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Lambda-scheme feedback spectroscopy

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Abstract. **We report a theoretical investigation of a control scheme of a single-Λ system in pump–probe feedback spectroscopy. The feedback controls the phase of one of the fields, and its action is triggered by a nontrivial unraveling of the quantum operation of detection of spontaneous photons emitted by an atom. Within the framework of Markovian master equations, it is shown that the shape and width of the 'dark' resonance of coherent population trapping can be controlled within very wide limits. In addition, sharp resonant-like structures in the field's work dependence on the Rabi frequencies are revealed and explained.**

Keywords: *coherent population trapping, 'dark' resonances, quantum feedback.*

1. Introduction

Nonlinear spectroscopy as part of nonlinear optics arose in the early 1960s [1] with the advent of lasers that made it possible to obtain sufficiently high-power and coherent light fields for observing nonlinear effects in various media. One of them is the effect of coherent population trapping (CPT) [2, 3], which plays an important role in spectroscopy of three-level systems with the Λ-configuration of energy levels [4].

Quantum control theory is a relatively new and rapidly developing field [5, 6], many applications of which have been stimulated by the development of technologies allowing experiments with single quantum objects, such as atoms or ions [7, 8]. Control of simple quantum-optical systems based on quantum feedback makes it possible to achieve such interesting effects as, for example, control of the steady state of a dissipative two-level system [9], enhancement of the squeezing of the fluorescence light emitted by a two-level atom [10] or stabilisation of a selected measurement outcome in a quantum system [11]. In systems of elementary radiators, i.e. atoms or molecules interacting with electromagnetic fields, the most natural type of feedback is the feedback based on photodetection, i.e., the control action is chosen based on the results of detecting the radiation emitted by the system. In our previous papers [12–15], we showed that the control of the field phase based on detected photoemission can substantially modify the spectrum and statistics of the resonant fluorescence of both a single atom and a pair of two-level atoms.

In this paper, we investigate a similar feedback system applied to the two-field spectroscopy scheme. The phase of one of the classical fields interacting with the system undergoes switching, depending on the type of registered spontaneous photons. Mainly we will be interested in the modification of the CPT phenomenon (the conditions of its emergence, as well as the width and shape of the 'dark' resonance) as a function of the choice of the type of events initiating the feedback action.

The concept of feedback spectroscopy was previously introduced in [16], but the idea proposed by its authors differed from that considered by us. Yudin et al. [16] put forward the idea of using feedback to stabilise the response of the system (as applied to the Λ-system, it was suggested to stabilise the level of resonance fluorescence) and to investigate the dependence of the controlled parameters on the frequencies of the fields interacting with the system.

2. Model

We consider a three-level system of Λ-configuration interacting with two classical electromagnetic fields (Fig. 1), with the energy level $|0\rangle$ set at zero. The feedback is organised in a fairly simple way: the phase of the field on one of the transitions (for definiteness the transition 0–1 is selected) is changed by π. The phase switching is initiated by detecting photoemission of a certain type, which allows the system to control its own evolution. Let us first consider the case without feedback. In its absence, the evolution of the density matrix of the system, $\hat{\rho}$, is given by the master equation:

$$\partial_t\hat{\rho} = -\mathrm{i}[\hat{H}_{\mathrm{tot}},\hat{\rho}] + \sum_{i=1,2}(\hat{L}_i\hat{\rho}\hat{L}_i^{+} - \tfrac{1}{2}\{\hat{L}_i^{+}\hat{L}_i,\hat{\rho}\}), \qquad (1)$$

where

$$\hat{L}_i = \sqrt{\gamma_i}\,|i\rangle\langle 0| \ \ (i = 1,2);\ \ \hat{H}_{\mathrm{tot}} = \hat{H}_{\Lambda} + \hat{V}_1 + \hat{V}_2;$$
$$\hat{H}_{\Lambda} = \sum_{i=1,2}\Delta_i\,|i\rangle\langle i|;\ \ \hat{V}_i = \Omega_i\,|0\rangle\langle i| + \Omega_i^{*}\,|i\rangle\langle 0|; \qquad (2)$$

