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Adiabatic phase-conserving processes for executing quantum operations with ultracold atoms

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Abstract. **We have studied the regimes of deterministic single-atom Rydberg excitation in the conditions of Rydberg blockade and the methods of compensation for the dynamic phase of the wave function during the adiabatic passage. Using these methods, we have proposed schemes of single-qubit and two-qubit quantum states with mesoscopic atomic ensembles containing a random number of atoms, considred as quibits. The double adiabatic passage of the Förster resonance for two interacting atoms with a deterministic phase shift can be used for the implementation of two-qubit gates with reduced sensitivity of the gate fidelity to the fluctuations of the interatomic distance.**

Keywords: *Rydberg atoms, quantum computing, adiabatic passage.*

1. Introduction

The experimental realisation of quantum computing is one of the most interesting problems of modern physics [1]. A number of quantum systems can be considered as qubits. Recently, a considerable progress has been achieved in quantum informatics using superconducting qubits [2, 3] and ultracold ions [4, 5]. At the same time, the scaling of such systems to a large number of qubits (10^3–10^4) is a rather challenging problem. A promising alternative in this respect is to use optical lattices of ultracold neutral atoms that satisfy all DiVincenzo criteria for the qubits of a quantum computer [6–8]. As logical states of such qubits, the long-lived hyperfine sublevels of the ground state of alkali metal atoms are used [9, 10]. To obtain a quantum register of arbitrary scale, the atoms are captured in the arrays of optical dipole traps [11, 12]. To initialise the register, the optical pumping to one of the hyperfine sublevels is used. Single-qubit gates are implemented using the interaction of atoms with microwave or laser radiation with addressing to individual trapped atoms [13]. To implement the two-qubit gates, the atoms are excited for a short time to the Rydberg states, which allows the control of their interaction and the preparation of quantum entangled states of the qubits [10, 14, 15]. The atomic quantum state can be efficiently measured using the resonance fluorescence (for atoms in short-lived low states) or selective field ionisation (for atoms in long-lived Rydberg states) [7, 16].

Recently, a significant progress has been made in the experimental realisation of quantum registers, comprising 50–100 individual ultracold atoms captured in the array of optical dipole traps [11, 12]. As an alternative, one can consider qubits based on arrays of atom ensembles (the so-called superatoms), an advantage of which is the smaller sensitivity to the loss of individual trapped atoms [7, 14]. Quantum registers with cold atoms are schematically illustrated in Fig. 1a. The two-qubit gates with ultracold atoms were successfully implemented in the experiment [15] using the dipole blockade effect [14]: the interaction of atoms with each other causes a shift of the collective energy levels of the system of two closely spaced atoms (Fig. 1b) and 'withdraws' such levels from the resonance with the exciting laser radiation. This determines the impossibility of exciting two closely spaced Rydberg atoms at once and allows the generation of the entangled states of two atoms. The dipole blockade effect can be used both for the implementation of two-qubit gates with single atoms, and for recording the quantum information in the collective states of a mesoscopic atomic ensemble (Fig. 1c). In the dipole blockade regime such an ensemble is a two-level system, where the ground state corresponds to all atoms being at the lower level, while the excited state is a symmetric superposition of all possible states, in which only one atom in the ensemble is excited to the Rydberg state. The frequency of the collective Rabi oscillations in this ensemble by $\sqrt{N}$ times exceeds the Rabi frequency of the single-atom laser excitation (N is the number of atoms in the ensemble). This makes the accuracy of quantum operations sensitive to the fluctuations of the number of atoms in the ensemble.

The main obstacle for quantum computing implementation with neutral atoms is the low accuracy of two-qubit gates that does not exceed 0.73 [17]. In this connection, the systems of two-qubit operations with reduced sensitivity to the fluctuation of parameters in the experiment are of great interest. We developed a number of systems of quantum gates with ultracold neutral atoms based on the double adiabatic passage with the conservation of the collective wave function phase [18–20]. In the present paper we consider the specific features of double adiabatic sequences in the case of single-photon adiabatic rapid passage (ARP) and two-photon stimulated Raman adiabatic passage (STIRAP) of optical

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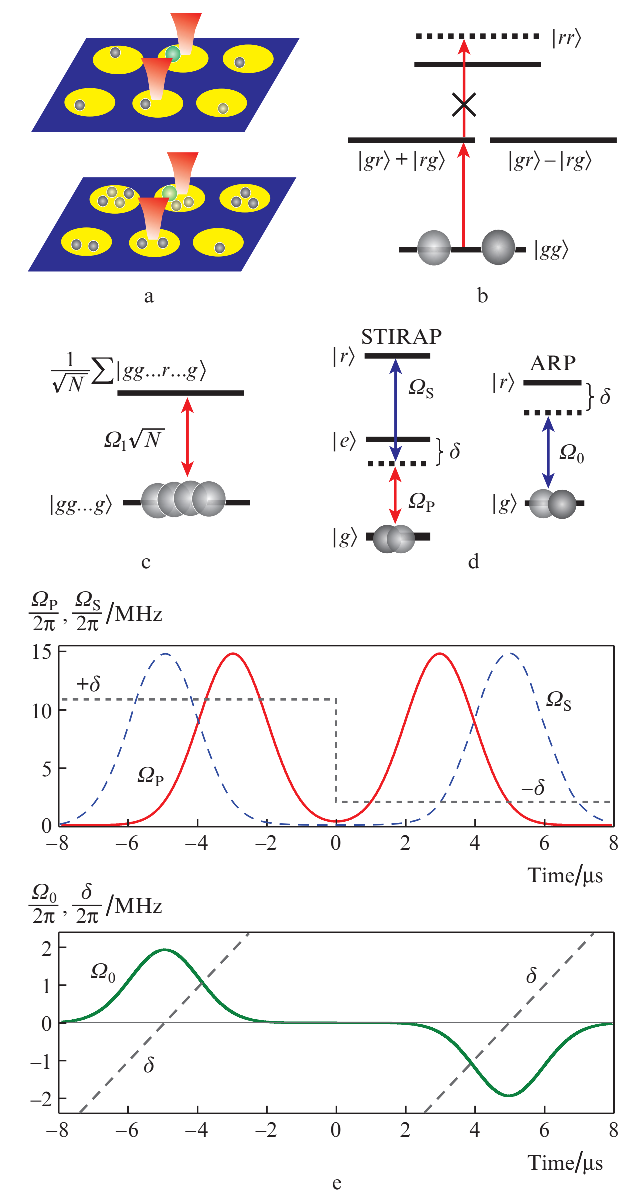

