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Kinetics of atoms in a bichromatic field formed by elliptically polarised waves

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Abstract. **The kinetics of atoms in a bichromatic field formed by waves with an arbitrary elliptical polarisation is considered. Analytical expressions are derived for the force and the friction and diffusion coefficients within the model of an atom with the $j_g = 1/2 \to j_e = 3/2$ optical transition. These expressions are presented in the general form as an expansion in gradients of light field parameters. Some specific features of the kinetics of atoms are found, which are due to the interference contributions to the friction and diffusion coefficients. The kinetics of atoms in the field of an optical lattice formed as a result of the interference of a wave with a uniform elliptical polarisation and a nonuniformly polarised dissipative field is analysed.**

Keywords: *elliptically polarised waves, kinetics of atoms, bichromatic field.*

1. Introduction

Laser cooling and trapping of neutral atoms is one of the most rapidly developing lines of research in atomic physics. Among numerous works in this field, the analysis of the kinetics of atoms in optical lattices (i.e., intense light fields forming a deep optical potential for trapping neutral atoms) can be selected. To date, optical lattices have become a basis for different experiments with ultracold atoms [1]. The studies related to quantum calculations [2–8], where ultracold atoms are used as objects for quantum information storage and processing, are being intensively developed.

Generally, when monochromatic light fields are used, strong localisation of atoms by laser cooling is a nontrivial problem. Since the optical potential and dissipative processes are determined by the same light field, some limitations arise, which hinder minimisation of the localisation sizes and atomic cooling temperature. In this case, e.g., upon sub-Doppler cooling in the lin⊥lin light field configuration (formed by two mutually orthogonal linearly polarised fields) and at optimal field parameters, the fraction of atoms not trapped into the optical potential is rather high: ~0.2 [9]. Indeed, superdeep cooling is obtained in resonance fields of relatively low intensity, in which the optical potential depth is also reduced [10, 11]. To attain a large fraction of atoms captured into an optical potential with a high degree of localisation in the latter, one can use an additional (second) field with a frequency significantly differing from that of the first field.

The first theoretical studies of the kinetics of atoms in the presence of two monochromatic fields were performed in [12, 13]. The works on the cooling in the Λ scheme [14–19] and cascade schemes [20–22] should also be noted; however, in these schemes, different light fields act on different optical transitions of an atom. In contrast, we will consider (as in [12, 13]) the case where both fields excite the same optical transition.

One of the ways in which the technique of laser cooling in multifrequency fields is developed is the resolved-sideband cooling [23–25], where atoms are cooled at vibrational sublevels in a deep optical potential, being excited by an additional light field. However, the theoretical consideration of this problem is generally performed with significant simplifications, within which the influence of a strong field on the kinetics of atoms is reduced to only the formation of optical shifts of sublevels (optical potentials) by this field and the induction of two-photon transitions between the sublevels, whereas the influence of the second field is reduced to only the control of the relaxation of both Zeeman and vibrational levels of the optical lattice [26, 27]. Generally, it is important to take into account the influence of both fields on the kinetics of atoms. For example, fluctuations of induced absorption and emission of nondissipative field photons may cause dipole force fluctuations and, therefore, significantly affect the diffusion of atoms in the optical potential and the kinetics of atoms as a whole.

We considered previously [28] the kinetics of atoms in a bichromatic field by an example of a model of a two-level atom. Analytical expressions for the force and the friction and diffusion coefficients were obtained, which allowed us to reveal a number of new effects in the kinetics of atoms. In particular, we found a strong localisation effect, caused by the interference contributions to the friction and diffusion coefficients.

In this study, we consider the kinetics of atoms in the framework of a two-level model with levels degenerate in the angular momentum projection in a bichromatic field. This model suggests low intensities of light waves, at which the shifts of light levels and the influence of other levels in the system can be neglected. This statement of the problem makes it possible to take into account the polarisation aspect of the interaction of atoms with the bichromatic field. It is well known that the degeneracy of atomic levels in the angular momentum projection in monochromatic light fields leads to the occurrence of the so-called sub-Doppler contributions to the friction force [29]. It is shown below that these mechanisms also occur in a

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bichromatic light field; moreover, they are accompanied by many new effects due to the interference contributions to the friction and diffusion coefficients.

2. Statement of the problem

Let us consider an atom with the closed optical transition $j_g \to j_e$ in a bichromatic field

$$\boldsymbol{E}(z,t) = \boldsymbol{E}_1(z)\exp(-\mathrm{i}\omega_1 t) + \boldsymbol{E}_2(z)\exp(-\mathrm{i}\omega_2 t) + \text{c.c.} \quad (1)$$

with frequencies close to the atomic resonance ω_0, i.e., with detunings $\delta_1 = \omega_1 - \omega_0$ and $\delta_2 = \omega_2 - \omega_0$, at which $|\delta_1|$ and $|\delta_2| \ll \omega_0$. The fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ have close wave vectors $k_2 \approx k_1 = k$; the relative spatial phase of the fields is a slowly varying function of coordinates:

$$\phi = \delta k z, \quad (2)$$

where $\delta k = k_2 - k_1$. The vector amplitudes of the fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ in the circular basis ($\boldsymbol{e}_\pm = \mp(\boldsymbol{e}_x \pm \mathrm{i}\boldsymbol{e}_y)/\sqrt{2}$),

$$\boldsymbol{E}_\kappa = E_\kappa^+ \boldsymbol{e}_+ + E_\kappa^- \boldsymbol{e}_-, \quad (3)$$

where $\kappa = 1,2$, have components E_κ^+ and E_κ^- for the fields with $\kappa = 1$ and 2, respectively.

Omitting the rapidly oscillating contributions ($\sim\exp(-\mathrm{i}2\omega_1)$, $\exp(-\mathrm{i}2\omega_2)$, and $\exp[-\mathrm{i}(\omega_1 \pm \omega_2)]$) and performing reduction in fields with small saturation parameters $S_\kappa = |\Omega_\kappa|^2/(\delta_\kappa^2 + \gamma^2/4) \ll 1$ ($\Omega_\kappa = -E_\kappa \hat{d}_{\mathrm{eg}}/\hbar$ is the Rabi frequency, $\hat{d}_{\mathrm{eg}}$ is the dipole moment of the atomic transition, and γ is the natural width), we obtain the following system of equations in the coordinate representation for the components of the atomic density matrix of the ground state in the coordinate representation $\hat{\rho}(z_1, z_2)$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(z_1,z_2) = \hat{\gamma}\{\hat{\rho}(z_1,z_2)\} - \sum_{\kappa=1,2} \frac{\mathrm{i}\delta_\kappa}{\gamma^2/2 + \delta_\kappa^2} \times[\hat{V}_\kappa^\dagger(z_1)\hat{V}_\kappa(z_1)\hat{\rho}(z_1,z_2) - \hat{\rho}(z_1,z_2)\hat{V}_\kappa^\dagger(z_2)\hat{V}_\kappa(z_2)]. \quad (4)$$

