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Optical control of interatomic interaction in a Bose condensate

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Abstract. **We propose to control the interatomic interaction in a Bose condensate by using optical beams forming an atomic trap. It is shown that for a Bose condensate in a double-well optical potential the nonresonant quantized radiation simulates a change in the interatomic interaction in the well. The magnitude and sign of this change can be effectively controlled by varying the frequency of the radiation source forming the trap.**

Keywords: *interatomic interaction, Bose condensate, atomic trap, atomic mode, photon mode.*

1. Introduction

The application of an atomic Bose condensate in future quantum technologies implies the use of two of its most important properties, namely, the macroscopic quantum coherence of the condensate and the presence of a substantial interatomic interaction [1]. The development of the arsenal of the means for controlling the interatomic interaction is very important. Currently, two methods of such control are used. The first method is based on the reconstruction of the shape of the trap potential, which changes the density of the condensate and, consequently, the average distance between the atoms. The second widespread and universal means for controlling the interatomic interaction is the Feshbach resonance [2]. This effect allows one to change, with the use of a constant magnetic field, the scattering length, which determines the interatomic interaction in the condensate.

In this paper, it is proposed to use optical beams forming an atomic trap to control the interatomic interaction in the condensate. As an example, a condensate in a double-well potential is used. The two-mode model of the atomic condensate corresponding to this form of the potential is intensively studied experimentally and theoretically [3–7]. We have shown that forming one of the wells by the field of the optical mode of the resonator can modify the parameter responsible for the interatomic interaction in this well. It is important to note that this control of the interatomic interaction is not related to the simple change in the shape of the potential mentioned above, and, as a consequence, the density of atoms. The closest analogue of the new phenomenon is the known 'optical spring' effect [8, 9].

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2. Model

The quantum kinetic equation for the statistical operator of a two-mode atomic condensate and a photon mode excited by an external harmonic source (see Fig. 1) can be written in a general form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = -\mathrm{i}[\hat{H}_{\mathrm{at}}^{(0)}, \hat{\rho}] + \Lambda_{\mathrm{ph}}[\hat{\rho}], \tag{1}$$

where the atomic Hamiltonian $\hat{H}_{\mathrm{at}}^{(0)}$ and the photon Liouvillian Λ_{ph} are introduced, which include both dynamic and dissipative terms. As shown in the figure, the first of the two atomic modes is formed in the waist of a conventional light beam of a travelling wave, and the second one – in the waist of a cavity mode beam. The Hamiltonian

$$\hat{H}_{\mathrm{at}}^{(0)} = \omega_1\hat{n}_1 + (\omega_2 - g\overline{n}_{\mathrm{ph}})\hat{n}_2 + \xi_1\hat{n}_1(\hat{n}_1 - 1) + \xi_2\hat{n}_2(\hat{n}_2 - 1) + \chi(\hat{b}_1^{\dagger}\hat{b}_2 + \hat{b}_2^{\dagger}\hat{b}_1) \tag{2}$$

for two bosonic modes, $\hat{b}_1$ ($\hat{b}_1^{\dagger}$) and $\hat{b}_2$ ($\hat{b}_2^{\dagger}$), i.e. atoms in wells 1 and 2, contains terms linear in the atomic number operators $\hat{n}_k = \hat{b}_k^{\dagger}\hat{b}_k$ ($k = 1, 2$) in the wells (ω_k are the depths of the wells)

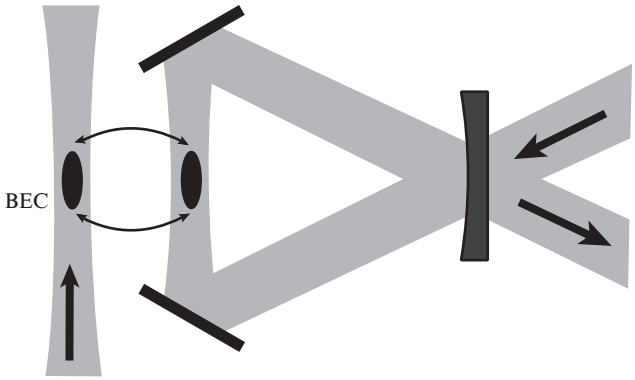

Figure 1. Scheme for the observation of optical control of the interatomic interaction in a Bose–Einstein condensate (BEC). Two parts of the Bose condensate are in the waist of the photon mode of the resonator and the free wave of approximately the same intensity and configuration. The curved arrows symbolise the tunnelling between two parts of the BEC.

and quadratic terms proportional to ξ_k, responsible for the interatomic interactions in the wells; the parameter χ specifies the tunnelling rate between the wells. The interaction between the atoms from different wells is neglected.