Figure 1. (a) Schematic of a quantum register based on the array of optical dipole traps with single atoms (top) or atom ensembles (bottom) in each node, (b) collective energy levels of two interacting atoms, (c) mesoscopic ensemble of N atoms, interacting with laser radiation in the dipole blockade regime, considered as a two-level system with the frequency of collective Rabi oscillations depending on the number of atoms, (d) energy level diagram for two-photon (STIRAP) and one-photon (ARP) adiabatic passage of resonances and (e) the double adiabatic sequence of laser pulses conserving the phase for the two-photon (top) and one-photon (bottom) adiabatic passage.

resonances, corresponding to the excitation to Rydberg states, as well as the adiabatic passage of Förster resonances in the interacting Rydberg atoms [21–26].

2. Deterministic excitation of single atoms in the regime of dipole blockade and double adiabatic sequences of controlling pulses

Although the dipole blockade effect forbids the excitation of more than one Rydberg atom in a mesoscopic ensemble, the fluctuations of the number of atoms, initially trapped in the optical dipole trap make it impossible to choose the exciting single-pulse 'area', necessary for coherent excitation of one Rydberg atom with the probability close to unity (the laser π-pulse). In our paper [27] it was shown that in a mesoscopic ensemble of strongly coupled Rydberg atoms in the dipole blockade regime, the deterministic adiabatic excitation of a single Rydberg atom, independent of the number of atoms in the ensemble, can be implemented. For this purpose, we considered the single-photon and two-photon adiabatic passage of the optical resonance. The relevant energy level diagrams are shown in Fig. 1d. In the case of a single-photon ARP the used pulses possess such a frequency chirp that during the laser pulse action the resonance detuning $\delta(t)$ changes the sign. In a STIRAP two partially overlapping laser pulses are used, acting in the inverse sequence on the transitions $|e\rangle \to |r\rangle$ and $|g\rangle \to |e\rangle$ [28], where $|g\rangle$ is the ground state, $|e\rangle$ is the intermediate excited state, and $|r\rangle$ stands for the Rydberg states. The radiation frequencies of two lasers must be tuned exactly to the two-photon resonance and have constant detuning δ from the intermediate excited state.

To implement quantum gates with atomic ensembles, we propose to use double adiabatic sequences of pulses, schematically shown in Fig. 1e. This allows the compensation of undesired dynamic phase shift of the collective states, accumulated after the first adiabatic sequence. In the two-photon adiabatic passage between two sequences the sign of the detuning is switched from $+\delta$ to $-\delta$, and in the single-photon adiabatic passage for the second pulse the sign of the Rabi frequency is changed, which corresponds to a phase shift of the laser pulse by π.

We have found that depending on the value of the detuning from the intermediate state δ, the regime of two-photon adiabatic passage in mesoscopic atomic ensembles essentially changes. In particular, when $\delta = 0$ the interaction of a single atom with the laser radiation leads to the excitation of the atom to the Rydberg state $|r\rangle$. For the ensemble of two interacting atoms in the dipole blockade regime the situation cardinally changes: after the end of the adiabatic passage, no atoms stay in the Rydberg state [29]. To explain this effect let us consider the interaction of the ensemble of atoms with laser radiation in the dipole blockade regime.

First, we consider a single atom with the energy levels $|g\rangle$, $|e\rangle$, and $|r\rangle$ (Fig. 1d). Let $\Omega_{\rm P}(t)$ be the Rabi frequency for the transition $|g\rangle \to |e\rangle$, and $\Omega_{\rm S}(t)$ – the Rabi frequency for the transition $|e\rangle \to |r\rangle$. Under the conditions of exact two-photon resonance, the Hamiltonian for the three-level system can be written in the form [29, 30]:

$$\hat{H}_{\rm STIRAP}(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_{\rm P}(t) & 0 \\ \Omega_{\rm P}(t) & 2\delta(t) & \Omega_{\rm S}(t) \\ 0 & \Omega_{\rm S}(t) & 0 \end{pmatrix}. \qquad (1)$$

Its eigenvalues are $\omega_{\rm d} = 0$ and $\omega_{\rm a,b} = \pm(\hbar/2)[\delta^2(t) + \Omega_{\rm P}^2(t) + \Omega_{\rm S}^2(t)]^{1/2}$. The eigenvalue $\omega_{\rm d} = 0$ corresponds to the 'dark' state $|D\rangle = \cos\theta(t)|g\rangle - \sin\theta(t)|r\rangle$, where $\theta(t)$ is the mixing angle $[\tan\theta(t) = \Omega_{\rm P}(t)/\Omega_{\rm S}(t)]$, from where one can find that $\cos\theta(t) = \Omega_{\rm S}(t)/\sqrt{\Omega_{\rm S}^2(t) + \Omega_{\rm P}^2(t)}$ and $\sin\theta(t) = \Omega_{\rm P}(t)/\sqrt{\Omega_{\rm S}^2(t) + \Omega_{\rm P}^2(t)}$.