Here, the operator $\mathrm{d}/\mathrm{d}t = \partial_t - \mathrm{i}\hbar/(M\partial_q\partial_z)$; $z = (z_2 + z_1)/2$; $q = z_1 - z_2$; and M is the atomic mass. The operator $\hat{\gamma}\{\hat{\rho}(z_1,z_2)\}$ describes the density matrix relaxation as a result of spontaneous emission of field photons, with allowance for the recoil effects (see, e.g., [10]). The operators of interaction with the field,

$$\hat{V}_\kappa = \Omega_\kappa \sum_{s=\pm 1} \hat{T}_s E_\kappa^s, \quad \hat{T}_s = C^{j_e,\mu}_{j_g,m;1,s}|j_e,\mu\rangle\langle j_g,m|, \quad (5)$$

are expressed in terms of the Clebsch–Gordan coefficients $C^{j_e,\mu}_{j_g,m;1,s}$. The solution of Eqn (4) is a nontrivial problem. The steady-state solution for the density matrix $\hat{\rho}(z_1,z_2)$ can be sought for based on the numerical methods proposed by us in [10, 11]. However, it is convenient to use the quasi-classical approach in order to perform a qualitative analysis and understand the effects of kinetics of neutral atoms in a bichromatic field. For example, within the quasi-classical approximation, at a small recoil parameter $\varepsilon_{\mathrm{rec}}$ [$\varepsilon_{\mathrm{rec}} = \omega_{\mathrm{rec}}/\gamma \ll 1$; $\hbar\omega_{\mathrm{rec}} = \hbar^2k^2/(2M)$ is the recoil energy], Eqn (4) for atoms with a sufficiently wide momentum distribution ($\Delta p \gg \hbar k$) can be reduced to the Fokker–Planck equation.

The expansion of the kinetic equation for the Wigner density matrix $\hat{\rho}(z,p)$ in the recoil parameter $\hbar k/\Delta p$ is equivalent to the expansion in powers of the parameter $-\mathrm{i}kq$ of the kinetic equation for the density matrix in the coordinate representation $\hat{\rho}(z_1,z_2)$ [30]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = \hat{\mathcal{L}}^{(0)}\{\hat{\rho}\} - \mathrm{i}kq\hat{\mathcal{L}}^{(1)} + (-\mathrm{i}kq)^2\hat{\mathcal{L}}^{(2)}\{\hat{\rho}\}\ldots. \quad (6)$$

Similar to the method reported by us in [30], Eqn (6) can be reduced to the Fokker–Planck equation for the Wigner distribution function $\mathcal{F}(z,p) = \mathrm{Tr}\{\hat{\rho}(z,p)\}$ with the following coefficients: gradient force (the force acting on an atom in rest), friction coefficient (contribution to the force linear in the atomic velocity) and diffusion coefficient. Then the expression for the force is determined by the first-order terms in expansion (6):

$$F = -\mathrm{Tr}\{\hat{\mathcal{L}}^{(1)}\{\hat{\sigma}\}\}, \quad (7)$$

where $\hat{\sigma}$ is the steady-state solution to the optical Bloch equation $\partial_t\hat{\sigma} = \hat{\mathcal{L}}^{(0)}\{\hat{\sigma}\}$. The expression for force (7) can be written (after simple transformations) as the sum of contributions from each field:

$$F = \hbar\sum_{\kappa=1,2}\mathrm{Tr}\{\hat{F}_\kappa\hat{\sigma}\}, \quad (8)$$

where

$$\hat{F}_\kappa = -\frac{1}{\gamma^2/4 + \delta_\kappa^2}\Big[\mathrm{i}\frac{\gamma}{2}\Big(\hat{V}_\kappa^\dagger\frac{\partial\hat{V}_\kappa}{\partial z} - \frac{\partial\hat{V}_\kappa^\dagger}{\partial z}\hat{V}_\kappa\Big) + \delta_\kappa\Big(\hat{V}_\kappa^\dagger\frac{\partial\hat{V}_\kappa}{\partial z} + \frac{\partial\hat{V}_\kappa^\dagger}{\partial z}\hat{V}_\kappa\Big)\Big] \quad (9)$$

are force operators.

The friction and diffusion coefficients, as in the case of a monochromatic field [30], can be found using an auxiliary matrix $\hat{\varphi}$ – a solution to the linear equation

$$\sum_{\kappa=1,2}\frac{1}{\gamma^2/4 + \delta_\kappa^2}\Big(\frac{\gamma}{2}\{\hat{V}_\kappa^\dagger\hat{V}_\kappa,\hat{\varphi}\} - \mathrm{i}\delta_\kappa[\hat{V}_\kappa^\dagger\hat{V}_\kappa,\hat{\varphi}] - \gamma\sum_s\hat{V}_\kappa^\dagger\hat{T}_s\hat{\varphi}\hat{T}_s^\dagger\hat{V}_\kappa\Big) = \sum_{\kappa=1,2}\frac{\widehat{\delta F_\kappa}}{\hbar}, \quad (10)$$

where $\widehat{\delta F_\kappa} = \hat{F}_\kappa - F$ is the force operator fluctuation. The friction coefficient is proportional to the spatial gradient $\hat{\sigma}$:

$$\xi = -\hbar\mathrm{Tr}\{\hat{\varphi}\partial_z\hat{\sigma}\}. \quad (11)$$

The diffusion coefficient can be presented as

$$D = \hbar[\mathrm{Tr}\{\hat{\mathcal{L}}^{(2)}\{\hat{\sigma}\}\} - \mathrm{Tr}\{\hat{\varphi}\hat{\mathcal{L}}^{(1)}\{\hat{\sigma}\}\} - \mathrm{Tr}\{\hat{\varphi},\hat{\sigma}\}F]. \quad (12)$$

It contains contributions from the spontaneous diffusion, which is the result of atomic momentum fluctuation during the spontaneous emission of photons, and from the induced diffusion, which occurs due to the fluctuation of light-pressure forces.

3. Results

As an example, we will consider the atoms with the $j_g = 1/2 \to j_e = 3/2$ optical transition in a bichromatic field. In the one-dimensional geometry, fields $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ have different intensities and, in the general case, elliptical polarisations, depending on the longitudinal coordinate z. The vector amplitude of each of the fields $\boldsymbol{E}_k$ is determined by four parameters: scalar amplitude $E_k = |\boldsymbol{E}_k|$, phase Φ_k, local ellipticity ε_κ and orientation angle ψ_κ of the polarisation ellipse with respect to the x axis. In the circular basis

$$\boldsymbol{E}_\kappa = E_\kappa^+ \boldsymbol{e}_+ + E_\kappa^- \boldsymbol{e}_-, \tag{13}$$

the circular components E_κ^+ and E_κ^- of the fields can be written as

$$E_\kappa^\pm = \mp E_\kappa \cos(\varepsilon_\kappa \mp \pi/4)\exp(\mathrm{i}\Phi_\kappa)\exp(\mp \mathrm{i}\psi_\kappa). \tag{14}$$

The steady-state solution for the density matrix of the ground state in zero order in the recoil parameter has the form

$$\hat{\sigma}_{\mp 1/2,\mp 1/2} = \frac{S \mp [S_1 \sin(2\varepsilon_1) + S_2 \sin(2\varepsilon_2)]}{2S}. \tag{15}$$

Here, S_κ are the saturation parameters, determined by the local values of light field amplitudes; and $S = S_1 + S_2$ is the total saturation parameter.