γ_i is the rate of spontaneous emission from level $|0\rangle$ to level $|i\rangle$; Δ_i and Ω_i are the detunings and Rabi frequencies of the corresponding transitions; and $\hat{L}_{1,2}$ are the operators describing the spontaneous decay into levels $|1\rangle$ and $|2\rangle$. The Hamiltonian is written in the rotating wave approximation. The CPT phenomenon consists in the fact that at equal detunings ($\Delta_1 = \Delta_2$)

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the system turns to be in the so-called dark state $|\Psi_{\rm dark}\rangle$, which does not interact with the field and is a certain superposition of the ground states. By analogy, we also introduce a 'bright' superposition of ground states $|\Psi_{\rm bright}\rangle$, orthogonal to $|\Psi_{\rm dark}\rangle$, from which the system is most rapidly excited to state $|0\rangle$. These states in the basis $\{|i\rangle\}_{i=0,1,2}$ have the form:

$$|\Psi_{\rm dark}\rangle = \frac{\Omega_2|1\rangle - \Omega_1|2\rangle}{\sqrt{\Omega_1^2+\Omega_2^2}},$$
$$|\Psi_{\rm bright}\rangle = \frac{\Omega_1|1\rangle + \Omega_2|2\rangle}{\sqrt{\Omega_1^2+\Omega_2^2}}. \quad (3)$$

Suppose now that instead of the operators $\{\hat L_i\}_{i=1,2}$, new operators are used, obtained by the transformation:

$$\hat L_+ = \alpha\hat L_1 + \beta\hat L_2,\;\; \hat L_- = -\beta^*\hat L_1 + \alpha^*\hat L_2,\;\; |\alpha|^2+|\beta|^2 = 1. \quad (4)$$

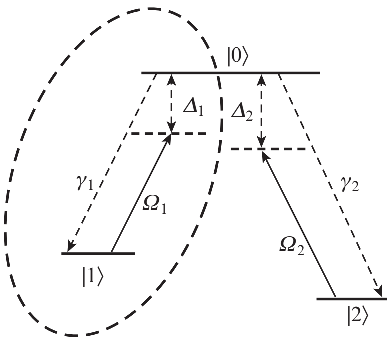

Figure 1. Scheme of two-field spectroscopy of the Λ-system. The phase of the field interacting with the 0–1 transition is controlled by feedback.

Such a transformation is sometimes called an alternative unraveling of the photodetection operation [17], since it corresponds to a different (nonstandard) photodetection process. As is known, experimentally investigated Λ-systems are always part of a more complex system of levels, for example, the transition $J=1\rightarrow J=1$, so that states $|0\rangle$ and $|1\rangle$ belong to different systems of magnetic sublevels. In this case, spontaneous photons emitted from the transitions 0–1 and 0–2 in the direction of the wave vectors of the external fields have orthogonal circular polarisations. The realisation of alternative unraveling can be accomplished by detecting two orthogonal elliptic polarisations. In the case of emission along the wave vector, a correctly rotated polarisation beam splitter mounted on the radiation path is sufficient; therefore, in what follows we will keep in mind the one-dimensional case. In the experiment, the effective one-dimensional geometry may be realised by a so-called holey fibres [18–20], suitable for channelling atoms [21–25].

It is easy to verify that equation (1) is invariant with respect to the choice of unraveling (4). However, as will be shown below, introducing a feedback of certain type makes the system sensitive to this choice.