Consider the state population dynamics during the time interval $(-T, T)$, where $T = 5$ μs, for Gaussian laser pulses (Figs 2a–2d) with the Rabi frequencies

$$\Omega_{\rm S}(t) = \Omega_0 \exp[(t - t_1)^2/(2w^2)],$$
$$\Omega_{\rm P}(t) = \Omega_0 \exp[(t - t_2)^2/(2w^2)]. \qquad (2)$$

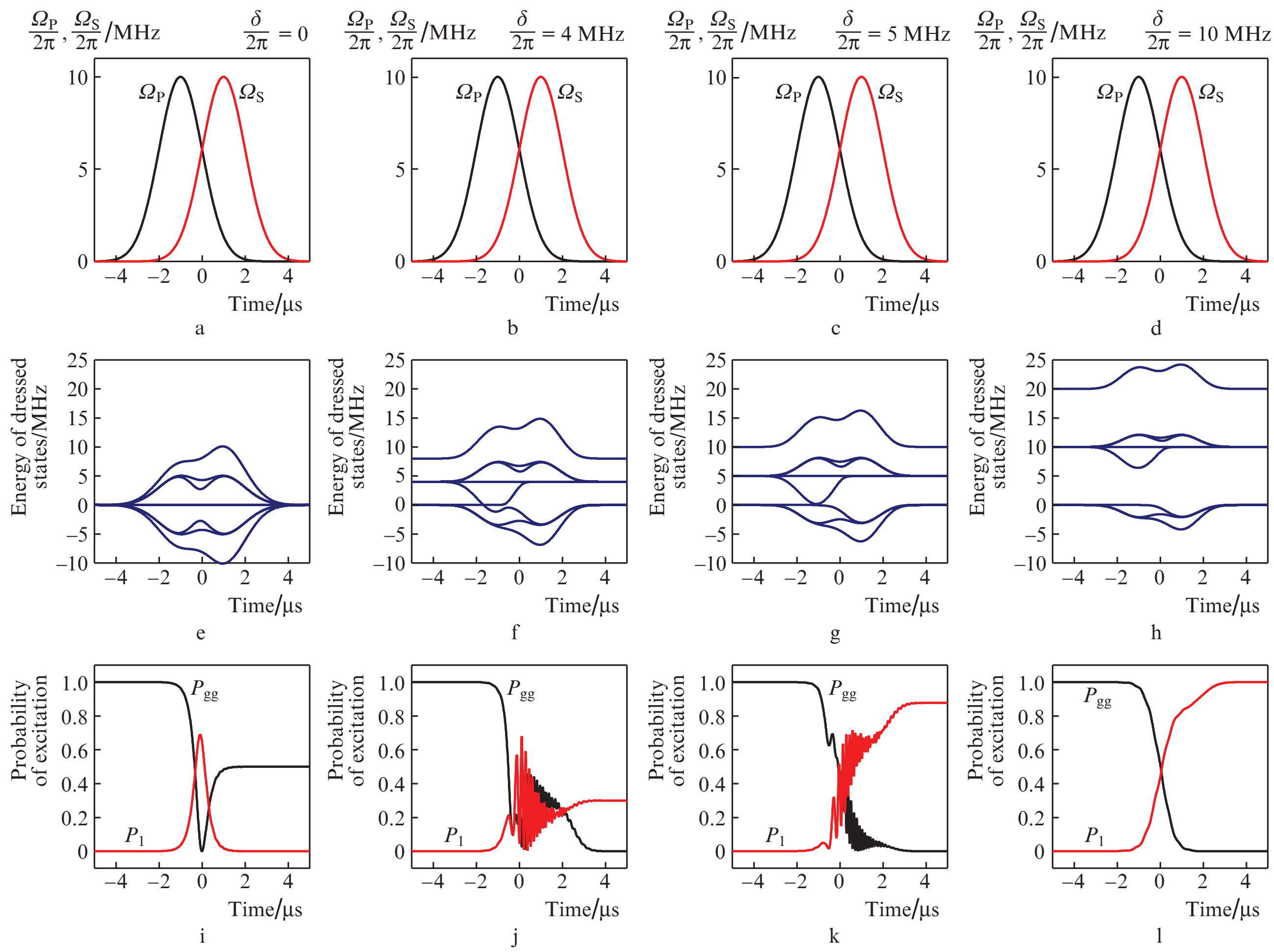

Figure 2. (a–d) STIRAP time sequences of laser pulses, (e–h) eigenvalues of the Hamiltonian for the system of two atoms interacting with laser radiation in the regime of dipole blockade and (i–l) numerically calculated probabilities P_{gg} to find two atoms in the ground state and the probability P_1 of the excitation of one Rydberg atom in the dipole blockade regime for different values of the detuning from the intermediate state $\delta/(2\pi)$.

Here $t_1 = -1$ μs; $t_2 = 1$ μs; and $\Omega_0/(2\pi) = 10$ MHz (these values correspond to the typical ones for experiments with Rydberg atoms). Initially $\cos\theta(t = -T) = 1$, $\sin\theta(t = -T) = 0$, and the ground state $|g\rangle$ coincides with the dark state $|D\rangle$. During the adiabatic passage of the resonance, the system stays in the dark state. However, after the end of the passage, the mixing angle changes: $\cos\theta(t = T) = 0$, $\sin\theta(t = T) = 1$, which corresponds to the transition of the atom to the state $|r\rangle$.