An essential point is the spatial configuration of the second atomic mode. The interatomic interaction parameter defined in this configuration for the second well, ξ_2, and the position of the lower vibrational level ω_2 are calculated for some average number of photons, $\overline{n}_{\rm ph}$, in the cavity mode. We will obtain below an equation for the average number of photons. For subsequent consideration, it is convenient to explicitly introduce into the Hamiltonian the product of $\overline{n}_{\rm ph}$ by the parameter g, which determines the interaction efficiency of the atomic and photon modes. This parameter, on the one hand, determines the contribution of a single photon to the trap potential for mode 2, and on the other hand, a shift of the eigenfrequency of the photon mode $\hat{a}(\hat{a}^\dagger)$ introduced by one atom. The latter circumstance is reflected in the structure of the photonic Liouvillian

$$\Lambda_{\rm ph}[\hat{\rho}] = -{\rm i}\Delta[\hat{a}^\dagger\hat{a},\hat{\rho}] - {\rm i}g[\hat{a}^\dagger\hat{a}\hat{n}_2,\hat{\rho}] + \Omega[\hat{a}^\dagger - \hat{a},\hat{\rho}]$$

$$+ \gamma(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}). \qquad (3)$$

Here, the explicit harmonic time dependence of the field of a classical external source with the amplitude Ω exciting the mode is excluded; Δ is the frequency detuning of the mode and the external source; the second term describes the above-mentioned frequency shift of the mode due to the nonresonance interaction with atoms from well 2; and the last three terms are responsible for the irreversible escape of photons from the cavity (γ is the escape rate). It should be noted that the term $(\omega_2 - g\overline{n}_{\rm ph})\hat{n}_2$ in the Hamiltonian $\hat{H}_{\rm at}^{(0)}$ in combination with the second term of the Liouvillian ensures the one-time account for the interaction of the atoms and the field, as well as the possibility of studying the effects of quantum fluctuations of the photon mode.

Interaction between atoms and radiation generates correlations (quantum entanglement) between the state of the photonic subsystem and the state of the system of atoms. Since the interaction is proportional to the number of atoms in well 2, in the Fock basis $\{|\,n_2\,\rangle\}_{n_2=0}^{\infty}$ for a given mode, the statistical operator of the whole system has the form

$$\hat{\rho} = \sum_{n_2,\,n_2'} \hat{\rho}_1(n_2,n_2')\otimes|\,n_2\,\rangle\langle\,n_2'\,|\otimes\hat{\rho}_{\rm ph}(n_2,n_2')\,. \qquad (4)$$

Here, there are operators $\hat{\rho}_{\rm ph}(n_2,n_2')$ of the photon subsystem, which explicitly reflect the presence of the mentioned correlations. The operator $\hat{\rho}_1(n_2,n_2')$ acts in the state space of the condensate mode 1.

We will assume that the photon subsystem is fast and its evolution adiabatically obeys the evolution of the slower subsystem of the atomic condensate. This means that the fast stage of evolution is described by the Liouvillian $\Lambda_{\rm ph}$ and terminates with the formation of a 'subordinate' photon state. To establish its form, we consider the action of the Liouvillian. Because it affects only the photon and the second atomic modes, it is sufficient to apply it to fragments of the terms from sum (4):

$$\Lambda_{\rm ph}[|\,n_2\,\rangle\langle\,n_2'\,|\otimes\hat{\rho}_{\rm ph}(n_2,n_2')] =$$

$$= |\,n_2\,\rangle\langle\,n_2'\,|\otimes\Lambda_{\rm ph}(n_2,n_2')[\hat{\rho}_{\rm ph}(n_2,n_2')]. \qquad (5)$$

The action of the [introduced in (5)] superoperator $\Lambda_{\rm ph}(n_2,n_2')$, which refers only to the photon subsystem, follows in an obvious way from structure (3). For a certain number of atoms $n_2 = n_2'$, evolution is completed by the Glauber coherent state:

$$\Lambda_{\rm ph}(n_2,n_2)[|\,\alpha(n_2)\,\rangle\langle\,\alpha(n_2)\,|] = 0\,. \qquad (6)$$

