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On the possibility of observing chaotic motion of cold atoms in rigid optical lattices

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Abstract. **The possibility of experimental observation of chaotic motion of cold atoms in a one-dimensional rigid optical lattice without any modulation of its parameters and additional impact is demonstrated theoretically and numerically. This effect of deterministic chaos arises near the optical resonance due to the nonlinear coupling of electronic and mechanical degrees of freedom in the atom. In a certain range of resonance detunings, at each crossing of the lattice-forming standing wave node by the atom, the atomic dipole moment changes pseudorandomly in a step-like manner, accompanied by an appropriate change in the atom momentum. This leads to the effect of chaotic wandering of atoms, possessing the properties of the classical deterministic chaos, namely, to the exponential sensitivity of atomic trajectories to small variations of the initial conditions and/or the control parameters. With time, part of atoms from the initial cloud changes the direction of their motion to the opposite one, which can be detected in a real experiment. Numerical experiments with lithium atoms with the spontaneous emission taken into account show that by varying the laser radiation intensity (normalised detuning of resonance) in the rigid optical lattice, one can implement the transition from the regular regime of the atomic motion to the chaotic wandering in a real experiment.**

Keywords: *cold atoms, rigid optical lattices, chaotic wandering.*

1. Introduction

One-, two-, and three-dimensional optical lattices (OLs) are produced using counterpropagating laser beams. In such lattice, neutral atoms are subject to the action of the dissipative force arising due to the absorption and emission of photons, and the gradient force caused by the interaction of the atomic electric dipole moment with a spatially nonuniform field of the standing light wave (SLW) [1, 2]. The gradient force is zero in the plane travelling wave and at exact optical resonance, when the frequency of the atomic transition ω_a is exactly equal to that of the SLW ω_f. In contrast to the dissipative force, it is not saturated with an increase in the Rabi frequency. The sufficiently cold atoms are concentrated in the optical potential minima of the OL, resembling a crystal lattice. As long ago as in 1962, Askaryan [3] showed that under the 'blue' detuning, $\omega_f - \omega_a > 0$, the gradient force acts towards a decrease in the field strength and pushes the atoms to the SLW nodes. Under the 'red' detuning, $\omega_f - \omega_a < 0$, this force acts towards an increase in the field strength and pushes the atoms to the SLW antinodes. Thus, sufficiently cold atoms are trapped by the optical potential and oscillate near the nodes or antinode of the SLW [4].

Cold atoms in OLs are a convenient object for studying many fundamental quantum phenomena, e.g., the Bose–Einstein condensation, quantum entanglement, quantum computing, quantum chaos, dynamic localisation and tunnelling, Bloch oscillations, etc. Among different practical applications, we would like to mention the atomic clock, atom lasers and interferometers, nanolithography, etc. As to the manifestations of quantum chaos of cold atoms in OLs, the most part of experiments [5–8] were carried out in the OLs with amplitude or frequency modulation of the parameters. To suppress spontaneous emission (SE) of atoms, the SLW frequency was tuned far from the frequency of the working atomic transition. Under such conditions the atoms can be modelled by quantum 'rotors' with periodic delta-like perturbations [9].

New possibilities appear when working near the optical resonance, where one has to take into account the variation of electronic (internal) degrees of freedom in the atom. Various dynamic effects have been analytically described and numerically found both in the semiclassical approximation and using the completely quantum description. Among these effects, we would like to mention such manifestations of Hamiltonian chaos, as chaotic Rabi oscillations [10–13], dynamic fractals [14–16], Lévy flights and anomalous diffusion [14, 16–18], proliferation of atomic wave packets at SLW nodes [19], quantum-classical correspondence [20], atomic chaos in 2D and 3D lattices [21, 22], and dissipative chaos with cold atoms [23, 24].