In our works [12–15], we investigated phase switching feedback of the fields in two-level systems. A similar scheme is considered in this work: a photodetection event of type '+', described by the operator $\hat L_+$ [i.e., performing a transformation on the density matrix $\hat\rho\rightarrow\hat L_+\hat\rho\hat L_+^+/{\rm Tr}(\hat L_+\hat\rho\hat L_+^+)$], sets the phase of the field interacting with the 0–1 transition equal to 0 ('+' state of the feedback loop); and the event of type '–', described the operator $\hat L_-$ sets the phase of the field equal to π. A system with a similar feedback type based on the switching of the classical parameter may be described in the framework of the formalism of hybrid systems developed in [26]. According to this approach, instead of the density matrix, a set of statistical operators with an index indicating a fixed state of the classical subsystem is introduced, i.e. the feedback loop: $\hat\rho\rightarrow\{\hat\rho^{(+)},\hat\rho^{(-)}\}$. The Hamiltonian of the system also depends on the position of the feedback loop:

$$\hat H^{(\pm)}_{\rm tot} = \hat H_\Lambda \pm \hat V_1 + \hat V_2. \quad (5)$$

Instead of a single master equation, a set of new statistical operators obeys the following system of master equations (a detailed derivation of this kind of equations through the formal quantisation of the classical subsystem can be found in [26]):

$$\partial_t\hat\rho^{(+)} = -{\rm i}[\hat H^{(+)}_{\rm tot},\hat\rho^{(+)}] + \hat L_+(\hat\rho^{(+)}+\hat\rho^{(-)})L_+^+ - \frac{1}{2}\sum_{\sigma=+,-}\{\hat L_\sigma^+\hat L_\sigma,\hat\rho^{(+)}\},$$
$$\partial_t\hat\rho^{(-)} = -{\rm i}[\hat H^{(+)}_{\rm tot},\hat\rho^{(-)}] + \hat L_-(\hat\rho^{(+)}+\hat\rho^{(-)})L_-^+ - \frac{1}{2}\sum_{\sigma=+,-}\{\hat L_\sigma^+\hat L_\sigma,\hat\rho^{(-)}\}. \quad (6)$$

3. Results and discussion

The steady-state solution of equations (6) can in general be obtained analytically, but we do not present it here due to its complexity. Instead, we consider a situation of almost equal detunings $\Delta_1\rightarrow\Delta_2$ (the case of mathematically exact resonance requires a separate consideration, but it is not feasible in the experiment). Since in the presence of feedback there are two possible values of the phase of the field on the 0–1 transition, there exist also two possible types of 'dark' and 'bright' states,

$$\left|\Psi^{(\pm)}_{\rm dark}\right\rangle = \frac{\Omega_2|1\rangle \mp \Omega_1|2\rangle}{\sqrt{\Omega_1^2+\Omega_2^2}},$$
$$\left|\Psi^{(\pm)}_{\rm bright}\right\rangle = \frac{\pm\Omega_1|1\rangle + \Omega_2|2\rangle}{\sqrt{\Omega_1^2+\Omega_2^2}}, \quad (7)$$

with the steady state of the system being their incoherent admixture (hereafter, without loss of generality we set $\alpha,\Omega_{1,2}\in\mathbb{R}$):

$$\hat\rho^{(\sigma)}_{\rm st} = p^{\rm (st)}_\sigma\left|\Psi^{(\sigma)}_{\rm dark}\right\rangle\left\langle\Psi^{(\sigma)}_{\rm dark}\right|,$$
$$p^{\rm (st)}_+ = \frac{|\beta|^2\gamma_2\Omega_1^2 + \alpha^2\gamma_1\Omega_2^2 - 2\Omega_1\Omega_2\sqrt{\gamma_1\gamma_2}\,\alpha\,{\rm Re}\beta}{\gamma_2\Omega_1^2+\gamma_1\Omega_2^2 - 4\Omega_1\Omega_2\sqrt{\gamma_1\gamma_2}\,\alpha\,{\rm Re}\beta},$$
$$p^{\rm (st)}_- = \frac{\alpha^2\gamma_2\Omega_1^2 + |\beta|^2\gamma_1\Omega_2^2 - 2\Omega_1\Omega_2\sqrt{\gamma_1\gamma_2}\,\alpha\,{\rm Re}\beta}{\gamma_2\Omega_1^2+\gamma_1\Omega_2^2 - 4\Omega_1\Omega_2\sqrt{\gamma_1\gamma_2}\,\alpha\,{\rm Re}\beta}. \quad (8)$$