Here

$$\alpha(n) = \frac{\Omega}{{\rm i}(\Delta + gn) + \gamma}. \qquad (7)$$

Since this is a pure quantum state, the above-mentioned entanglement between the atomic and the photonic subsystems makes it natural to consider the action $\Lambda_{\rm ph}(n_2,n_2')$ at $n_2 \neq n_2'$ on the photon operators $|\alpha(n_2)\,\rangle\langle\,\alpha(n_2')\,|$, pretending to the place of

$$\hat{\rho}_{\rm ph}(n_2,n_2') = |\,\alpha(n_2)\,\rangle\langle\,\alpha(n_2')\,|$$

in (4). They turn out to be eigenoperators for $\Lambda_{\rm ph}(n_2,n_2')$:

$$\Lambda_{\rm ph}(n_2,n_2')\,[|\,\alpha(n_2)\,\rangle\langle\,\alpha(n_2')\,|]$$

$$= \frac{-{\rm i}\Omega^2 g(n_2 - n_2')}{[\gamma + {\rm i}(\Delta + gn_2)][\gamma - {\rm i}(\Delta + gn_2')]}\,|\,\alpha(n_2)\,\rangle\langle\,\alpha(n_2')\,|\,. \qquad (8)$$

The eigenvalue is a function of the numbers n_2 and n_2' and vanishes, as expected, for $n_2 = n_2'$.

In the limit $gn_2,\, gn_2' \ll |\gamma + {\rm i}\Delta|$ we have

$$\frac{-{\rm i}\Omega^2 g(n_2 - n_2')}{[\gamma + {\rm i}(\Delta + gn_2)][\gamma - {\rm i}(\Delta + gn_2')]}$$

$$\approx \frac{-{\rm i}\Omega^2 g(n_2 - n_2')}{\Delta^2 + \gamma^2}\left[1 - \frac{g\Delta(n_2 + n_2')}{\Delta^2 + \gamma^2} - \frac{{\rm i}g\gamma(n_2 - n_2')}{\Delta^2 + \gamma^2}\right]. \qquad (9)$$

Transforming the terms from the right-hand side to the operator form, we obtain the quantum kinetic equation for the state of the atomic subsystem:

$$\frac{\rm d}{{\rm d}t}\hat{\rho}_{\rm at} = -{\rm i}[\hat{H}_{\rm at},\hat{\rho}_{\rm at}] + 2\nu\hat{n}_2\hat{\rho}_{\rm at}\hat{n}_2 - \nu\hat{n}_2^2\hat{\rho}_{\rm at} - \nu\hat{\rho}_{\rm at}\hat{n}_2^2. \qquad (10)$$

Here we introduce the statistical operator of atoms

$$\hat{\rho}_{\rm at} = Tr_{\rm ph}\hat{\rho} = \sum_{n_2,n_2'}\hat{\rho}_1(n_2,n_2')\,\langle\,\alpha(n_2')\,|\,\alpha(n_2)\,\rangle\otimes|\,n_2\,\rangle\langle\,n_2'\,|;$$

$\hat{H}_{\rm at}$ is the Hamiltonian describing the change in the depth of well 2 and containing the interaction parameter of atoms in this well:

$$\hat{H}_{\rm at} = \hat{H}_{\rm at}^{(0)} + \left(\frac{\Omega^2 g}{\Delta^2 + \gamma^2} + \delta\xi_2\right)\hat{n}_2 + \delta\xi_2\hat{n}_2(\hat{n}_2 - 1), \qquad (11)$$

where

$$\delta\xi_2 = -\Omega^2 g^2\Delta/(\Delta^2 + \gamma^2)^2.$$

The resulting correction to the interatomic interaction in well 2 changes the sign along with the detuning Δ of the frequency of the photon mode and the frequency of the external source. The three-term Lindblad structure in (10) describes the destruction of the coherence between the states with different atomic numbers in well 2 and, consequently, phase relations between atomic modes 1 and 2. The reason is the quantum entanglement between atoms and photons. Since the photon subsystem exchanges information with the environment through the channel of irreversible loss of quanta, there is a potential possibility for spectral measurements of emitted radiation. The spectral shift depends on the number of atoms in the beam, and so there is a natural process of measuring the number of atoms in well 2, which is reflected in (10) as dephasing. The rate of dephasing is

$$\nu = \Omega^2 g^2 \gamma / (\Delta^2 + \gamma^2)^2.$$

The dephasing effect accompanies any scenario of optical probing of atomic ensembles in individual minima of the trap potential [10].

The presence of an irreversible process leads to the existence of a stationary solution $\hat{\rho}_{\rm at}^{\rm (st)}$ in Eqn (10). The average number of photons,

$$\overline{n}_{\rm ph} = \sum_{n_2, n_2'} \langle n_2 | \, Tr_1 \hat{\rho}_{\rm at}^{\rm (st)} | \, n_2' \rangle \, \alpha(n_2')^* \alpha(n_2), \qquad (12)$$

corresponds to this stationary solution. Since the parameters of equation (10) and the configuration of the second atomic mode are implicitly dependent on the choice of $\overline{n}_{\rm ph}$, formula (12) actually represents a complicated equation for the average number of photons. For small deviations of $\hat{\rho}_{\rm at}$ from $\hat{\rho}_{\rm at}^{\rm (st)}$, the solution of equation (12) can be used as the initial parameter in the kinetic equation.