The aim of the present paper is to show the possibility of observing experimentally the chaotic motion of cold atoms in a one-dimensional rigid optical lattice without any modulation of its parameters and additional impact on the atoms. Based on the standard Hamiltonian of interaction between a two-level atom and a single-mode plane one-dimensional SLW, we derive the equations of motion in the semiclassical approximation for the coordinate and momentum of a point atom and for the Bloch vector components. The obtained system of nonlinear ordinary differential equations is analysed and solved numerically with the spontaneous emission taken into account using the stochastic wave function method. As a result, it is shown numerically that the histograms of the atomic coordinate and momentum distributions essentially differ from each other for different regimes of the motion. Varying the laser radiation intensity and, thus, varying the normalised resonance detuning in a rigid OL, it is possible to implement the transition from the regular regime of the atomic motion to the chaotic wandering in a real experiment.

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2. Results

For definiteness, we consider the transition $2^2S_{1/2}-2^2P_{3/2}$ with the wavelength λ_a = 671 nm and the frequency $\omega_a = 2\pi 4.47\times10^{14}$ rad s^{-1} in lithium-6 atoms with the mass $m_a = 1.15\times10^{-23}$ g. The lifetime of the excited level is $T_{sp} = 2.7\times10^{-8}$ s, the photon emission recoil velocity is v_r = 8.87 cm s^{-1}, and the corresponding recoil frequency is ν_r = 63 kHz.

The one-dimensional OL is formed by counterpropagating beams of laser radiation having the wavelength λ_f and the wavenumber k_f. The Hamiltonian of the two-level atom in the reference frame, rotating at the laser frequency ω_f, has the standard form

$$\hat{H} = \frac{P^2}{2m_a} + \frac{\hbar}{2}(\omega_a - \omega_f)\hat{\sigma}_z - \hbar\Omega(\hat{\sigma}_- + \hat{\sigma}_+)\cos k_f X - \frac{i\hbar\Gamma}{2}\hat{\sigma}_+\hat{\sigma}_-, \qquad (1)$$

where $\hat{\sigma}_{\pm,z}$ are the Pauli operators for the electronic degree of freedom of the atom; X and P are the atom coordinate and momentum; Ω is the amplitude value of the Rabi frequency; and $\Gamma = 1/T_{sp}$ is the relaxation coefficient. The motion of not too cold atoms can be considered in the semiclassical approximation, in which the electronic degree of freedom is described quantum mechanically, and the translational motion is treated as classical.

The wave function of the electronic degree of freedom has the form $|\Psi(t)\rangle = a(t)|2\rangle + b(t)|1\rangle$, where $a \equiv A + i\alpha$ and $b \equiv B + i\beta$ are complex-valued amplitudes of the probability to detect the atom in the excited ($|2\rangle$) and the ground ($|1\rangle$) state. The equations of motion for the real and imaginary parts of these amplitudes, the coordinate and the momentum of the point atom have the form:

$$\begin{aligned}
&\dot{x} = \omega_r p, \quad \dot{p} = -2(AB + \alpha\beta)\sin x, \\
&\dot{A} = \frac{1}{2}(\omega_r p^2 - \Delta)\alpha - \frac{1}{2}\gamma A - \beta\cos x, \\
&\dot{\alpha} = -\frac{1}{2}(\omega_r p^2 - \Delta)A - \frac{1}{2}\gamma\alpha + B\cos x, \qquad (2)\\
&\dot{B} = \frac{1}{2}(\omega_r p^2 + \Delta)\beta - \alpha\cos x, \\
&\dot{\beta} = -\frac{1}{2}(\omega_r p^2 + \Delta)B + A\cos x,
\end{aligned}$$

where $x \equiv k_f X$; $p \equiv P/\hbar k_f$; and the differentiation is carried out with respect to the dimensionless time $\tau \equiv \Omega t$. Equations (2) comprise several normalised control parameters, namely, the atomic recoil frequency, $\omega_r \equiv \hbar k_f^2/(m_a\Omega)$; the detuning from the resonance, $\Delta \equiv (\omega_f - \omega_a)/\Omega$; and the decay rate of the atomic excited level, $\gamma = \Gamma/\Omega$.