Note that expression (8) is valid only for the case $\Delta_1 \neq \Delta_2$. From a more detailed analysis of the analytical solution, it follows that there exist certain values of the system parameters at which the feedback loop ceases to act, i.e., the phase of the field 'freezes' in one of two positions. This occurs at the following parameters:

$$\frac{\alpha}{\beta} = \frac{\Omega_1\sqrt{\gamma_2}}{\Omega_2\sqrt{\gamma_1}}:\ \hat{\rho}^{(+)} = 0,\ \hat{\rho}^{(-)} = |\Psi^{(-)}_{\rm dark}\rangle\langle\Psi^{(-)}_{\rm dark}|,$$
$$\frac{\alpha}{\beta} = \frac{\Omega_2\sqrt{\gamma_1}}{\Omega_1\sqrt{\gamma_2}}:\ \hat{\rho}^{(-)} = 0,\ \hat{\rho}^{(+)} = |\Psi^{(+)}_{\rm dark}\rangle\langle\Psi^{(+)}_{\rm dark}|. \tag{9}$$

To explain this effect, it is necessary to introduce the states $|\Psi^{(\sigma)}_{\rm post}\rangle$, in which the system emerges after different types of photoemissions:

$$\frac{\hat{L}_\sigma\hat{\rho}L^+_\sigma}{{\rm Tr}(\hat{L}_\sigma\hat{\rho}L^+_\sigma)} = |\Psi^{(\sigma)}_{\rm post}\rangle\langle\Psi^{(\sigma)}_{\rm post}|\ (\sigma = +, -),$$
$$|\Psi^{(+)}_{\rm post}\rangle = \frac{\alpha\sqrt{\gamma_1}\,|1\rangle + \beta\sqrt{\gamma_2}\,|2\rangle}{\sqrt{\alpha^2\gamma_1 + |\beta|^2\gamma_2}}, \tag{10}$$
$$|\Psi^{(-)}_{\rm post}\rangle = \frac{-\beta^*\sqrt{\gamma_1}\,|1\rangle + \alpha\sqrt{\gamma_2}\,|2\rangle}{\sqrt{\alpha^2\gamma_2 + |\beta|^2\gamma_1}}.$$

Let us start with the analysis of the 'dark' and 'bright' states in the first formula in (9). After the photodetection of the '+'-type event, the system is in the $|\Psi^{(+)}_{\rm post}\rangle$ state, which coincides with the 'bright' state for the '+' configuration of the feedback

$$\frac{\alpha}{\beta} = \frac{\Omega_1\sqrt{\gamma_2}}{\Omega_2\sqrt{\gamma_1}}:\ |\Psi^{(+)}_{\rm post}\rangle = |\Psi^{(+)}_{\rm bright}\rangle, \tag{11}$$

from which the system is rapidly excited to state $|0\rangle$, so that on average the system resides in the '+' configuration for a short time. Similar simple arguments can be made for the second formula in (9):

$$\frac{\alpha}{\beta} = \frac{\Omega_2\sqrt{\gamma_1}}{\Omega_1\sqrt{\gamma_2}}:\ |\Psi^{(-)}_{\rm post}\rangle = |\Psi^{(-)}_{\rm bright}\rangle. \tag{12}$$

It would seem that in order to freeze the feedback loop action it is sufficient that after the corresponding type of photodetection (for example, of '+' type) the system is in the 'dark' state corresponding to a new position of the feedback loop (in this case, a positive sign of the Rabi frequency Ω_1). This occurs for the following relations between the parameters α and β:

$$\frac{\alpha}{\beta} = -\frac{\Omega_2\sqrt{\gamma_2}}{\Omega_1\sqrt{\gamma_1}}:\ |\Psi^{(+)}_{\rm post}\rangle = |\Psi^{(+)}_{\rm dark}\rangle,$$
$$\frac{\alpha}{\beta} = -\frac{\Omega_1\sqrt{\gamma_1}}{\Omega_2\sqrt{\gamma_2}}:\ |\Psi^{(-)}_{\rm post}\rangle = |\Psi^{(-)}_{\rm dark}\rangle. \tag{13}$$