It is convenient to present the expressions for the force and the friction and induced diffusion coefficients in the form of expansions in gradients of parameters of the fields E_1 and E_2, specifically, in $\nabla_z \Lambda_\kappa = \nabla_z \ln E_\kappa$, $\nabla_z \varepsilon_\kappa$, $\nabla_z \Phi_\kappa$ and $\nabla_z \psi_\kappa$. For example, the force F acting on an immobile atom can be written as the sum

$$F = \sum_\kappa \sum_\beta f_{\beta_\kappa} \nabla_z \beta_\kappa, \tag{16}$$

where the expansion coefficients f_{β_κ} are expressed in terms of the local field parameters:

$$\begin{aligned}
f_{\Lambda_1} &= -\frac{2}{3}\frac{S_1\delta_1}{S}[2S + S_1 \sin^2(2\varepsilon_1) + S_2 \sin(2\varepsilon_1)\sin(2\varepsilon_2)],\\
f_{\varepsilon_1} &= -\frac{2}{3}\frac{S_1\delta_1}{S}\cos(2\varepsilon_1)[S_1 \sin(2\varepsilon_1) + S_2 \sin(2\varepsilon_2)],\\
f_{\Phi_1} &= \frac{\gamma}{3}\frac{S_1}{S}[2S + S_1 \sin^2(2\varepsilon_1) + S_2 \sin(2\varepsilon_1)\sin(2\varepsilon_2)],\\
f_{\psi_1} &= -\frac{\gamma}{3}\frac{S_1}{S}[3S_1 \sin(2\varepsilon_1) + S_2 \sin(2\varepsilon_2) + 2S_2 \sin(2\varepsilon_1)].
\end{aligned} \tag{17}$$

The coefficients f_{β_κ} for the second field can be obtained by replacing the subscripts $\kappa = 1$ with $\kappa = 2$ and vice versa. The friction and induced diffusion coefficients are also expanded in contributions proportional to quadratic combinations of gradients of light field parameters:

$$\xi = \hbar \sum_{\kappa,\kappa'} \sum_{\beta,\beta'} \chi_{\beta_\kappa \beta'_{\kappa'}} \nabla_z \beta_\kappa \nabla_z \beta'_{\kappa'}, , \tag{18}$$

$$D^{(i)} = \gamma\hbar \sum_{\kappa,\kappa'} \sum_{\beta,\beta'} \mathcal{D}_{\beta_\kappa \beta'_{\kappa'}} \nabla_z \beta_\kappa \nabla_z \beta'_{\kappa'}. \tag{19}$$

Since the expressions for $\chi_{\beta\beta'}$ and $\mathcal{D}_{\beta\beta'}$ are rather cumbersome, we omit them. Note that in the limit under consideration, $S_\kappa \ll 1$, the friction coefficient is nonzero both in fields with spatially inhomogeneous polarisation and in fields with spatially inhomogeneous intensity (for elliptically polarised fields), in contrast to the case of a monochromatic field, where a necessary condition is the presence of spatial inhomogeneity of the field polarisation (ellipticity, polarisation vector orientation).

The spontaneous-diffusion coefficient can be explicitly selected from (12):

$$\begin{aligned}
D^{(\mathrm{s})} &= \hbar^2 k^2 \sum_\kappa \mathrm{Tr}\Big\{\frac{\gamma}{\gamma^2/4 + \delta_\kappa^2} \sum_{s=0,\pm 1} K_s \hat{T}_s^\dagger \hat{V}_\kappa \hat{\sigma} \hat{V}_\kappa^\dagger \hat{T}_s\Big\}\\
&= \frac{\hbar^2 k^2}{5S}\Big[\Big(1 - \frac{7}{18}\cos^2(2\varepsilon_1)\Big)S_1^2 + \Big(\frac{11}{9} + \frac{7}{9}\sin(2\varepsilon_1)\sin(2\varepsilon_2)\Big)S_1 S_2\\
&\quad + \Big(1 - \frac{7}{18}\cos^2(2\varepsilon_2)\Big)S_2^2\Big].
\end{aligned} \tag{20}$$

Note that, in the absence of one of the fields, the expressions for the force and the friction and diffusion coefficients take the form known for the atoms with the $j_g = 1/2 \to j_e = 2/3$ optical transition in a monochromatic field [29, 31].

3.1. Examples of optical lattices

As a specific example, we will consider an optical lattice formed by a standing-wave field $\boldsymbol{E}_1(z) = 2E_{01}\boldsymbol{e}_1 \cos(kz)$ with a uniform polarisation and large detuning ($|\delta_1| \gg |\delta_2|$), forming a deep optical potential, and a field $E_2(z)$ with a nonuniform polarisation, providing spatially inhomogeneous optical pumping of levels. In other words, we assume that

$$|\delta_1| S_1 \gg |\delta_2| S_2, \;\; S_1 \ll S_2. \tag{21}$$

The standing-wave field is formed by counterpropagating waves with the same amplitude E_{01} and uniform polarisation $\boldsymbol{e}_1 = -\cos(\varepsilon_1 - \pi/4)\boldsymbol{e}_+ + \cos(\varepsilon_1 + \pi/4)\boldsymbol{e}_-$, where the ellipticity parameter ε_1 is independent of coordinate z ($\varepsilon_1 = 0$ corresponds to a linear polarisation of the field and $\varepsilon_1 = \pm\pi/4$ corresponds to circular polarisations). The standing-wave field is characterised by a spatial gradient of only one parameter:

$$\nabla_z \Lambda_1 = -\, k \tan(kz).$$

Correspondingly, $S_1 = 4S_{01}\cos^2(kz)$, where S_{01} is the saturation parameter per one of the counterpropagating waves with an amplitude E_{01}.

The second field (pump field with a nonuniform polarisation) may generally contain all spatial gradients of parameters $\{\Lambda_2, \varepsilon_2, \Phi_2, \psi_2\}$. This field can be written as a sum of two counterpropagating waves with elliptical polarisations ε_{2+} and ε_{2-}:

$$\boldsymbol{E}_2(z) = E_{02}\mathcal{E}\exp(-\mathrm{i}\omega_2 t) + \mathrm{c.c.}, \tag{22}$$

where E_{02} is the amplitude of each of the waves; the complex vector $\mathcal{E} = a_+\boldsymbol{e}_+ + a_-\boldsymbol{e}_-$ with cyclic components $a_+(z)$ and $a_-(z)$ determines the local polarisation ellipse of the light field and the variation in its amplitude along the z axis:

$$a_+ = -\cos(\varepsilon_{2+} - \pi/4)\exp[\mathrm{i}(kz + \phi)] - \cos(\varepsilon_{2-} - \pi/4)\exp[-\mathrm{i}(kz + \phi)]\exp(-\mathrm{i}\theta),$$
$$a_- = \cos(\varepsilon_{2+} - \pi/4)\exp[\mathrm{i}(kz + \phi)] + \cos(\varepsilon_{2-} + \pi/4)\exp[-\mathrm{i}(kz + \phi)]\exp(+\mathrm{i}\theta); \quad (23)$$

