\rm sp}:S_{\rm sub}:S_{\rm coat} \propto \left(\frac{L}{R^2}\frac{\varphi_{\rm sp}}{E_{\rm sp}}\right):\left(\frac{1}{w_0}\frac{\varphi_{\rm sub}}{E_{\rm sub}}\right):\left(\frac{d}{w_0^2}\frac{\varphi_{\rm coat}}{E_{\rm coat}}\right). \qquad (4)$$

Recently, the attention of researchers has been attracted by the possibility of the formation of highly reflective coatings based on multi-layered crystal structures. Particularly, for GaAs/AlGaAs crystalline coatings deposited onto substrates of fused silica, a rather low level of thermal noises was demonstrated [15]. In our calculations, a record-low frequency instability caused by thermal noises has been attained for a resonator with a body and mirror substrates manufactured from monocrystalline silicon, and with highly reflective crystalline (GaAs/AlGaAs) mirror coatings.

We have also examined a sample with a single-crystal GaAs being the material of the body and mirror substrates. The use of this material could significantly simplify the fabrication technology of mirror coatings based on GaAs/AlGaAs. Besides, the zero point temperature of GaAs is less than that of crystalline silicon, and is located in the vicinity of 56 K. Unfortunately, there are no reliable measurements of the Q-factor for this material in the literature; therefore, additional experiments are needed to draw valid conclusions regarding its prospects.

3. Development of an ultrastable laser based on a Fabry – Perot silicon resonator

Our group at LPI have started work on the design of a frequency stabilisation system for a narrow-band erbium fibre laser with a wavelength of 1.5 μm using a highly stable resonator made of monocrystalline silicon with dielectric reflecting mirrors. Presently, the experiment is in the phase of equipment adjustment. To cool the silicon sample down to 124 K, a vacuum nitrogen cryostat developed at LPI is used (Fig. 2). A general scheme of the cryogenic system is shown in Fig. 3. One of its features is the presence of an Elan 2 generator capable of providing the system with liquid nitrogen in the operating mode. Temperature stabilisation is ensured by a heater controlling the temperature of one of the external screens. An additional internal screen and deep vacuum (at a level of 5×10^{-9} mbar) provide the filtration of high-frequency temperature fluctuations, so that the system time constant is about 10 h. The system also allows the possibility of vibrational decoupling of the cryostat section (on which the interferometer is mounted) from a nitrogen screen being potentially a source of vibrations caused by boiling of liquid nitrogen.

During the first test experiments, it was found out that the resonator cannot be cooled to the zero point temperature $T_0 = 124$ K of silicon (Fig. 4a). Under high vacuum, the heat exchange between the nitrogen screen and the screen with a heater is performed by thermal radiation, but its power turned out lower than the heat supply through the legs on which the internal parts of the cryostat are installed. To increase the heat exchange with the internal screen at the expense of reducing the nitrogen container albedo, its nickel-plated surface was subjected to blackening, which was per-