However, this type of unraveling does not show a significant difference between the stationary populations of the 'dark' states, $p^{(\rm st)}_+$ and $p^{(\rm st)}_-$, as follows from (8). The reason for this is the structure of the state of the system after photodetection, which in the first formula of (13) (for the second formula similar arguments apply) has the form $|\Psi^{(-)}_{\rm post}\rangle \propto \Omega_1\gamma_1|1\rangle + \Omega_2\gamma_2|2\rangle$. This state has a significant coherent admixture of $|\Psi^{(-)}_{\rm dark}\rangle$, so that the entire system can appear in it after any photodetection of the '–'-type event.

In the experiment, the CPT effect manifests itself in the form of 'dark' resonances [2, 3], namely dips in the frequency dependences of the fields' work observed in the vicinity of $\Delta_1 = \Delta_2$. The value of the field work at the bottom of the dip is zero, since the 'dark' states are completely excluded from the interaction with the field. In the absence of feedback, the field work performed per unit time is given by the expression [27]

$$A^{(\rm nf)}_i \sim -{\rm iTr}([|i\rangle\langle i|, \hat{D}_i]E_i\hat{\rho}_{\rm st}) \sim {\rm Re}({\rm i}\Omega_i\rho_{0i}), \tag{14}$$

where ρ_{01} and ρ_{02} are the matrix elements of the steady-state solution (1); E_i are the amplitudes of the external fields; and $\hat{D}_i = d_i|0\rangle\langle i|$ + h.c. are the dipole moments of the $0-i$ transitions. In the presence of feedback, the situation changes significantly due to phase switching:

$$A_1 \sim {\rm Re}[{\rm i}\Omega_1(\rho^{(+)}_{01} - \rho^{(-)}_{01})],$$
$$A_2 \sim {\rm Re}[{\rm i}\Omega_2(\rho^{(+)}_{02} + \rho^{(-)}_{02})], \tag{15}$$
$$A_1/\gamma_1 = A_2/\gamma_2.$$

The last relation follows from the form of the steady-state solution (6). Since the fields' works are proportional, it is sufficient to investigate only the normalised value $A = A_1/\gamma_1 = A_2/\gamma_2$. Its dependence on the detunings of the fields in comparison with the case without feedback is presented in Fig. 2. It is easy to see that the width and shape of the 'dark' resonance can be effectively controlled by changing the unraveling parameters of the photodetection operation. In addition, after a detailed study of the steady-state solution, it was found that when the relations between the parameters of the problem

$$\frac{\Omega_1\sqrt{\gamma_2}}{\Omega_2\sqrt{\gamma_1}} = 1, \quad \alpha = \beta = \frac{1}{\sqrt{2}} \tag{16}$$

are fulfilled, the 'dark' resonance completely disappears in the case of an exact equality of detunings, $\Delta_1 = \Delta_2$, or becomes infinitely narrow in the case of $\Delta_1 \to \Delta_2$. The stages of this narrowing are shown in Fig. 2. At the last stage, the equality $|\Psi^{(\pm)}_{\rm post}\rangle = |\Psi^{(\pm)}_{\rm bright}\rangle$ is met and therefore CPT is impossible in this situation, and in a small neighbourhood (16) $|\Psi^{(\pm)}_{\rm post}\rangle$ contains only a small admixture $|\Psi^{(\pm)}_{\rm dark}\rangle$ which makes the 'dark' resonance narrower than in the absence of feedback.