Now let us consider the Hamiltonian for two three-level atoms in the dipole blockade regime, which is written for eight collective states $|gg\rangle$, $|ge\rangle$, $|gr\rangle$, $|eg\rangle$, $|ee\rangle$, $|er\rangle$, $|rg\rangle$, and $|re\rangle$ of the quasi-molecule consisting of two interacting atoms:

$$\hat{H}_{2\mathrm{STIRAP}}(t) = \frac{\hbar}{2}\times$$

$$\times\begin{pmatrix} 0 & \Omega_P(t) & 0 & \Omega_P(t) & 0 & 0 & 0 & 0 \\ \Omega_P(t) & 2\delta(t) & \Omega_S(t) & 0 & \Omega_P(t) & 0 & 0 & 0 \\ 0 & \Omega_S(t) & 0 & 0 & 0 & \Omega_P(t) & 0 & 0 \\ \Omega_P(t) & 0 & 0 & 2\delta(t) & \Omega_P(t) & 0 & \Omega_S(t) & 0 \\ 0 & \Omega_P(t) & 0 & \Omega_P(t) & 4\delta(t) & \Omega_S(t) & 0 & \Omega_S(t) \\ 0 & 0 & \Omega_P(t) & 0 & \Omega_S(t) & 2\delta(t) & 0 & 0 \\ 0 & 0 & 0 & \Omega_S(t) & 0 & 0 & 0 & \Omega_P(t) \\ 0 & 0 & 0 & 0 & \Omega_S(t) & 0 & \Omega_P(t) & 2\delta(t) \end{pmatrix}. \quad (3)$$

Here we took into account the dipole blockade effect, excluding from the consideration the collective state $|rr\rangle$, which corresponds to two simultaneously excited Rydberg atoms. The numerically calculated eigenvalues of Hamiltonian (3) for the laser pulses described by Eqn (2) are presented in Figs 2e–3h for four values of the detuning from the intermediate state: $\delta/(2\pi) = 0$, 4, 5, and 10 MHz. The numerically calculated time dependences for the population of the ground state and the collective state with one Rydberg atom are presented in Figs 2i–2l. If the detuning from the intermediate state $\delta/(2\pi) = 0$, than the dark state exists that corresponds to the zero eigenvalue of energy (Fig. 2e). After the end of the adiabatic passage, the systems stays in the dark state, and the probability to find any of the atoms in the Rydberg state is zero (Fig. 2i). Such states are thoroughly considered in Ref. [29]. In Ref. [31] it was shown that in the absence of phase shifts between the laser pulses in the STIRAP, the geometric phase is zero. Thus, it is sufficient to study the effect of the dynamic phase.

For the nonzero detuning from the intermediate state, there are no eigenstates with zero energy. The transition to the excitation of a single Rydberg atom occurs in the vicinity of $\delta/(2\pi) = 5$ MHz (Figs 2j and 2k). For $\delta/(2\pi) = 10$ MHz the deterministic single-atom excitation is observed (Fig. 2l).

The absence of the dark state in Fig. 2h leads to the accumulation of the dynamic phase during the time of the adia-

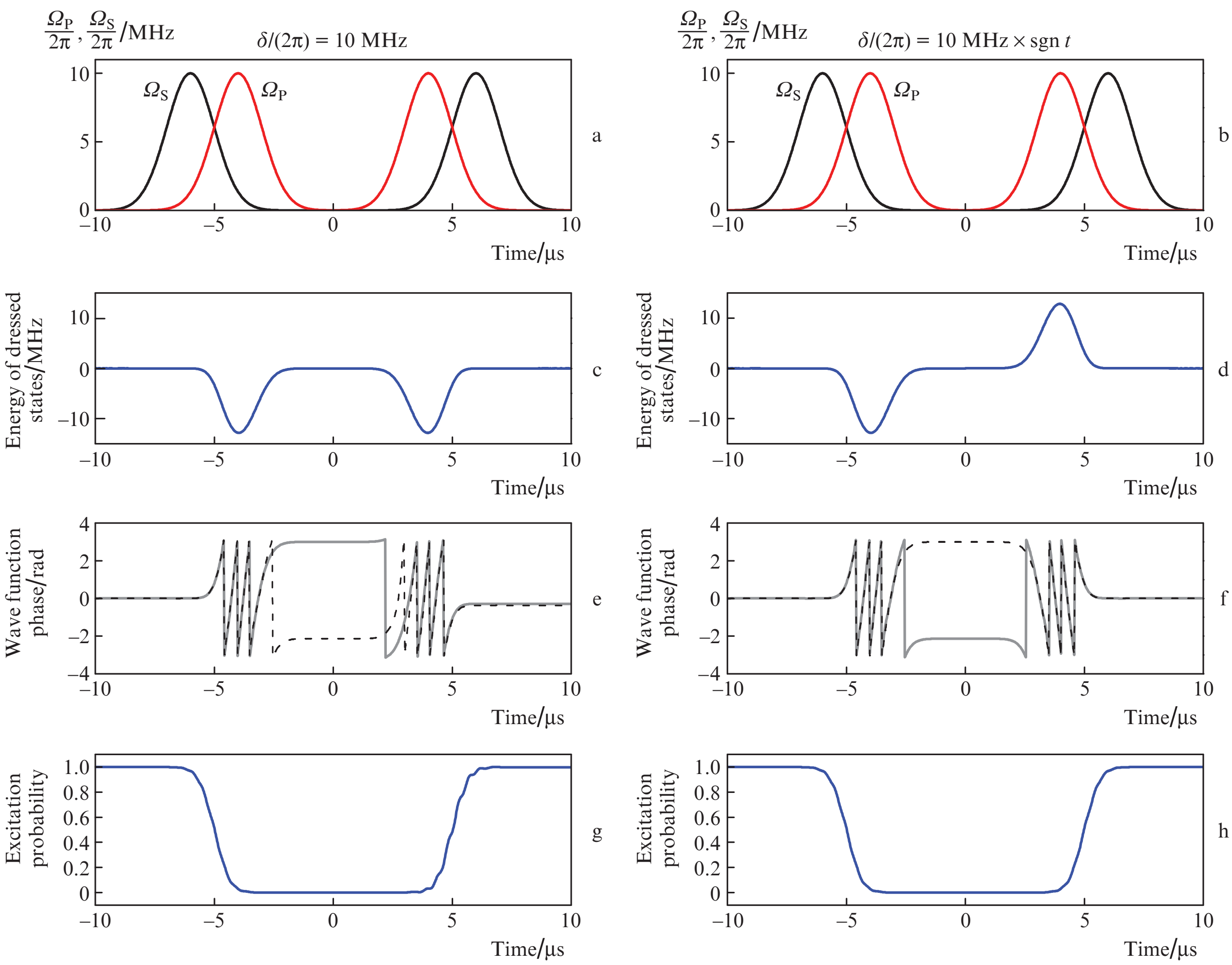

Figure 3. (a, b) Sequences of laser pulses in the case of double adiabatic passage, (c, d) time dependences of the Hamiltonian eigenvalues for the system of two interacting atoms in the dressed state, corresponding to the initial ground state of the system $|gg\rangle$, (e, f) numerically calculated time dependence of the phase of the collective state $|gg\rangle$ (solid curves) and the results of calculations in the adiabatic approximation (dashed curves), as well as (g, h) numerically calculated time dependences of the population of the ground state $|gg\rangle$ for two detunings from the intermediate state $\delta/(2\pi)$.