θ is the angle between the semiaxes of the polarisation ellipses of counterpropagating waves. For simplicity, we will consider two cases: the lin⊥lin configuration, formed by counterpropagating waves with linear polarisations making an angle of 90° (i.e., $\varepsilon_{2+} = \varepsilon_{2-} = 0$ and $\theta = \pi/2$), and the $\sigma_+ - \sigma_-$ configuration, formed by counterpropagating circularly polarised waves (i.e., $\varepsilon_{2+} = \pi/4$ and $\varepsilon_{2-} = -\pi/4$). These field configurations contain only one nonzero gradient: $\nabla_z \varepsilon_2 = -k$ in the former case and $\nabla_z \psi_2 = -k$ in the latter case. The parameter ϕ in (23) is the relative phase (2), characterising the relative spatial configurations of $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$. Since condition (21) implies that $|\delta_1| \gg |\delta_2|$, the relative phase $\phi = (\delta_2 - \delta_1)cz$ is an increasing function of coordinates at $\delta_1 < 0$ and, correspondingly, a decreasing function at $\delta_1 > 0$.

3.2. Lin⊥lin configuration

The field of the lin⊥lin configuration contains one spatial gradient: $\nabla_z \varepsilon_2 = -k$. The field $\boldsymbol{E}_2$ at each point has an elliptical polarisation with the ellipticity parameter determined by the relation $\sin(2\varepsilon_2) = -\sin(2kz + 2\phi)$. The other parameters (intensity, phase and polarisation ellipse orientation) remain constant. The pump-field saturation parameter at any point is

$$S_2 = 2S_{02},$$

where the saturation parameter S_{02} is determined per each counterpropagating wave with amplitude E_{02}. Under conditions (21), the force is determined by the contribution proportional to the gradient of the lattice field intensity:

$$F = -\hbar\frac{2}{3}S_1\delta_1[2 + \sin(2\varepsilon_1)\sin(2\varepsilon_2)]\nabla_z \Lambda_1. \quad (24)$$

On the scales of the order of wavelength, where the change in the relative field phase ϕ can be neglected, the optical potential takes the form

$$U = \frac{\hbar\delta_1 S_{01}}{3}\left\{4\cos(2kz) + \sin(2\varepsilon_1) \times \left[2kz\cos(2\phi) - \frac{1}{2}\sin(4kz + 2\phi)\right]\right\}. \quad (25)$$

In the general case, the expressions for the optical potential and force differ from the known expressions in the case of the monochromatic field [31]. Nevertheless, the optical potential also has a period of $\lambda/2$, and the positions of local minima and maxima coincide with the positions of nodes and antinodes of the lattice field at blue ($\delta_1 > 0$) and red ($\delta_1 < 0$) detunings. Correspondingly, the additional contribution, proportional to $\sin(2\varepsilon_1)$, leads to the 'rectification' effect in the force, i.e., to a nonzero force after spatial averaging over wavelength:

$$\langle F\rangle = -\frac{\hbar k 2S_{01}\delta_1\sin(2\varepsilon_1)\cos(2\phi)}{3}. \quad (26)$$

Thus, in the case of an elliptically polarised lattice field, this contribution leads to modulation of the optical potential on a macroscopic scale with a period $L = \pi/(\delta k)$, greatly exceeding the wavelength (Fig. 1). Here, the regions of global minima and maxima of the optical potential correspond to the relative phase of the fields $\phi = \pm\pi/4$.

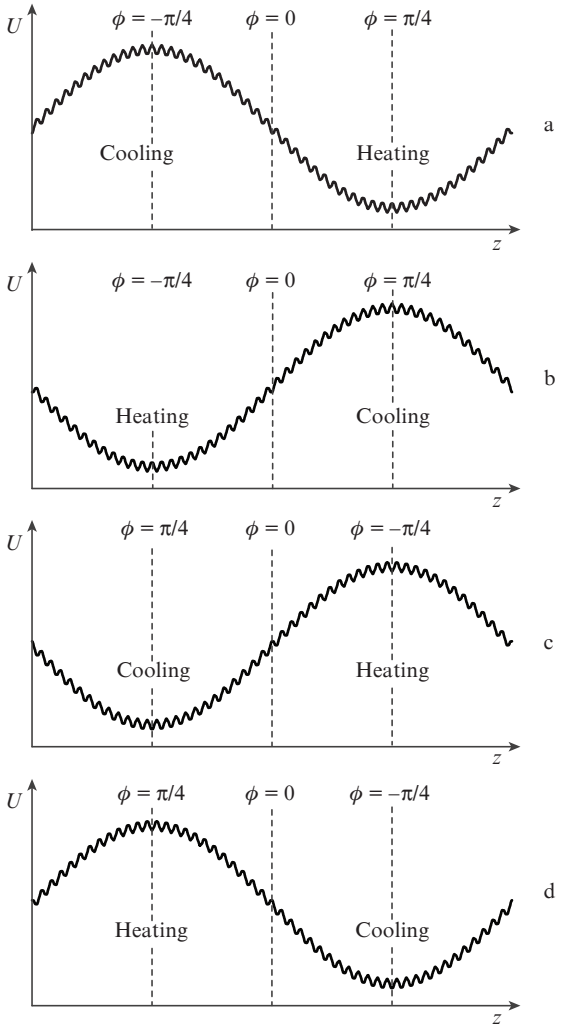

Figure 1. Schematic dependence of the optical potential in the interval $z = [-\pi/(2\delta k)\ldots\pi/(2\delta k)]$ and the positions of the heating and cooling regions for atoms at different signs of lattice field ellipticity and detuning: (a) $\delta_1 < 0$, $\varepsilon_1 > 0$, (b) $\delta_1 < 0$, $\varepsilon_1 < 0$, (c) $\delta_1 > 0$, $\varepsilon_1 > 0$ and (d) $\delta_1 > 0$, $\varepsilon_1 < 0$.