The dependence of the width of the 'dark' resonance on the choice of the unraveling method is shown in Fig. 3 for $\alpha = 1/\sqrt{2}$. One can see that this width reaches a minimum at point $\alpha = \beta$ and a maximum at point $\alpha = -\beta$. The calculations showed that these points also coincide with the points of maximum and minimum of $|\langle\Psi^{(\sigma)}_{\rm post}|\Psi^{(\sigma)}_{\rm bright}\rangle|^2$, which characterises the closeness of the state of the system after photodetection and the corresponding 'bright' state. This explains the behaviour of the function in Fig. 3: the closer the $|\Psi^{(\sigma)}_{\rm post}\rangle$ to $|\Psi^{(\sigma)}_{\rm bright}\rangle$, the narrower the 'dark' resonance. It is also seen that by varying the parameters α and β, one can control the resonance width over a wide range of values.

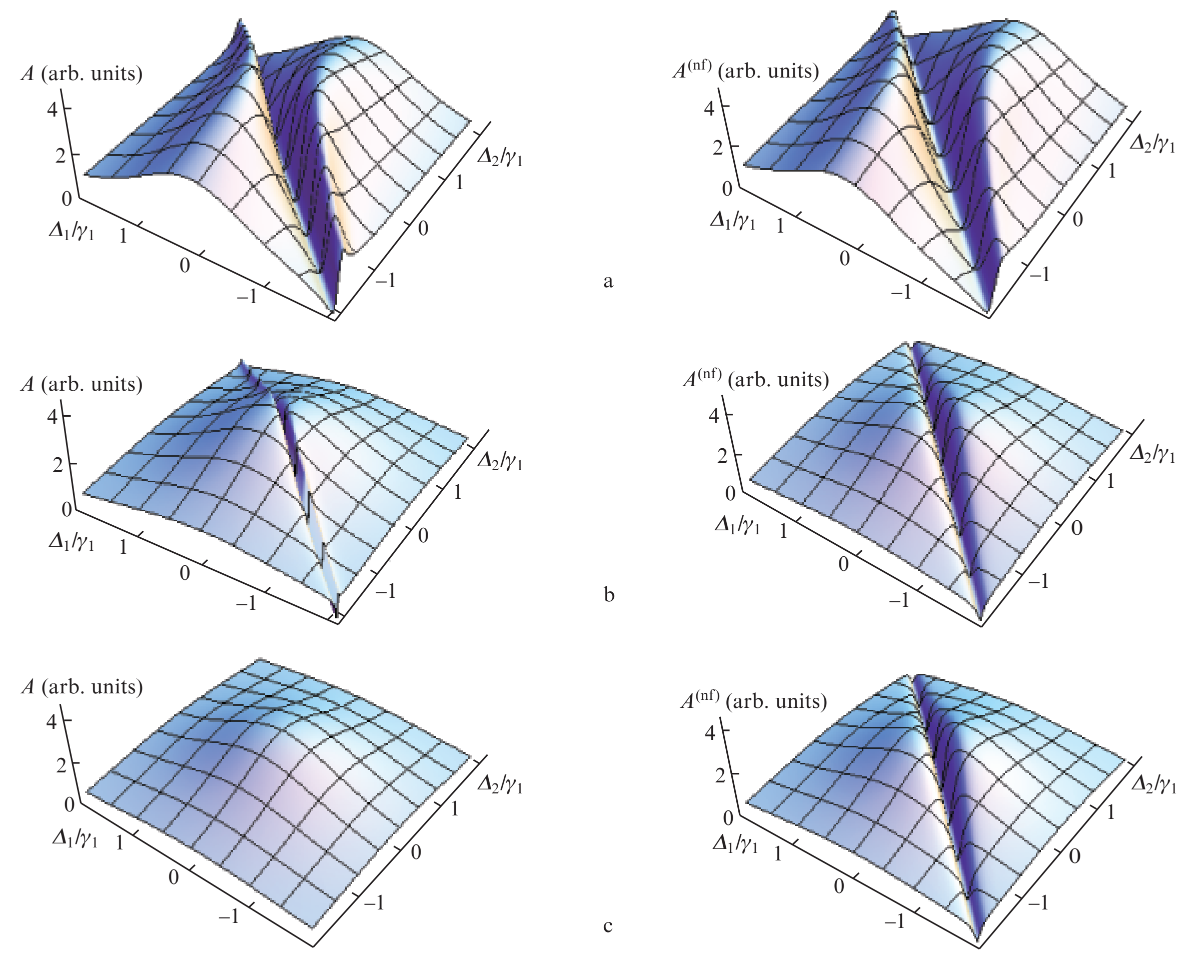

Figure 2. Field work per unit time with and without feedback (left and right, respectively) for $\alpha = \beta = 1/\sqrt{2}$, $\gamma_1 = \gamma_2$ and (a) $\Omega_1 = 0.25\gamma_1$, $\Omega_2 = 0.67\gamma_1$; (b) $\Omega_1 = 0.25\gamma_1$, $\Omega_2 = 0.3\gamma_1$; and (c) $\Omega_1 = \Omega_2 = 0.25\gamma_1$.

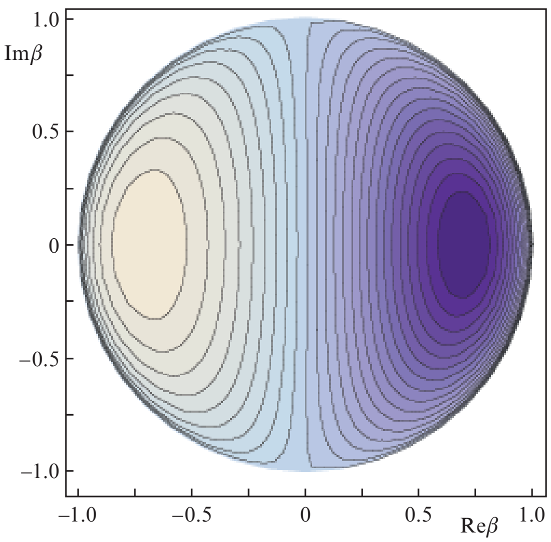

Figure 3. Width of the 'dark' resonance as a function of the parameter β calculated for $\Omega_1 = \gamma_1$, $\Omega_2 = 4\gamma_1$, $\gamma_2 = 2\gamma_1$. A lighter colour corresponds to larger values of the function.