batic passage. This phase is sensitive to the Rabi frequencies and the number of atoms in the ensemble, which is undesirable for quantum informatics with mesoscopic atom ensembles. In our previous papers we have found that the switching of the detuning sign from the resonance with the intermediate state between two adiabatic sequences allows the elimination of this undesirable phase incursion [18–20]. This can be explained in the following way: let us consider the double adiabatic sequence of pulses in the interval $(-T, T')$ at $T = 10$ μs (Figs 3a and 3b) with the Rabi frequencies

$$\Omega_S(t) = \Omega_0 \exp[(t-t_1)^2/(2w^2)] + \Omega_0 \exp[(t+t_1)^2/(2w^2)],$$
$$\Omega_P(t) = \Omega_0 \exp[(t-t_2)^2/(2w^2)] + \Omega_0 \exp[(t+t_2)^2/(2w^2)], \quad (4)$$

where $\Omega_0/(2\pi) = 10$ MHz; $t_1 = -6$ μs; and $t_2 = -4$ μs. The detuning from the intermediate excited state $\delta(t)/(2\pi)$ is constant for the left-hand column in Fig. 3 and amounts to 10 MHz. For the right-hand column in Fig. 3 the sign of the detuning is changed for the opposite between two sequences: $\delta(t)/(2\pi) = 10\ \text{MHz} \times \text{sgn}\, t$. The numerically calculated energy eigenvalue $E(t)/(2\pi\hbar)$ for Hamiltonian (1), corresponding to the initial state of the system $|gg\rangle$, is shown in Fig. 3c (the detuning is constant) and Fig. 3d (the detuning changes the sign). The numerically calculated time dependences of the phase of the ground state $|gg\rangle$ and the phase in the adiabatic approximation

$$\arg\left[\exp\left(-\frac{i}{\hbar}\int_{-T}^{t} E(t')\,dt'\right)\right]$$

are shown in Figs 3e and 3f. Good agreement with the adiabatic approximation is observed in the region, where the probability amplitude of populating the ground state is different from zero. The dynamics of populations, presented in Figs 3g and 3h, does not change with the switching of the detuning sign.

Under the double fast adiabatic excitation using the chirped laser pulses (Fig. 4) we have found two regimes of deterministic accumulation of the dynamic phase. If the sign of the Rabi frequency is unchanged (the left-hand column of Fig. 4), then the phase shift is also deterministic and equals π. In the case of switching the Rabi frequency sign during the interval between the pulses (the right-hand column of Fig. 4), the dynamic phase is compensated for, as in the case of two-photon fast adiabatic passage with the change of the detuning sign from resonance. We considered the phase dynamics of a