The main contribution to the friction coefficient in the field of the lin⊥lin configuration is determined by the coefficients $\chi_{\varepsilon_2\varepsilon_2}$ and $\chi_{\Lambda_1\varepsilon_2}$; under conditions (21), this contribution takes the form

$$\frac{\xi}{\hbar k^2} = 6\delta_2\cos^2(2\varepsilon_2)\frac{\nabla_z\varepsilon_2\nabla_z\varepsilon_2}{k^2} + 6\frac{\delta_1 S_1}{S_2}\sin(2\varepsilon_1)\cos(2\varepsilon_2) \times\frac{\nabla_z\Lambda_1\nabla_z\varepsilon_2}{k^2} = 6\delta_2\cos^2(2kz + 2\phi) + 6\frac{\delta_{01}S_{01}}{S_{02}} \times\sin(2\varepsilon_1)\sin(2kz)\cos(2kz + 2\phi). \quad (27)$$

The first term is the known expression for the friction coefficient in the monochromatic field of the lin⊥lin configuration [29, 31], while the second term is an additive from the lattice field, which dominates in the case of an elliptically polarised standing wave ($\varepsilon_1 \neq 0$). The friction coefficient averaged over the spatial period,

$$\frac{\langle \xi \rangle}{\hbar k^2} = 3\delta_2 - 3\frac{\delta_1 S_{01}}{S_{02}} \sin(2\varepsilon_1)\sin(2\phi), \tag{28}$$

determines the direction of the kinetic process (heating or cooling). The friction coefficient sign is determined by not only the sign of the 'cooling' field detuning, δ_2, but also the sign of the lattice field detuning, δ_1, as well as the ellipticity ε_1 and relative phase ϕ. The induced-diffusion coefficient, determined by the three main contributions, $\mathcal{D}_{\Lambda_1\Lambda_1}$, $\mathcal{D}_{\varepsilon_2\varepsilon_2}$ and $\mathcal{D}_{\Lambda_1\varepsilon_2} + \mathcal{D}_{\varepsilon_2\Lambda_1}$, under the conditions (21) is reduced to

$$\begin{aligned}
\frac{D^{(i)}}{\hbar^2 k^2 \gamma} &= 2\sin^2(2\varepsilon_1)\cos^2(2\varepsilon_2)\frac{\delta_1^2 S_1^2}{\gamma^2 S_2}\frac{(\nabla_z \Lambda_1)^2}{k^2} \\
&+ 4\frac{\delta_2 \delta_1 S_1}{\gamma^2}\sin(2\varepsilon_1)\cos^3(2\varepsilon_2)\frac{\nabla_z \Lambda_1 \nabla_z \varepsilon_2}{k^2} \\
&+ \left[2\frac{\delta_2^2}{\gamma^2}\cos^4(2\varepsilon_2) + \frac{1+\cos^2(2\varepsilon_2)}{6}\right] S_2 \frac{\partial_z \varepsilon_2 \partial_z \varepsilon_2}{k^2} \\
&= 4\sin^2(2\varepsilon_1)\frac{\delta_1^2 S_{01}^2}{\gamma^2 S_{02}}\cos^2(2kz+2\phi)\sin^2(2kz) \\
&+ 8\sin(2\varepsilon_1)\frac{\delta_2 \delta_1 S_{01}}{\gamma^2}\cos^3(2kz+2\phi)\sin(2kz) \\
&+ \left[4\frac{\delta_2^2}{\gamma^2}\cos^4(2kz+2\phi) + \frac{1+\cos^2(2kz+2\phi)}{3}\right] S_{02}.
\end{aligned} \tag{29}$$

The third term is the known result for the induced-diffusion coefficient in the monochromatic field of the lin⊥lin configuration [29, 31]; however, the first and second terms dominate if the standing-wave field polarisation differs from linear; i.e., $\varepsilon_1 \neq 0$. The induced-diffusion coefficient, averaged over the spatial period, has the form

$$\begin{aligned}
\frac{\langle D^{(i)} \rangle}{\hbar^2 k^2 \gamma} &= \sin^2(2\varepsilon_1)\frac{\delta_1^2 S_{01}^2}{\gamma^2 S_{02}}\left[\frac{3}{2} - \cos^2(2\phi)\right] \\
&+ \frac{S_{02}}{2}\left(3\frac{\delta_2^2}{\gamma^2} + 1\right) - 3\frac{\delta_2 \delta_1 S_{01}}{\gamma^2}\sin(2\varepsilon_1)\sin(2\phi).
\end{aligned} \tag{30}$$

The spontaneous-diffusion coefficient [under conditions (21)] is determined by only the pump field,

$$D^{(s)} = \hbar^2 k^2 \gamma \frac{2S_{02}}{5}\left[1 - -\frac{7}{18}\cos^2(2kz+2\phi)\right], \tag{31}$$

and corresponds exactly to the result for an atom in the monochromatic field of the lin⊥lin configuration.

As follows from the expressions for the force and the friction and diffusion coefficients, the optical potential under conditions (21) is determined by the lattice field $\boldsymbol{E}_1$, whereas the friction and diffusion coefficients are determined by the pump field $\boldsymbol{E}_2$ only in the case of the linearly polarised field $\boldsymbol{E}_1$ ($\varepsilon_1 = 0$). A variation in phase ϕ leads to only a relative shift of the spatial dependences of the optical potential and the friction and diffusion coefficients. Figure 2 shows as an example the results of calculating the stationary distribution of atoms in lattices for different phases, for red lattice field detunings ($\delta_1 < 0$). The calculation was performed based on the numerical solution of the quantum kinetic equation for the atomic density matrix, with complete consideration of the recoil quantum effects, by the method proposed in [10, 11]. In particular, in the vicinity of $\phi = 0$, the optical potential minimum coincides with the minimum for the friction coefficient, which leads to a stronger localisation and lower temperatures upon laser cooling of the atoms in lattices. It should be noted again that, in the case of a linearly polarised lattice field, under conditions (21), a change in the sign of the lattice field detuning δ_1 does not affects much the cooling of the atoms and leads only to the optical potential inversion, which is equivalent to the spatial shift of localisation domains by $\lambda/4$. Hence, the necessary condition for cooling is $\delta_2 < 0$.

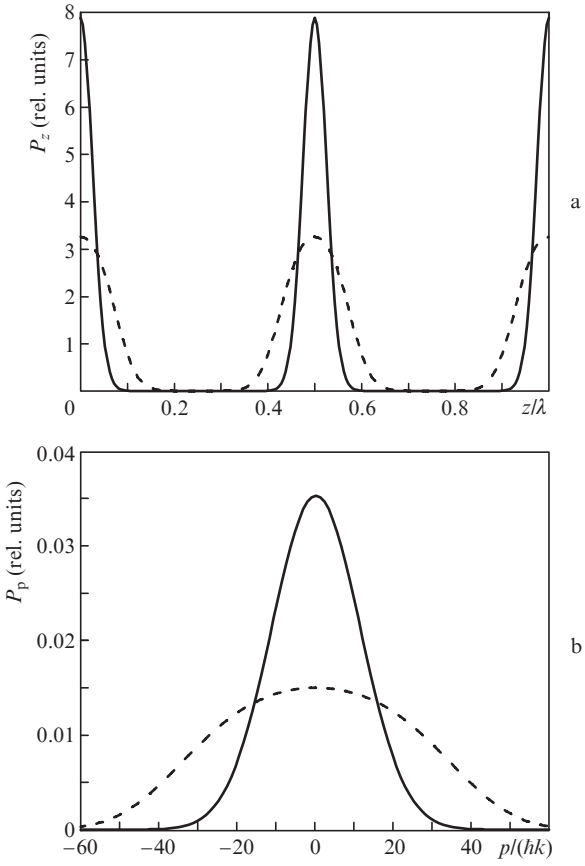

Figure 2. (a) Spatial and (b) momentum distributions of the steady-state solution for the atomic density matrix, with the recoil effects completely taken into account, at relative field phases of $\phi =$ (solid line) 0 and (dashed line) $\pi/4$ and linearly polarised lattice field ($\varepsilon_1 = 0$). The wave saturation parameters are $S_{01} = 0.05$ and $S_{02} = 0.5$, the detunings are $\delta_1/\gamma = -200$ and $\delta_2/\gamma = -2$ and $\varepsilon_{rec} = 0.005$. The corresponding mean kinetic energies of stationary distributions are $E_{kin} \simeq 0.65\hbar\gamma$ ($\phi = 0$) and $E_{kin} \simeq 2.75\hbar\gamma$ ($\phi = \pi/4$).