3. Discussion

The proposed mechanism of local optical modification of the parameter responsible for the interatomic interaction does not change the scattering length of the atoms. For this reason it is natural to talk about the simulation control of this interaction. An accompanying phenomenon and inevitable price is the emergence of a process that destroys the phase correlations between atomic modes. It modifies the effect of interwell tunnelling and can in some cases turn out to be an undesirable phenomenon. Therefore, the proposed mechanism for controlling the interatomic interaction should not be considered universal.

Let us prove the direct relationship between the effective change in the interatomic interaction and the phenomenon of the 'optical spring' in optomechanics [8]. As is known, the basic optomechanical model explores the interaction of the oscillations of a movable mirror of the resonator and the photon mode through the dependence of the fundamental frequency of the mode on the varying length of the resonator. In the case of a relatively small change in the length of the resonator during the motion of the mirror, the corresponding term in the Hamiltonian has the form $G\hat{Q}\hat{a}^\dagger\hat{a}$. Here G is a constant (in $\rm cm^{-1}\ s^{-1}$), and $\hat{Q}$ is the operator of the mirror coordinate. This operator appears in place of the number of atoms in well 2 in our model. The correction to the equation for the coordinate density matrix $\langle Q | \hat{\rho}_{\rm m} | Q' \rangle$ of the moving mirror (in the limit of the adiabatic subordination of the fast photon evolution to the slow mechanical system) acquires, by analogy with the second term in parentheses in Eqn (9), a term proportional to $\Delta(Q^2 - Q'^2)$. This is equivalent to an effective change in the stiffness of the suspension of the movable mirror, i.e. the 'optical spring'. In reality, there is no change in the elastic properties of the suspension. In a similar way, the effect proposed in this work is not related to a real change in the nature of interatomic interactions.

We estimate the change in the energy of the interatomic interaction of the ^{87}Rb atoms, caused by the change in ξ_2 [Eqn (11)]. Let the optical traps be produced by radiation with $\lambda = 0.96$ μm and the waist radii equal to 10 μm. The ring resonator (10 cm perimeter) is formed by an input mirror with a transmission $\tau = 0.01$ (in intensity) and two dense mirrors. The rate of escape of photons from such a resonator is $\gamma = 3\times10^7\ \rm s^{-1}$, and the optical potential of an atom with a single photon in the cavity is $g = -0.1\ \rm s^{-1}$. It is convenient to express the parameter Ω in terms of the number $N_{\rm ph}$ of photons in the resonator:

$$\Omega^2 = N_{\rm ph}(\Delta^2 + \gamma^2). \qquad (13)$$

Let the intensity of the input radiation be chosen so that the number of photons in the resonator is $N_{\rm ph} = 10^8$. The ^{87}Rb atoms in the field of such an optical trap have radial and axial oscillation frequencies: 2.6 kHz and 47 Hz, respectively. If the Bose condensate contains 10^6 ^{87}Rb atoms, then the energy of the interatomic interaction without the 'optical spring' effects is $8\times10^4\ \rm s^{-1}$ per atom, according to the Thomas–Fermi model. Taking into account the 'optical spring' effects changes ξ_2 according to Eqn (11). With an optimal detuning of the radiation frequency from the resonator frequency, $\Delta = +\gamma(-\gamma)$, the effect of the 'optical spring' decreases (increases) the energy of the interatomic interaction by approximately 20%. The rate of the condensate dephasing at these parameters is $\nu \approx 0.02\ \rm s^{-1}$.

4. Conclusions

The results of the work can be summarised as follows. Firstly, the emergence of quantum correlations (entanglement) is shown between the mode of the ring resonator, which forms a trap, and an atomic condensate localised in this trap. To do this requires to introduce into the considered model an explicit form of the source of mode photons and the mechanism of irreversible escape of photons from the resonator. Secondly, the arising correlations can effectively simulate the interatomic interaction in a localised condensate and control its sign and magnitude by changing the frequency of the external radiation source. This phenomenon basically reveals a resemblance to the 'optical spring'. The above estimates show a significant variation of the effective interatomic interaction in comparison with the original one. In this case, inevitable dephasing, accompanying the appearance of the effective interaction, is sufficiently small in the scale of the duration of a typical experiment with a condensate. It is possible to make an address change of the effective interaction in one of the potential wells of the trap of complex configuration (as, for example, in the figure).

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