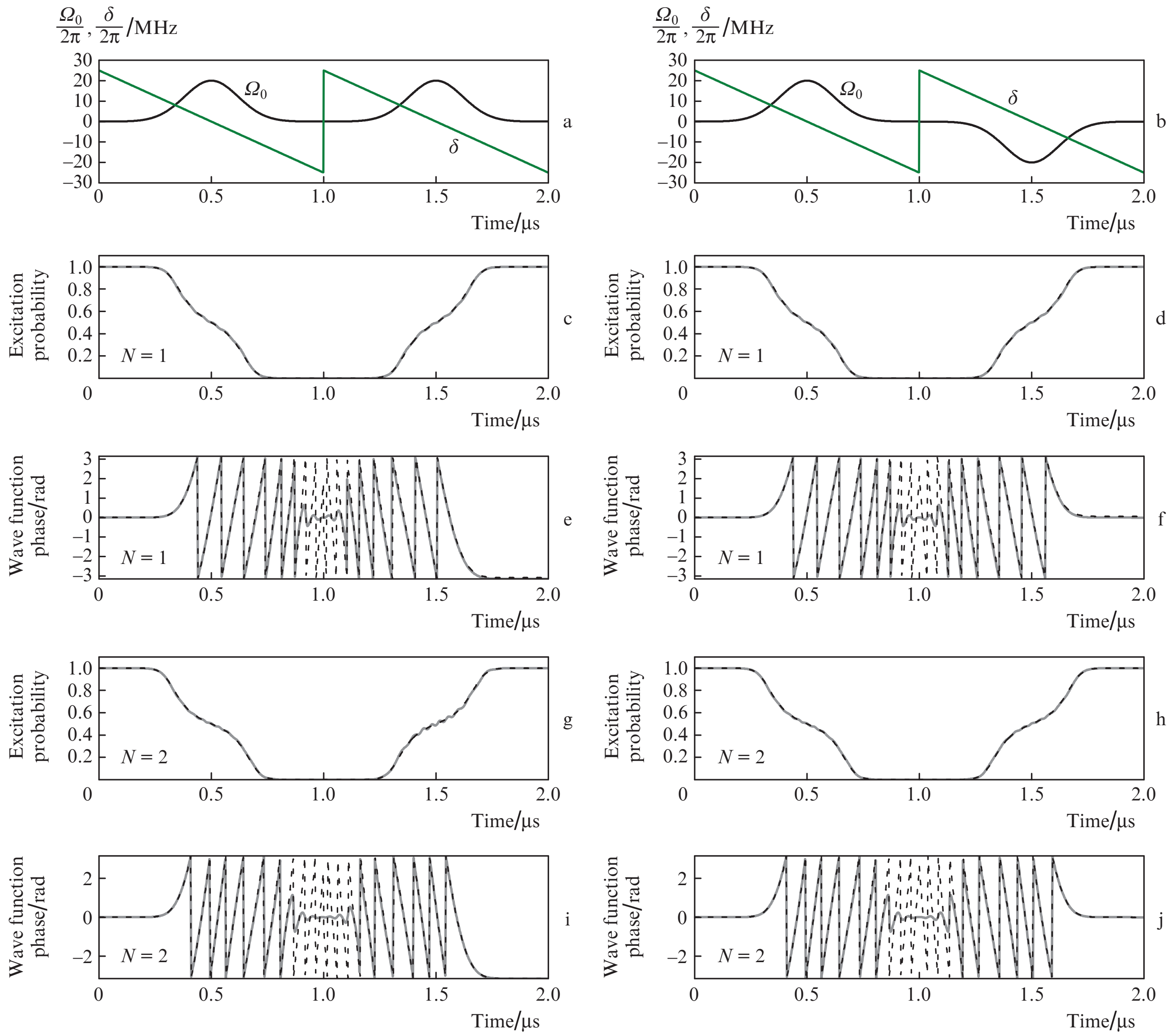

Figure 4. (a, b) Time dependences of the Rabi frequency Ω_0 and detuning δ, numerically calculated time dependences of (c, d) the population and (e, f) phase of the probability amplitude of the ground state $|g\rangle$ (solid curves) in comparison with the results of calculation in the adiabatic approximation (dashed curves), as well as the numerically calculated time dependences of (g, h) the population and (i, j) the phase of the probability amplitude of the ground collective state of the system of two interacting atoms in the regime of dipole blockade $|gg\rangle$ (solid curves) in comparison with the results of calculation in the adiabatic approximation (dashed curves) at the phase shift, equal to π for the left-hand column and zero for the right-hand column.

two-level system in Ref. [21]. The Hamiltonian of the two-level atom interacting with the chirped laser pulse can be written in the form [30, 32]:

$$\hat{H}_{\rm chirp}(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_0(t) \\ \Omega_0(t) & 2\delta(t) \end{pmatrix}. \qquad (5)$$

Its eigenvalues are

$$\frac{\hbar}{2}\Omega_{\pm} = \frac{\hbar}{2}[\delta(t) \pm \sqrt{\Omega_0^2(t) + \delta^2(t)}],$$

and the eigenstates are the semiclassical 'dressed' states

$$|I(t)\rangle = \cos\theta(t)|g\rangle - \sin\theta(t)|r\rangle,$$
$$|II(t)\rangle = \sin\theta(t)|g\rangle + \cos\theta(t)|r\rangle. \qquad (6)$$

Here $\theta(t)$ is the time-dependent mixing angle, for which the condition $\tan[2\theta(t)] = \Omega_0(t)/\delta(t)$ holds. The wave function of the atom in the basis of dressed states can be presented as the superposition $|\psi\rangle = \tilde{c}_1(t)|I(t)\rangle + \tilde{c}_2(t)|II(t)\rangle$, and the Hamiltonian has the diagonal form:

$$\hat{H}_{\rm d}(t) = \frac{\hbar}{2}\begin{pmatrix} \Omega_-(t) & 0 \\ 0 & \Omega_+(t) \end{pmatrix}. \qquad (7)$$

The solution of the Schrödinger equation $i\hbar\dot{\tilde{\boldsymbol{c}}} = \hat{H}_{\rm d}\tilde{\boldsymbol{c}}$ yields

$$\tilde{c}_1(t) = \tilde{c}_1(0)\exp\left[-\frac{i}{2}\int_0^t \Omega_-(t)\,dt\right],$$
$$\tilde{c}_2(t) = \tilde{c}_2(0)\exp\left[-\frac{i}{2}\int_0^t \Omega_+(t)\,dt\right]. \qquad (8)$$

Consider the double adiabatic sequence of pulses beginning at $t = 0$. The initial positive detuning from the resonance $\delta(t = 0) > 0$ corresponds to $\theta(t = 0) = 0$. The system is initially in the state $|g\rangle$, which corresponds to the dressed state $|I(t)\rangle$ with the probability amplitude $\tilde{c}_1(t = 0) = 1$. After the first adiabatic passage we have $\delta(T) < 0$ and $\theta(T) = \pi/2$, and the atom is excited to the state $-|r\rangle$, the accumulated dynamic phase of the state $|I(t)\rangle$ being

$$-\frac{1}{2}\int_0^t \Omega_-(t)\,\mathrm{d}t.$$

For the second adiabatic sequence, let us use a prime with the mixing angle. The initial conditions for this sequence are such: the detuning from resonance is positive, $\delta(t = T) > 0$ and, therefore, the mixing angle $\theta'(T) = 0$. The system is initially in the state $|2\rangle$, which corresponds to the dressed state $|II(t)\rangle$. After the second adiabatic sequence the mixing angle becomes $\theta'(2T) = \pi/2$. For the conditions of Fig. 3a the dynamic phase accumulated during both sequences is

$$-\frac{1}{2}\left(\int_0^T \Omega(t)dt + \int_T^{2T} \Omega_+(t)dt\right) = 0.$$

Keeping in mind the sign of the state $-|r\rangle$, excited after the first adiabatic passage, we get the final state of the atom $-|g\rangle$. This corresponds to the return of the atom into the initial state with the accumulated phase shift π.

Switching the sign of the Rabi frequency will correspond to such a mixing angle that $\tan[2\theta'(t)] = -\Omega_0(t)/\delta(t)$. At the end of the second pulse the mixing angle is $\theta'(2T) = -\pi/2$. Keeping in mind the sign of the state $-|r\rangle$ excited after the first pulse, we obtain the final state of the system $|g\rangle$. Thus, the atom returns to the initial state without a phase shift.