Two conditions should be satisfied to make the semiclassical approximation valid. First, the atom recoil energy should be much smaller than the atom kinetic energy and the energy of the atom–field interaction, which for Eqns (2) has the form of a simple condition $\omega_r \ll 1$. Second, the atom momentum should be much larger than that of the photon, i.e., $p \gg 1$. Choosing the amplitude value of the Rabi frequency to be $\Omega_0/2\pi = 125\times10^6$ Hz, we arrive at the normalised recoil frequency of the atom, $\omega_r = 10^{-3} \ll 1$, and the normalised decay rate of the excited level, $\gamma = 5\times10^{-2}$.

As a result of SE recoil, the momentum of the atom changes by a certain random value lying in the interval $|\Delta p| \leqslant 1$. Besides that, the quantity $2(AB + \alpha\beta)\sin x$ varies as the atom moves through the lattice. The variations are caused not only by the SLW field gradient, but also by the behaviour of the component of the atom electric dipole moment $u \equiv 2(AB + \alpha\beta)$.

The initial cloud of N noninteracting cold atoms has the Gaussian coordinate and momentum distribution

$$\rho(x,p) = \frac{1}{2\pi\sigma_x\sigma_p}\exp\left(-\frac{|x-x_0|^2}{2\sigma_x^2} - \frac{|p-p_0|^2}{2\sigma_p^2}\right),$$

where we put $x_0 = 0$, $p_0 = -10$ and $\sigma_x = \sigma_p = 2$. All atoms are in the ground state: $A(0) = \alpha(0) = \beta(0) = 0$, $B(0) = 1$. The laser is switched on at a time $\tau = 0$. The coordinates and the momenta of atoms are calculated at a certain fixed moment of time. Note that the mean value of the normalised momentum in the atomic cloud, $p_0 = -10$, is greater than the momentum of a photon by an order of magnitude and has a negative value, i.e., all atoms initially fly along the negative OL semiaxis.

To prove that an abrupt pseudorandom change in the atomic dipole moment u at the time of crossing the SLW node by the atom is the key cause of its chaotic wandering in the rigid OL, we first perform a numerical experiment in the absence of SE, i.e., we put $\gamma = 0$ in Eqn (2). Let us introduce the Bloch vector components

$$u \equiv 2\mathrm{Re}(ab^*), \quad v \equiv -2\mathrm{Im}(ab^*), \quad z \equiv |a|^2 - |b|^2, \qquad (3)$$

where u and v are the components of the atomic dipole moment; and z is the population inversion of the atomic level. The equations of motion in the absence of dissipation can be written in the form

$$\begin{aligned}
&\dot{x} = \omega_r p, \quad \dot{p} = -u\sin x, \quad \dot{u} = \Delta v, \\
&\dot{v} = -\Delta u + 2z\cos x, \quad \dot{z} = -2v\cos x.
\end{aligned} \qquad (4)$$

These equations have two integrals of motion, namely, the total energy

$$H \equiv \frac{\omega_r}{2}p^2 - u\cos x - \frac{\Delta}{2}z \qquad (5)$$

and the Bloch vector length $u^2 + v^2 + z^2 = 1$.

Under condition of exact resonance, we have $u(\tau)$ = const = $u(0)$ = 0. Therefore, the optical potential is absent, and p = const = $p(0)$ for every atom. Thus, the cloud of atoms flies in the negative direction of the x axis, conserving its initial shape. This is confirmed by the histogram of the coordinate distribution of 10^4 atoms in Fig. 1a, calculated at a fixed moment of time $\tau = 10^4$.