Quite a different pattern is observed when the lattice field polarisation differs from linear. Specifically, for the circular and elliptical field polarisations (in the ranges $\phi \neq \pi n$, $n = 0, \pm 1, \pm 2\ldots$), additional contributions (determined by the field $\boldsymbol{E}_1$) begin to dominate in the friction and diffusion coeffi-

cients. Moreover, the rectifying contribution to force (24) leads to a macroscopic modulation of the optical potential (see Fig. 1). As a result, the spatial regions where $\phi = \pi n$ ($n = 0, \pm1, \pm2\ldots$) and the regions where $\phi = \pm\pi/4$ can be selected. In the former case, a significant effect of the dipole force rectification occurs, which leads to pushing the atoms out; the latter undergo above-barrier motion in the region with $\phi = \pm\pi/4$ (depending on the sign of detuning δ_1 and the field ellipticity ε_1), where the dipole force rectification is absent (24). These regions correspond to the global minima of the optical potential; the additional contributions (determined by the lattice field) to the friction and diffusion coefficients dominate in them, significantly changing the kinetics pattern. The choice of the sign of the pump field detuning δ_2 barely affects the results of cooling the atoms in lattices, whereas the direction of kinetic process depends exclusively on the lattice field detuning δ_1, the sign of ellipticity ε_1 and the sign of phase ϕ in Eqn (28).

At $\delta_1 < 0$, the relative phase of fields ϕ under conditions (21) increases with an increase in z (see Fig. 1). The regions of the global minima of optical potential (25) correspond to the relative phase $\phi = \pi/4$ at $\varepsilon_1 > 0$ and to $\phi = -\pi/4$ at $\varepsilon_1 < 0$. However, these conditions, as follows from (28), exclude the possibility of cooling atoms in lattices. Cooling in the regions of the global minima of the optical potential both at $\varepsilon_1 > 0$ (the global minimum position corresponds to $\phi = \pi/4$) and at $\varepsilon_1 < 0$ (the global minimum position corresponds to $\phi = -\pi/4$) can be implemented only at $\delta_1 > 0$.

Let us consider in more detail the mechanism of dipole force rectification in this field configuration. To this end, it is convenient to apply the concept of dressed states. Recall that the dressed (adiabatic) states are the eigenstates of the Hamiltonian of an atom in a field with the translational motion disregarded. For a two-level atom, there is only a pair of dressed states [32]. In the case under consideration (atoms with the $j_g = 1/2 \to j_e = 3/2$ optical transition), there are six such states: $|1\rangle, |2\rangle, \ldots |6\rangle$. Each of them is a superposition of the wave functions of the Zeeman sublevels of the ground and excited states of the atom. The eigenvalues corresponding to the dressed states are adiabatic potentials:

$$U_1 = -\hbar\delta_1/2 + \mathcal{E}_-,\ U_3 = -\hbar\delta_1/2 - \mathcal{E}_-,\ U_5 = -\hbar\delta_1/2,$$
$$U_2 = -\hbar\delta_1/2 + \mathcal{E}_+,\ U_4 = -\hbar\delta_1/2 - \mathcal{E}_+,\ U_6 = -\hbar\delta_1/2, \quad (32)$$

where

$$\mathcal{E}_\mp = \hbar\frac{1}{2}\sqrt{\delta_1^2 + \frac{4}{3}|\Omega_1(z)|^2[2 \mp \sin(2\varepsilon_1)]}\,. \quad (33)$$

Figure 3 shows schematically the spatial dependence of the optical potentials in the monochromatic field of a standing wave. At positive detunings ($\delta_1 > 0$) and in the limit of low light field intensities ($\Omega_1 \ll \gamma$), the states with the optical potentials U_1 and U_2 correspond to the Zeeman sublevels of the ground state ($|j_g, -1/2\rangle$ and $|j_g, 1/2\rangle$, respectively), whereas the populations of the other states are negligible. In the monochromatic field of a standing wave, the populations of the $|1\rangle$ and $|2\rangle$ states are set by a symmetric spatial function with respect to the nodes and antinodes of the lattice field, which leads to a zero force averaged over the spatial period.

An addition of the field of the lin⊥lin configuration leads to an additional spatially modulated optical pumping

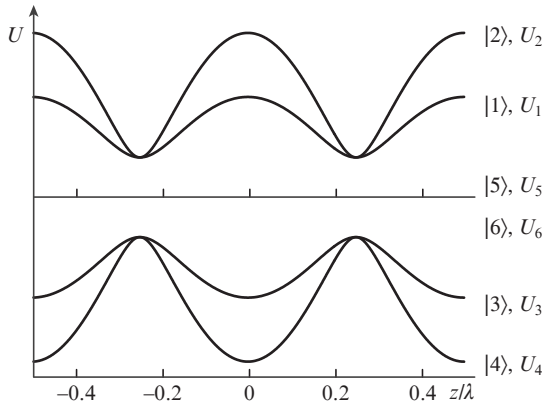

Figure 3. Spatial dependence of the adiabatic potentials in the monochromatic field of the standing wave with elliptical polarisation ($\varepsilon_1 > 0$) at blue light field detunings ($\delta_1 > 0$) for atoms with the $j_g = 1/2 \to j_e = 3/2$ optical transition.

of the ground-state sublevels. Let us consider the region with $\phi = 0$, where, according to (26), the effect of dipole force rectification is most pronounced. The ellipticity parameter of the pump field, which is determined by the relation $\sin(2\varepsilon_2) = -\sin(2kz)$, takes zero values (corresponding to the linearly polarised field) at the nodes and antinodes of the lattice field ($z = n\lambda/4$, $n = 0, \pm1, \pm2\ldots$; Fig. 3). Note that the ellipticity parameter of the pump field, ε_2, exhibits an asymmetric dependence in the symmetric (with respect to the lattice field maxima) intermediate intervals. This asymmetry leads to an asymmetric additive in the spatial distribution of the populations of adiabatic states $|1\rangle$ and $|2\rangle$. As a result, the force averaged over the spatial period takes nonzero values.