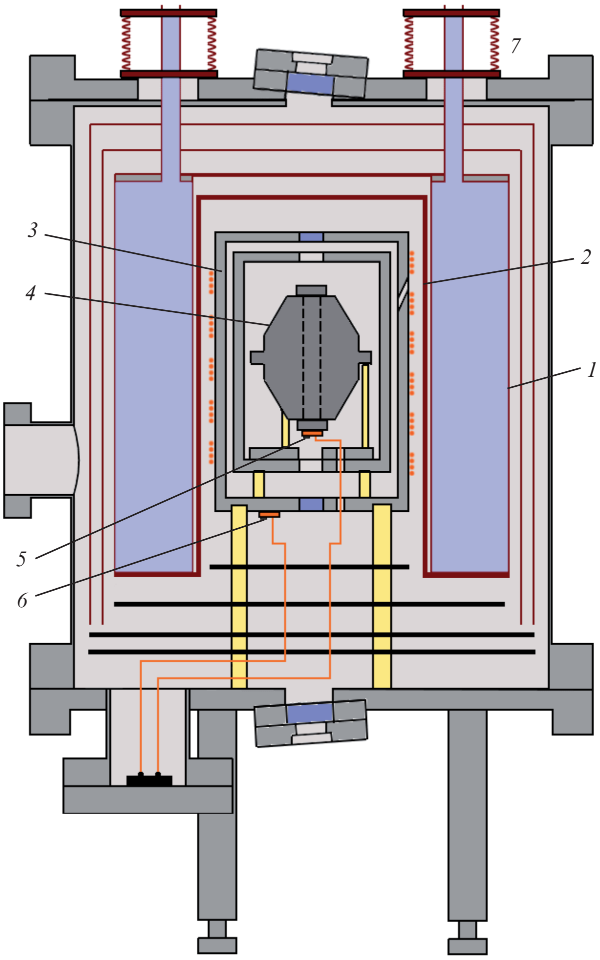

Figure 2. Vacuum cryostat cross-section: (*1*) container with liquid nitrogen; (*2*) internal surface of the container subjected to blackening; (*3*) thermal screen with a heater; (*4*) silicon resonator (or, in the course of tests, its model); (*5*, *6*) temperature sensors on the resonator model and on the thermal screen (*3*), respectively; the container with nitrogen has a separate support point and is connected to the vacuum chamber only by means of soft bellows (*7*).

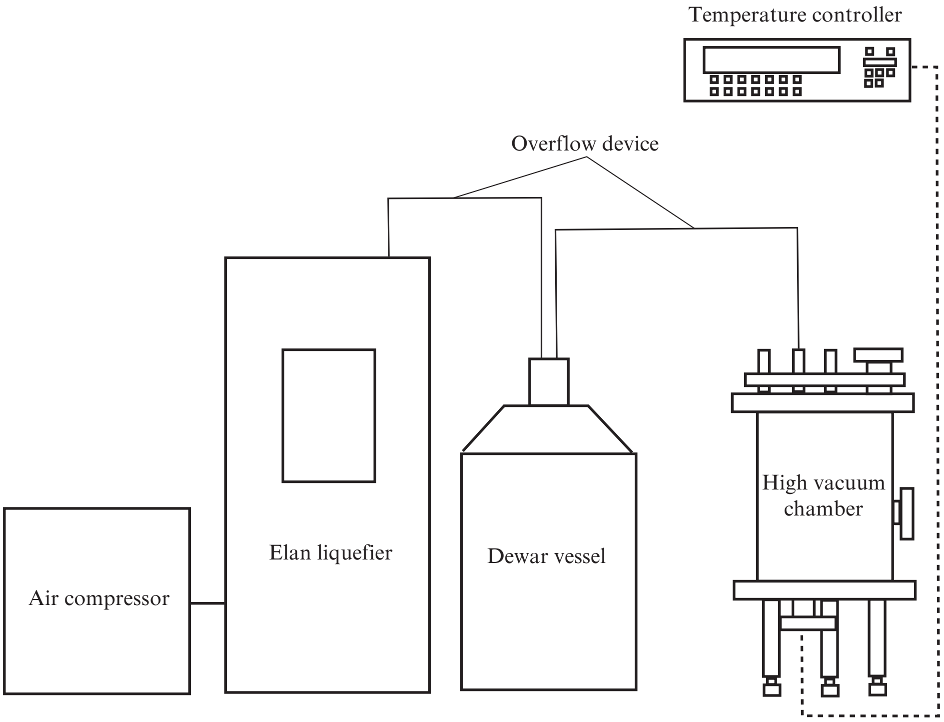

Figure 3. Scheme of a cryosystem for continuous cooling.

formed by coating the container surface with the VGE-7031 cryogenic varnish.

In view of the large time constant of the system (over 9 h), establishing of the predetermined temperature occurs for a long time which may constitute up to several days. Figure 4b shows the time dependences of the temperatures of the external screen with a heater and a model resonator fabricated from duralumin (Fig. 1). Fluctuations in the resonator body temperature are less than 0.01 K on the daily interval, which is sufficient to achieve the required radiation frequency stability of a system being developed.