In the presence of detuning from the resonance, the atomic motion is not so trivial, but also regular. For the system of equations (4) it was shown in Ref. [20], that the maximal Lyapunov exponent (the quantitative measure of deterministic chaos) differs from zero in the interval of detunings $|\Delta| <$ 0.8, i.e., at relatively large absolute values of the detuning the motion is regular. The histogram of the coordinate distribution at a time moment $\tau = 10^4$ is shown in Fig. 1b for $\Delta = 1$. For such values of detuning, the majority of atoms appear to be trapped in the optical potential wells, nearest to the centre of the atomic cloud. Only a small part of atoms with the initial momenta exceeding the potential barrier, are able to fly through a significant distance in the negative direction of the x axis. Moreover, far from resonance the efficiency of the

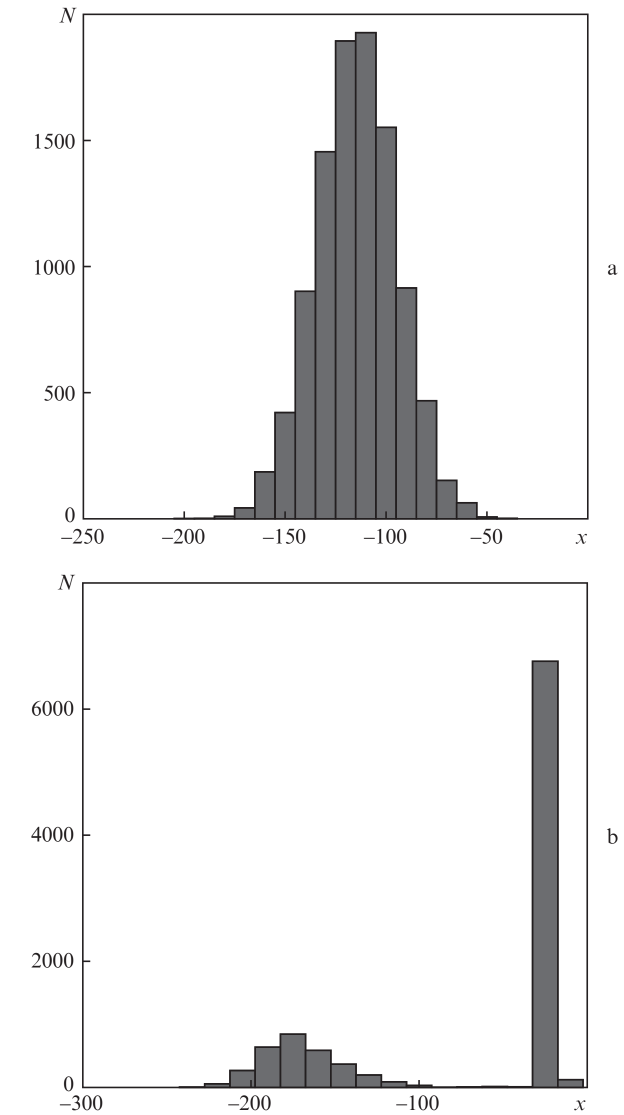

Figure 1. Histograms of atomic coordinate distributions at a fixed time moment without SE taken into account: (a) the ballistic flight of atoms through the SLW at exact resonance ($\Delta = 0$); (b) the trapping of a part of the atomic cloud in the optical potential wells and the ballistic flight of the other part of the cloud through the SLW at $\Delta = 1$.