3.3. σ_+–σ_- configuration

Let us consider the second case, where the pump field is formed by counterpropagating circularly polarised waves, i.e., the σ_+–σ_- field configuration. This field contains only one nonzero gradient: the gradient of the polarisation vector orientation, $\nabla_z \psi_2 = -k$. The field $\boldsymbol{E}_2$ is linearly polarised at each point, i.e., $\varepsilon_2 = 0$. As well as for the lin⊥lin configuration, the pump field saturation parameter at any point z is

$$S_2 = 2S_{02}.$$

Here, the saturation parameter S_{02} is determined per each counterpropagating wave with amplitude E_{02}. The force is decomposed into two main contributions, which take the following form under conditions (21):

$$F = -\hbar\frac{4}{3}\delta_1 S_1 \nabla_z \Lambda_1 - \hbar\frac{\gamma}{3} S_1 \sin(2\varepsilon_1)\nabla_z \psi_2\,. \quad (34)$$

The second term has a small weight; however, it does not disappear when averaging over the field spatial period:

$$\langle F\rangle = \frac{\hbar k\gamma 2 S_{01}\sin(2\varepsilon_1)}{3}. \quad (35)$$

This contribution arises due to the unbalance of light-pressure forces from the counterpropagating pump field waves. Indeed, a deviation of the lattice field polarisation from linear

leads to population anisotropy in the ground state and to different scattering probabilities for photons with σ_+ and σ_- polarisations from the counterpropagating pump field waves. The optical potential corresponding to the force,

$$U = \frac{\hbar 8\delta_1 S_{01}\cos^2(kz)}{3} - \frac{\hbar\gamma S_{01}\sin(2\varepsilon_1)[\sin(2kz) + 2kz]}{3}, \quad (36)$$

is independent of the relative phase of the fields, which, in contrast to the lin⊥lin configuration of the pump field, does not lead to a periodic dependence on the scales of $\pi/(\delta k)$. The main contribution to the friction coefficient in the pump field of the $\sigma_+ - \sigma_-$ configuration under conditions (21) is determined by the contribution from $\chi_{\Lambda_1\Lambda_1}$; the coefficient $\chi_{\psi_2\Lambda_1}$ yields a small additive

$$\frac{\xi}{\hbar k^2} = 6\delta_1\frac{S_1^2}{S_2^2}\sin^2(2\varepsilon_1)\frac{\nabla_z\Lambda_1\nabla_z\Lambda_1}{k^2} + 3\gamma\frac{S_1}{S_2}\sin(2\varepsilon_1)\frac{\nabla_z\psi_2\nabla_z\Lambda_1}{k^2} = 6\delta_1\frac{S_{01}^2}{S_{02}^2}\sin^2(2\varepsilon_1)\sin^2(2kz) + 3\gamma\frac{S_{01}}{S_{02}}\sin(2\varepsilon_1)\sin(2kz). \quad (37)$$

The induced-diffusion coefficients are determined by the contributions from $\mathcal{D}_{\Lambda_1\Lambda_1}$, $\mathcal{D}_{\psi_2\psi_2}$ and $\mathcal{D}_{\Lambda_1\psi_2} + \mathcal{D}_{\psi_2\Lambda_1}$, which take the following form under conditions (21):

$$\frac{D^{(i)}}{\hbar^2k^2\gamma} = 2\frac{\delta_1^2S_1^2}{\gamma^2S_2}\frac{\nabla_z\Lambda_1\nabla_z\Lambda_1}{k^2} + \frac{8}{3}\frac{\delta_1S_1}{\gamma}\sin(2\varepsilon_1)\frac{\nabla_z\Lambda_1\nabla_z\psi_2}{k^2} + \frac{7S_2}{6}\frac{\nabla_z\psi_2\nabla_z\psi_2}{k^2} = \frac{\delta_1^2S_{01}^2}{\gamma^2S_{02}}\sin^2(2\varepsilon_1)\sin^2(2kz) + \frac{8}{3}\frac{\delta_1S_{01}}{\gamma}\sin(2\varepsilon_1)\sin(2kz) + \frac{7S_{02}}{3}. \quad (38)$$

The last term is the known contribution, which determines the diffusion coefficient for an atom in the monochromatic field of the $\sigma_+ - \sigma_-$ configuration. The expression for the spontaneous-diffusion coefficient,

$$D^{(s)} = \gamma\hbar^2k^2S_{02}\frac{11}{45}, \quad (39)$$

also coincides with that for an atom in the monochromatic field of the $\sigma_+ - \sigma_-$ configuration.

Note that the nonzero friction coefficient in this light field configuration is related to the presence of interference contributions to the coefficients $\chi_{\Lambda_1\Lambda_1}$ and $\chi_{\psi_2\Lambda_1}$; it becomes zero in the absence of one of the fields. Cooling can be implemented only at $\delta_1 < 0$ and a lattice field polarisation different from linear. Under these conditions, the stationary distribution of atoms in the momentum space (see Fig. 4) is asymmetric as a result of the unbalance of spontaneous light-pressure forces of the pump field waves with $\sigma_+ - \sigma_-$ polarisations. Cooling can be implemented at any sign of detuning δ_2, in particular, under exact-resonance conditions for the pump field.

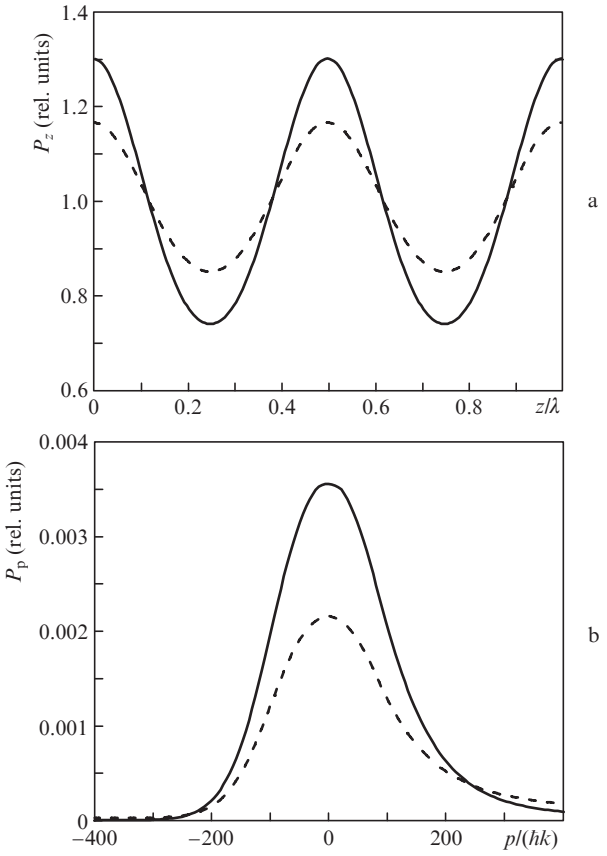

Figure 4. (a) Spatial and (b) momentum distributions of the steady-state solution for the atomic density matrix, with the recoil effects completely taken into account, at the optical lattice ellipticity parameter ε_1 = (solid line) $\pi/4$ and (dashed line) $\pi/8$. The wave saturation parameters are $S_{01} = 0.05$ and $S_{02} = 0.2$, the detunings are $\delta_1/\gamma = -400$ and $\delta_2/\gamma = 0$, and $\varepsilon_{rec} = 0.005$. The corresponding temperatures of stationary distributions are $k_BT = 133\hbar\gamma$ ($\varepsilon_1 = \pi/4$) and $226\hbar\gamma$ ($\varepsilon_1 = \pi/8$).