The dependence of the 'dark' resonance width on the ratio between the Rabi frequencies $\Omega_{1,2}$ (see Fig. 2) leads to an unusual behaviour of the field work as a function of the Rabi frequencies (Fig. 4). The relation between the unraveling parameters is chosen in the form $\beta = \alpha\exp(\mathrm{i}\pi/6)$, and detunings are such that the system can appear both outside and inside the 'dark' resonance (depending on its width). At a fixed frequency Ω_1 and a small value of Ω_2, the work is also small, but it increases with increasing Ω_2. The sharpest growth of Ω_2 is observed when the system is initially located outside of the 'dark' resonance. For $\Omega_2 > \Omega_1\sqrt{\gamma_2/\gamma_1}$ (the condition of the minimum resonance width), the difference in the detunings $\Delta_1 - \Delta_2$ will be near the bottom of the broadening 'dark' resonance, so that the field work again decreases (Fig. 4c). A similar situation occurs for two other maxima: if we change the phase β to π, then the minimum resonance width will be observed for $\Omega_1\sqrt{\gamma_2} = -\Omega_2\sqrt{\gamma_1}$. The case $\alpha \approx \pm\beta$ deserves a special consideration. Here, changing the parameters along the straight lines, $\Omega_1\sqrt{\gamma_2} = \pm\Omega_2\sqrt{\gamma_1}$, will no longer drive the state of the system away from the state corresponding to the minimum width condition. As a result, the dependences of the work of the fields on the Rabi frequencies have narrow and extended 'walls', the fore-runners of which are observed in Fig. 4d.