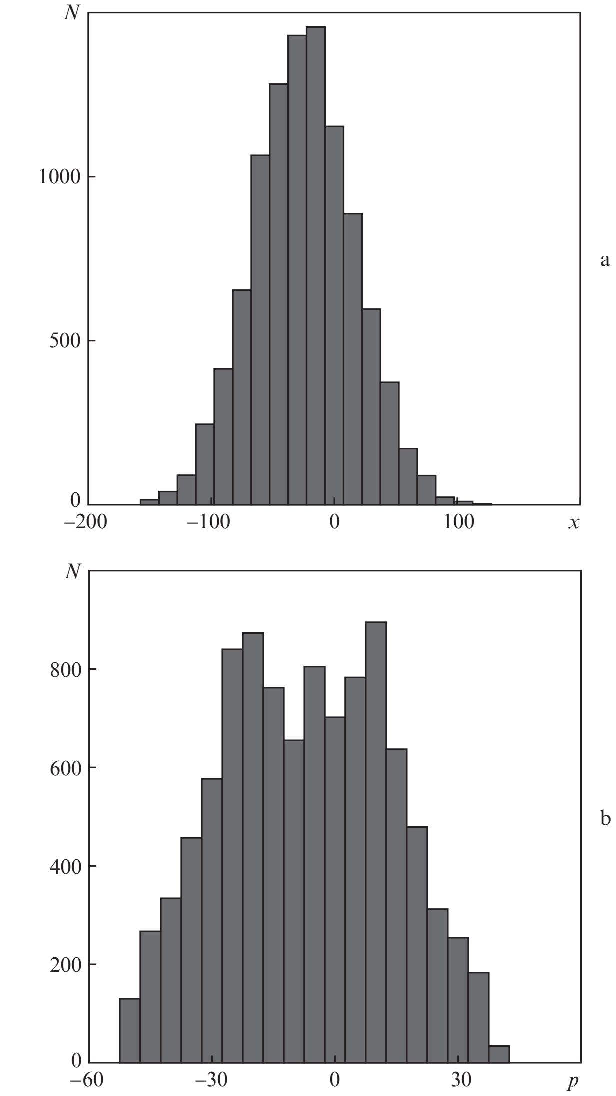

Figure 2. Chaotic wandering of atoms in the SLW near the resonance ($\Delta = 0.15$) without SE taken into account. The histograms of (a) coordinate and (b) momentum distributions of atoms.

atom excitation by the laser field falls. We emphasise that both for exact resonance and far enough from it all atoms of the initial cloud at the time moment $\tau = 10^4$ have negative values of the coordinate.

At relatively small detunings, the atomic motion dramatically changes. From the histograms of Fig. 2, it follows that a significant part of atoms for $\Delta = 0.15$ have changed their direction of motion. At a fixed time moment $\tau = 10^4$ the atoms of the cloud have a nearly Gaussian coordinate distribution in the interval from –150 to 100, and with respect to the momenta in the interval from –50 to 45. The reason for a change in the atomic motion direction is the chaotic wandering of atoms, which, in turn, is caused by a stepwise change in the component of the atomic electric dipole moment u at the time of crossing the SLW nodes by the atom. As shown in Ref. [16], the value of u experiences pseudorandom changes in the range $[-1, 1]$ when crossing each node. From the equations of motion (4) it follows that these changes cause an appropriate pseudorandom change in the atomic momentum at crossing each node. As a result, with time part of atoms acquire positive values of the momenta, and the atomic cloud splits.

The direct calculation of the maximal Lyapunov exponent using the method of Ref. [25] yields the value 0.03 for $\Delta = 0.15$, confirming quantitatively the fact of chaotic motion of atoms in the sense of exponential sensitivity to the small changes in the initial conditions. In other words, two atoms with a very small initial difference of their positions in the cloud can begin with time to move in opposite directions. On average, the velocity of their separation is close to exponential.

From the expression of the normalised atomic energy (5), one can deduce the condition, at which the atom will change the sign of p after crossing the mth node of the SLW. The result depends on the parity of the number m of crossings. The atom will continue moving in the same direction, if $(-1)^{m+1}u_m < H$, and will change the direction for the opposite one, if $(-1)^{m+1}u_m > H$. In the case of exact equality, $(-1)^{m+1}u_m = H$, the motion is separatrix-like. The chaotic wandering is possible, if the energy of the atom lies in the interval $0 < H < 1$. At $H < 0$ the atoms do not reach even the nearest node and are trapped by the optical potential well, while at $H > 1$ the values of u always correspond to the motion in the initial direction. Thus, if $|u| < H$, then the atoms continue moving in the same direction, and when $|u| > H$, then, depending on the sign of $\cos x$ within the considered time interval, they either continue moving in the same direction, or change it for the opposite one.