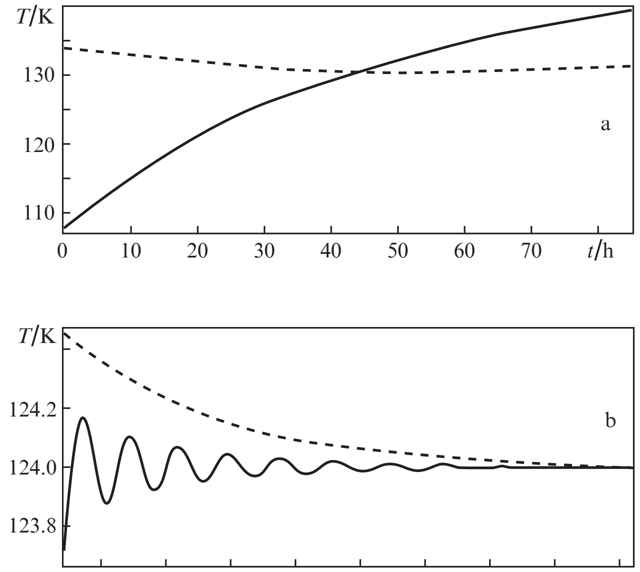

Figure 4. Time dependences of screen with heater temperature (solid curves) and temperature of the resonator model (dashed curves) when using a nitrogen container with (a) nickel-plated and (b) blackened surfaces. To reach an initial temperature below 120 K, the gas was supplied into the chamber at a pressure of 10^{-2} mbar, which ensured a rapid heat exchange with the nitrogen reservoir, after which the gas was pumped out and the temperature dynamics measurements began.

4. Conclusions

A brief review of the works on the up-to-date direction of laser physics – the development and design of ultrastable laser systems being widely used in precision measurements – is presented. The greatest attention is paid to the problem of thermal noises as a fundamental factor limiting the radiation frequency stability in these systems. The calculations performed show that the relative radiation frequency instability of the order of 10^{-17} can be achieved by using compact monocrystalline silicon resonators with crystal mirrors based on AlGaAs/GaAs multilayer structures. A new material – GaAs crystal, which has never been used before in the design of resonators, is examined. The necessity of measuring mechanical losses in the crystal, which depend substantially on the presence of impurities and dislocations, is noted. The resonators made of GaAs can compete with relevant silicon-based samples, provided their mechanical *Q*-factor reaches a level of 10^7.

The vacuum chamber design of a cryogenic silicon resonator is described, and the results of experiments on cooling and temperature stabilisation of a resonator model fabricated of duralumin are presented. It is shown that the resonator is cooled to a temperature corresponding to the zero temperature point of silicon (124 K); the residual temperature fluctuations are less than 0.01 K on a daily basis. The characteristics achieved provide the required frequency stability parameters of the laser system at λ = 1.5 μm, which will be locked to the transmission peak frequency of the resonator. We hope to attain the relative radiation frequency instability better than 10^{-16} at an averaging time of 1 – 100 s.

Acknowledgements. The work was supported by the Russian Foundation for Basic Research (Grant Nos. 16-29-11723 and 15-02-03936).

References

1. Bloom B.J. et al. *Nature*, **506**, 71 (2014).
2. Chou C.W., Hume D.B., Koelemeij J.C.J., Wineland D.J., Rosenband T. *Phys. Rev. Lett*., **104**, 070802 (2010).
3. Riehle F. *Frequency Standards: Basics and Applications* (Weinheim: Wiley-VCH, 2004; Moscow: Fizmatlit, 2009).
4. Alnis J., Matveev A., Kolachevsky N., Udem T., Hänsch T.W. *Phys. Rev. A*, **77**, 053809 (2008).
5. Kessler T., Hagemann C., Grebing C., Legero T., Sterr U., Riehle F., Martin M.J., Chen L., Ye J. *Nat. Photonics*, **6**, 687 (2012).
6. Häfner S., Falke S., Grebing C., Vogt S., Legero T., Merimaa M., Lisdat C., Sterr U. *Opt. Lett*., **40**, 2112 (2015) .
7. Matei D.G. et al. arXiv:1702.04669 (2017).
8. Braginsky V.B., Gorodetsky M.L., Vyatchanin S.P. *Phys. Lett. A*, **264**, 1 (1999).
9. Levin Yu. *Phys. Rev. D*: *Part. Fields*, **57**, 659 (1998).
10. Numata K., Kemery A., Camp J. *Phys. Rev. Lett*., **93**, 250602 (2004).
11. Nawrodt R., Zimmer A., Nietzsche S., Thurk M., Vodel W., Seidel P. *Cryogenics*, **46** (10), 718 (2006).
12. http://www.corning.com/media/worldwide/csm/documents/D20FD2EA-7264-43DD-B544-E1CA042B486A.pdf.
13. Khabarova K.Yu. et. al. *Quantum Electron*., **42**, 1021 (2012) [*Kvantovaya Elektron*., **42**, 1021 (2012)].
14. Vishnyakova G.A. et. al. *Usp. Fiz. Nauk*, **186**, 176 (2016).
15. Cole G.D., Zhang W., Martin M.J., Ye J., Aspelmeyer M. *Nat. Photonics*, **7**, 644 (2013).