A distinctive feature of the problem with feedback is the sharp asymmetry of the work values with positive and negative signs of the Rabi frequency. In addition, with sufficiently close field detunings, unusual narrow structures are observed that are fundamentally different from those in the case without feedback.

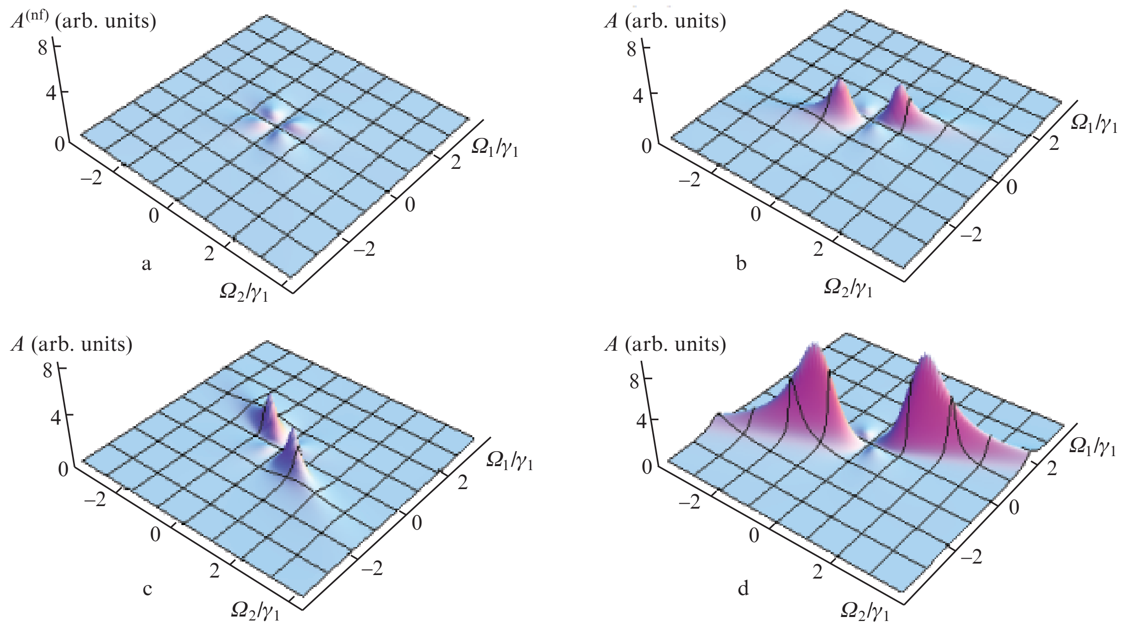

Figure 4. Field work per unit time as a function of the Rabi frequencies with (a) feedback and (b, c, d) without feedback for $\gamma_2 = 4\gamma_1$, $\Delta_1 = 0$, $\Delta_2 = -\gamma_1/10$ and (b) $\beta = \alpha\exp(i\pi/6)$, (c) $\beta = \alpha\exp(7i\pi/6)$ and (d) $\beta = \alpha\exp(i\pi/20)$.

4. Conclusions

We have theoretically investigated the feedback loop scheme applied to a three-level Λ-configuration system interacting with two classical electromagnetic fields. The spontaneous emission of the system has been detected, and the phase of one of the fields has been varied as a function of the type of the detected photoemission using a feedback loop. It has been shown that by choosing the appropriate types of photodetection events (i.e., realising a proper unraveling of the master equation) in a steady-state regime, it is possible to almost completely stop the phase switching. The possibility of controlling the width and shape of the 'dark' CPT resonance over a wide range has been also demonstrated by a fairly simple modification of the photodetection equipment. In addition, the dependences of the work of the fields on the Rabi frequencies exhibited narrow structures suitable for amplitude stabilisation, which is similar to the known method of frequency stabilisation.

The obtained results allow us to conclude that even relatively simple types of feedback based on the phase switching of the fields interacting with single quantum systems can significantly modify the properties of these systems and lead to the discovery of new effects.

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