To illustrate the above model, we calculated numerically the time dependence of the probability amplitudes in the case of a two-level atom, interacting with two chirped laser pulses having the Rabi frequencies $\Omega_j(t) = \Omega_0\exp[-(t - t_j)^2/(2w^2)]$ and the detunings $\delta_j(t) = s_1(t - t_j)$, where $j = 1, 2$ (Fig. 4a). The peak Rabi frequency is $\Omega_0/(2\pi) = 20$ MHZ, the detuning slope is $s_1/(2\pi) = -50$ MHz μs^{-1}, and the pulse width is $w = 0.12$ μs. The intensity maxima are achieved at the time moments $t_1 = 0.5$ μs and $t_2 = 1.5$ μs. In Fig. 4b the conditions are similar, but the sign of the Rabi frequency is opposite. The time dependences of the probability P_1 to find the atom in the state $|g\rangle$ calculated numerically are shown in Figs 4c and 4d together with the adiabatic dependence of $\cos^2\theta(t)$. Figures 4e and 4f present the numerically calculated phases of the probability amplitudes of the state $|g\rangle$ in comparison with the adiabatic approximation. Good agreement of results is observed in the domain, where the probability amplitudes are nonzero.

Now let us proceed to the phase dynamics in the case of a fast adiabatic passage of the resonance in the ensemble of two interacting Rydberg atoms in the dipole blockade regime. The Hamiltonian for two two-level atoms interacting with the chirped laser pulse in the dipole blockade regime can be written in the form:

$$\hat{H}_{2\mathrm{chirp}}(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_0(t) & \Omega_0(t) \\ \Omega_0(t) & 2\delta(t) & 0 \\ \Omega_0(t) & 0 & 2\delta(t) \end{pmatrix}. \quad (9)$$

Here we consider the collective states $|gg\rangle$, $|gr\rangle$, $|rg\rangle$, and exclude the state $|rr\rangle$ due to the dipole blockade. The eigenvalues of such a Hamiltonian are

$$\frac{\hbar}{2}\left[\delta(t) \pm \sqrt{2\Omega_0^2(t) + \delta^2(t)}\right] \text{ и } \hbar\delta(t).$$

The initial state $|gg\rangle$ corresponds to the eigenvalue

$$\frac{\hbar}{2}\left[\delta(t) - \sqrt{2\Omega_0^2(t) + \delta^2(t)}\right].$$

Thus, the dynamics of collective states of two interacting atoms in the dipole blockade regime is reduced to the dynamics of a two-level system described above by the replacement $\Omega_0(t) \to \Omega_0(t)\sqrt{2}$. The results of numerical calculation of the probability to find the system in the collective state $|gg\rangle$ and the phase of the probability amplitude, as well as the results of calculations in the adiabatic approximation are presented in Fig. 4.

3. Quantum gates

The schemes of quantum gates based on the adiabatic passage of resonances with the conservation of phase were proposed and studied in our previous papers [18–21]. The quantum gates with qubits based on mesoscopic atom ensembles are schematically shown in Figs 5a and 5b. The collective states of the atom ensemble $|\bar{0}\rangle = |00\ldots0\rangle$ and

$$|\bar{1}\rangle = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}|00\ldots1_j\ldots0\rangle$$

are considered as the qubit logical levels, where the states $|0\rangle$ and $|1\rangle$ are the hyperfine sublevels of rubidium or caesium atoms, and N is the unknown number of atoms, randomly captured in the optical dipole trap. The ensemble is initially prepared in the state $|\bar{0}\rangle$. In the course of interaction with the laser radiation tuned to the transition $|0\rangle \to |r\rangle$ in a single atom, only the symmetric collective state can be excited due to the dipole blockade with one Rydberg excitation

$$|\bar{r}\rangle = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}|00\ldots r_j\ldots0\rangle.$$

Then the state $|\bar{r}\rangle$ can be transferred into the state $|\bar{1}\rangle$ by one laser π pulse, acting on the transition $|r\rangle \to |1\rangle$.

The frequency of collective Rabi oscillations for the transition $|\bar{0}\rangle \to |\bar{r}\rangle$ in the regime of dipole blockade amounts to $\Omega_1\sqrt{N}$, where Ω_1 is the Rabi frequency of the transition $|0\rangle \to |r\rangle$ in a single atom. If the number of atoms in the ensemble is not exactly known due to their random loading into the optical dipole traps, the frequency of the collective Rabi oscillations becomes indefinite and the exact quantum gates with atom ensembles cannot be implemented. We proposed to overcome this difficulty using the adiabatic passage in the regime of dipole blockade. The schematic diagram of a single-qubit gate is presented in Fig. 5a. Two auxiliary Rydberg states $|r_0\rangle$ and $|r_1\rangle$ are used. The phase-conserving adiabatic sequences, providing the excitation and deexcitation of the collective state

$$|\bar{r}_0\rangle = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}|00\ldots r_{0j}\ldots0\rangle$$

with switching the sign of the detuning from the intermediate state in the case of two-photon excitation or the sign of the Rabi frequency in the case of one-photon excitation, are denoted as π_+ and π_- sequences, where the sign '+' or '–' cor-