In reality, the coherent evolution of lithium atoms interacting with the laser field near the resonance is interrupted by the SE events at random time moments. As a result of each event, the atomic momentum changes by a random value from the interval [–1, 1]. The numerical solution of Eqn (2) is implemented using the stochastic wave function method (see, e.g., [26, 27]), i.e., the following algorithm. The integration time is partitioned into a large number of small intervals $\Delta\tau = 10^{-5}$. The random number generator chooses the number ε_1 from the interval [0, 1]. At the end of the first time interval $\tau = \tau_1$, the probability of the atom SE is calculated using the formula

$$s_1 = \gamma\Delta\tau\,|a_{\tau_1}|^2/(|a_{\tau_1}|^2 + |b_{\tau_1}|^2).$$

Then, depending on the relation between the numbers ε_1 and s_1, two cases are possible.

1. If $s_1 < \varepsilon_1$, then the integration of equations (2) is continued at the next time interval. Since the coherent evolution occurs with a decrease in the wave function norm, the state vector should be renormalised immediately after the end of the first interval at $\tau = \tau_1^+$:

$$a_{\tau_1^+} = a_{\tau_1}/\sqrt{|a_{\tau_1}|^2 + |b_{\tau_1}|^2}, \quad b_{\tau_1^+} = b_{\tau_1}/\sqrt{|a_{\tau_1}|^2 + |b_{\tau_1}|^2}.$$

This coherent evolution with the decreasing norm of the wave function describes the continuous relaxation of the atomic dipole moment.

2. If $s_1 \geqslant \varepsilon_1$, then the SE occurs, and the atom finds itself in the ground state at the time moment $\tau = \tau_1$: $A_{\tau_1} = \alpha_{\tau_1} = \beta_{\tau_1} = 0$, $B_{\tau_1} = 1$. The atomic momentum changes in a stepwise manner: $p_{\tau_1} = p_{\tau_1^-} + l$, where l is a random number from the interval [–1, 1], describing the change in the atomic momentum as a result of photon SE (recoil effect). The procedure is repeated at the next time intervals.

The main result of the numerical modelling with SE taken into account is that SE significantly facilitates the observation of the chaotic wandering effect in a real experiment. From the comparison of Fig. 2a (without SE) and Fig. 3a (with SE) it

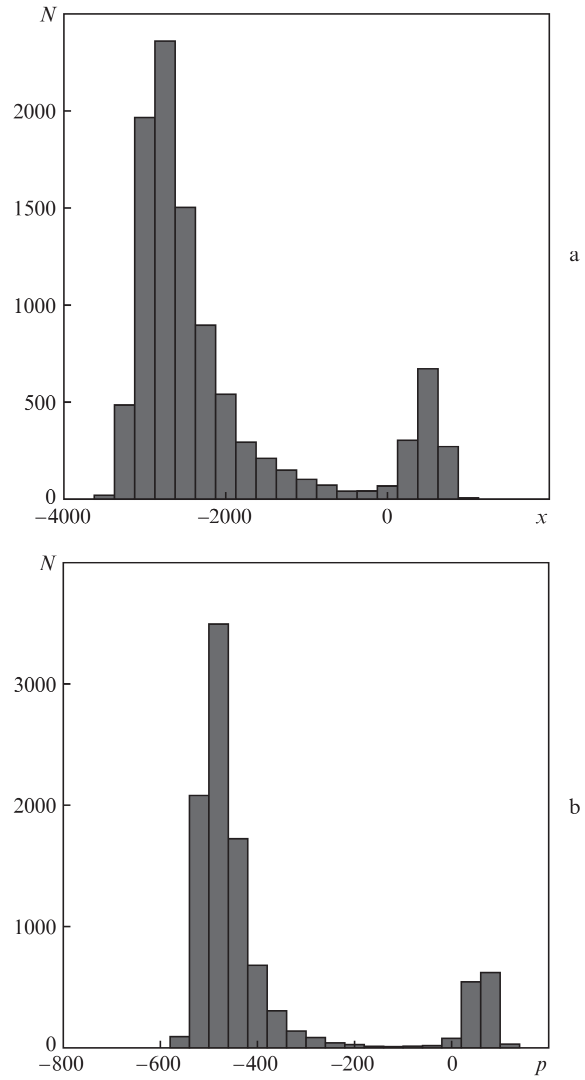

Figure 3. Chaotic wandering of atoms near the resonance ($\Delta = 0.15$) with SE taken into account. The histograms of (a) coordinate and (b) momentum distributions of atoms.