4. Conclusions

The kinetics of atoms in a field formed by two monochromatic waves with different frequencies, close to that of the optical resonance of a two-level atom, was considered. General expressions for the kinetic coefficients of the Fokker–Planck equation and expressions for the force and friction and diffusion coefficients of atoms were obtained within a simple model of atoms having degenerate (with respect to the angular momentum projection) levels with closed $j_g = 1/2 \to j_e = 3/2$ optical transition.

The expressions are presented in the general form of expansion in the gradients of light field parameters. For the two-level model of an atom, these are the gradients of the light field amplitude and phase. Generally, it was shown that the friction and diffusion coefficients can be written as expansions in quadratic combinations of these gradients, including their cross combinations. Two limiting cases were considered additionally, in which E_1 is the field of a standing wave that is strongly detuned from the atomic resonance and forms a deep optical potential and the field E_2 is resonant with the optical transition and is used for optical pumping of levels. The cases where the resonance field is the field of either standing or travelling wave were analysed. The following features of the kinetics of atoms for the lin⊥lin configuration of the pump field were revealed:

(i) In the case of a linearly polarised lattice field, the cooling is independent of the sign of field detuning δ_1. The friction and diffusion coefficients take the form known for the atoms in the monochromatic field of the lin⊥lin configuration. Correspondingly, the direction of the kinetic process is determined by the sign of pump field detuning: red field detunings ($\delta_2 < 0$) are necessary to implement cooling.

(ii) When the lattice field is elliptically or circularly polarised ($\varepsilon_1 \neq 0$), an effect of dipole force 'rectification' arises, which leads to an additional spatial modulation of the optical potential with a period greatly exceeding the wavelength $\pi/(\delta k)$. Under these conditions, additional contributions to the friction and diffusion coefficients begin to dominate in the regions of global minima and maxima of the optical potential, as a result of which the kinetics of atoms significantly changes. For example, cooling of atoms in the regions of optical potential minima can be implemented only at blue lattice field detunings ($\delta_1 > 0$), while the sign of the pump field detuning is of little importance. In particular, the pump field detuning may be zero.

The $\sigma_+ - \sigma_-$ configuration of the pump field is characterised by the following features:

(i) The kinetics of atoms is independent of the relative phase of the lattice and pump fields.

(ii) Cooling can be implemented at only red lattice field detunings ($\delta_1 < 0$) and any signs of pump field detunings (including $\delta_2 = 0$). A necessary condition for the cooling of atoms with the $j_g = 1/2 \rightarrow j_e = 3/2$ optical transition in this light field configuration is the nonzero ellipticity of the lattice field polarisation vector: $\varepsilon_1 \neq 0$.

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References

1. Grynberg G., Robilliard C. *Phys. Reports*, **355**, 335 (2001).
2. Jaksch D., Briegel H.-J., Cirac J.I., Gardiner C.W., Zoller P. *Phys. Rev. Lett*., **82**, 1975 (1999).
3. Brennen G.K., Caves C.M., Jessen P.S., Deutsch I.H. *Phys. Rev. Lett*., **82**, 1060 (1999).
4. Jaksch D., Cirac J.I., Zoller P., Rolston S.L., Cote R., Lukin M.D. *Phys. Rev. Lett*., **85**, 2208 (2000).
5. Jessen P.S., Deutsch I.H., Stock R. *Quantum Inf. Process*, **3**, 91 (2004).
6. Daley A.J., Boyd M., Ye M.J., Zoller P. *Phys. Rev. Lett*., **101**, 170504 (2008).
7. Bloch I. *Nature*, **453**, 1016 (2008).
8. Brickman Soderberg K.-A., Gemelke N., Chin C. *New J. Phys*., **11**, 055022 (2009).
9. Castin Y., Dalibard J. *Europhys. Lett*., **14**, 761 (1991).
10. Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *Zh. Eksp. Teor. Fiz*., **131**, 963 (2007).
11. Prudnikov O.N., Il'enkov R.Ya., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *Zh. Eksp. Teor. Fiz*., **139**, 1074 (2011).
12. Kazantsev A.P., Krasnov I.V. *Pis'ma Zh. Eksp. Teor. Fiz*., **46**, 264 (1987).
13. Grimm R., Ovchinnikov Y.B., Sidorov A.I., Letokhov V.S. *Phys. Rev. Lett*., **65**, 1415 (1990).
14. Hemmer P.R., Shahriar M.S., Prentiss M.G., Katz D.P., Berggren K., Mervis J., Bigelow N.P. *Phys. Rev. Lett*., **68**, 3148 (1992).
15. Gupta R., Xie C., Padua S., Batelaan H., Metcalf H. *Phys. Rev. Lett*., **71**, 3087 (1993).
16. Drewsen M. *Phys. Rev. A*, **51**, 1407 (1995).
17. Javanainen J. *Phys. Rev. Lett*., **64**, 519 (1990).
18. Kosachiov D.V., Rozhdestvensky Yu.V., Nienhuis G. *J. Opt. Soc. Am. B*, **14**, 535 (1997).
19. Prudnikov O.N., Arimondo E. *J. Opt. Soc. Am. B*, **20**, 909 (2003).
20. Grove T.T., Gould P.L. *Laser Phys*., **4**, 957 (1994).
21. Pu H., Cai T., Bigelow N.P., et al. *Opt. Commun*., **118**, 261 (1995).
22. Dunn J.W., Thomsen J.W., Greene C.H., Cruz F.C. *Phys. Rev. A*, **76**, 011401(R) (2007).
23. Hamann S.E., Haycock D.L., Klose G., Pax P.H., Deutsch I.H., Jessen P.S. *Phys. Rev. Lett*., **80**, 4149 (1998).
24. Vuletic V., Chin C., Kerman A.J., Chu S. *Phys. Rev. Lett*., **81**, 5768 (1998).
25. Kerman A.J., Vuletic V., Chin C., Chu S. *Phys. Rev. Lett*., **84**, 439 (2000).
26. Taichenachev A.V., Tumaikin A.M., Yudin V.I., Hollberg L. *Phys. Rev. A*, **63**, 033402 (2001).
27. Domenico G.Di., Castanga N., Mileti G., Thoman P., Taichenachev A.V., Yudin V.I. *Phys. Rev. A*, **69**, 063403 (2004).
28. Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *Zh. Eksp. Teor. Fiz*., **144**, 262 (2013).
29. Dalibard J., Tannoudji C.C. *J. Opt. Soc. Am. B*, **6**, 2023 (1989).
30. Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *Zh. Eksp. Teor. Fiz*., **125**, 499 (2004).
31. Bezverbnyi A.V., Prudnikov O.N., Taichenachev A.V., Tumaikin A.M., Yudin V.I. *Zh. Eksp. Teor. Fiz*., **123**, 437 (2003).
32. Kazantsev A.P., Surdutovich G.I., Yakovlev V.P. *Mekhanicheskoe deistvie sveta na atomy* (Mechanical Action of Light on Atoms) (Moscow: Nauka, 1991).