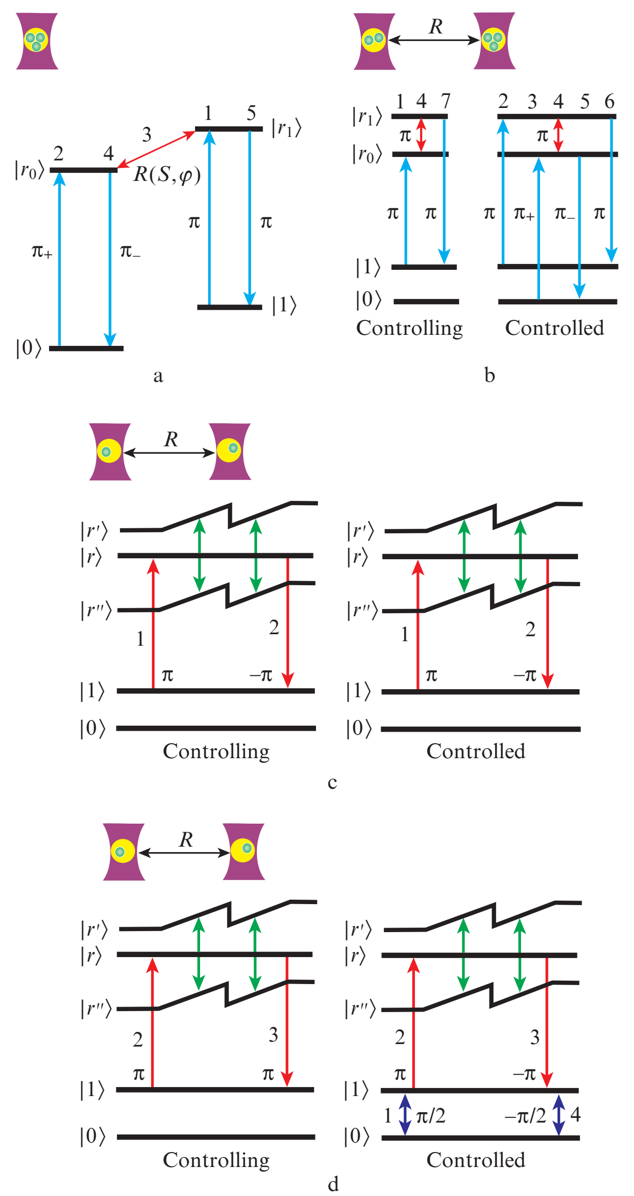

Figure 5. Schematics of quantum gates based on double adiabatic passage of a phase-conserving resonance: (a) single-qubit rotation in a mesoscopic atomic ensemble, (b) two-qubit CNOT gate with two mesoscopic ensembles considered as qubits, (c) two-qubit CZ gate with two atoms based on adiabatic passage of the Förster resonance in interacting Rydberg atoms and (d) two-qubit CNOT gate with tw atoms based on adiabatic passage of the Förster resonance. Figures 1–7 correspond to the numbers of incident pulses.

responds to the sign of detuning or Rabi frequency. The area S and the phase φ for the single-qubit rotation are determined by the parameters of the microwave transition between the auxiliary Rydberg states $|r_0\rangle$ and $|r_1\rangle$. The laser π-pulses at the transition $|r_1\rangle \leftrightarrow |1\rangle$ are used to record the quantum information in the long-lived hyperfine sublevel of the ground state.

The two-qubit operation 'controlled NOT' (CNOT) is schematically presented in Fig. 5b. Two mesoscopic ensembles are separated by such a distance from each other that due to the dipole blockade the excitation of only one Rydberg atom in the entire system is possible. The sequence of pulses acting on the controlled qubit inverts its state, if in the controlling ensemble no Rydberg atoms have been excited. Otherwise, the excitation of Rydberg states in the controlled ensemble will be blocked and no inversion will occur.

Unlike the schemes using the dipole blockade effect, the quantum operations based on phase accumulation due to dipole–dipole interaction of Rydberg atoms do not require large interaction energies. At the same time, these schemes are sensitive to the fluctuations of interatomic separation that determines the interaction energy. We proposed a method to reduce this sensitivity essentially at the expense of the adiabatic passage of Förster resonances with Stark tuning for the interacting Rydberg atoms. The schemes of two-qubit operations 'controlled phase shift' (CZ) with single cold atoms are presented in Figs 5c and 5d. Two atoms are captured in the optical dipole traps separated from each other by the distance R. The atoms are simultaneously excited to the Rydberg state $|r\rangle$ by the laser π-pulse, denoted by figure 1. Due to the Stark effect, the time-dependent electric field shifts the collective energy levels so that the system passes twice the Förster resonance ($|rr\rangle \to |r'r''\rangle$, the condition of resonance being the equality of energies of the collective levels $|rr\rangle$ and $|r'r''\rangle$, where $|r'\rangle$ and $|r''\rangle$ are the adjacent Rydberg states). Due to the resonance dipole–dipole interaction, the system transits from the state $|rr\rangle$ to the state $|r'r''\rangle$. After the end of the Förster resonance adiabatic passage, the atoms are returned to the initial state by the laser π-pulse 2, having the phase shift π with respect to pulse 1. Due to the dipole–dipole interaction, the system acquires the deterministic phase shift, but only in the case when both atoms were initially prepared in the state $|1\rangle$ and then excited to the Rydberg state. To implement the two-qubit CNOT gate, shown in Fig. 5d, we completed the scheme with two rotations of the controlled qubit through the angle $\pi/2$ about the y axis of the Bloch sphere in the opposite directions (which is implemented by means of the pulses 1 and 4 in Fig. 5d). The sequence of pulses acting on the controlled qubit leaves its state unchanged, if the controlling qubit was not excited to the Rydberg state, and the dipole–dipole interaction did not lead to the phase shift. Otherwise, the state of the controlled qubit will be inverted, which is required to implement the CNOT gate.

4. Conclusions

Thus, we have studied double one-photon and two-photon adiabatic sequences of controlling laser pulses, conserving the phase both for one atom, interacting with the laser radiation, and two Rydberg atoms in the regime of dipole blockade. It has been shown that the switching of the sign of the detuning from resonance with the intermediate state leads to the compensation of the dynamical phase in the double two-photon adiabatic passage of the resonance because of the change in the Hamiltonian eigenvalues. We have also shown that the fast adiabatic passage of resonance under the conditions of dipole blockade in the ensemble of two interacting atoms can be reduced to the dynamic of one two-level system. The obtained results can be used to implement quantum computing with registers of qubits, represented by mesoscopic ensembles with unknown number of atoms, which arise in the process of loading optical dipole traps and lattices.

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