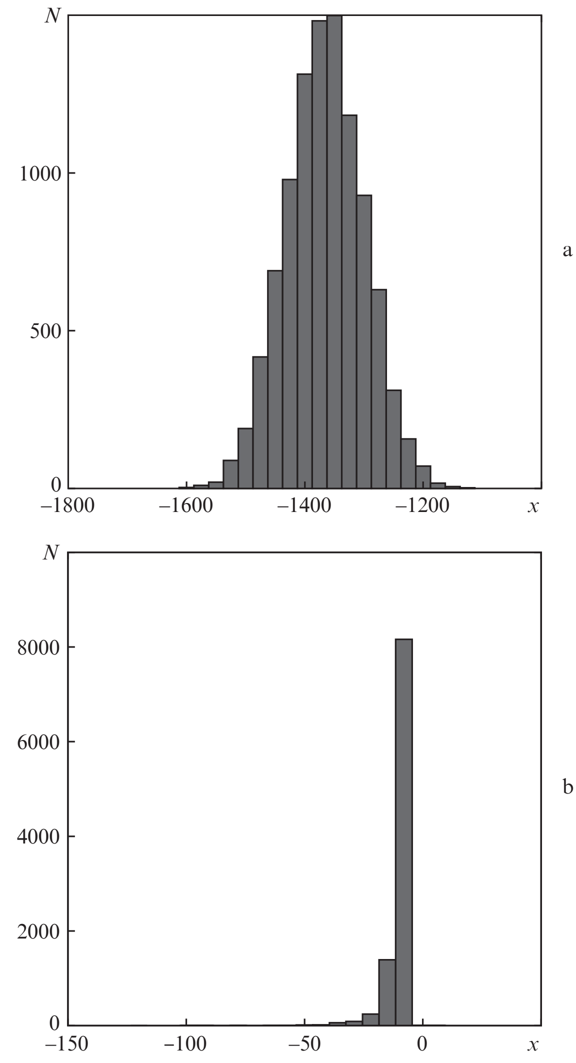

Figure 4. The same as in Fig. 1 with SE taken into account.

follows that in the latter case one can observe not simply an increase in the initial atomic cloud size, but also its splitting with an increase in atomic path lengths both in the positive and in the negative direction of the x axis. A fraction of the atomic cloud moves in the positive direction of the x axis, achieving the distance of $x \simeq 1000$ at a time moment $\tau = 10^4$. At the chosen values of the parameters for lithium atoms, this corresponds to the shift by $x = 100$ μm during 10^{-5} s, which can be detected in a real experiment. The momentum distribution of atoms with SE taken into account also has the double-humped shape (see Fig. 3b) with the momentum values greater by an order of magnitude than that in coherently evolving atoms (see Fig. 2b). For comparison, Fig. 4 shows the histograms of coordinate distributions of atoms with SE taken into account under the condition of exact resonance (Fig. 4a) and far from the resonance (Fig. 4b), where the coordinates of all atoms are negative. The difference from Fig. 3a is apparent.

3. Conclusions

In the paper, a new effect is described that arises under the interaction of cold atoms with the laser beams, forming a SLW. It is shown theoretically and numerically that in a certain range of resonance detunings the atoms can wander chaotically in a rigid OL without any modulation of the optical lattice parameters or additional external action on the atoms. This effect of deterministic chaos arises near the optical resonance due to the nonlinear coupling of electronic and mechanical degrees of freedom of the atom, as a result of which each crossing of the SLW node by the atom is accompanied by a stepwise pseudorandom change in the atomic dipole moment with an appropriate change in the atomic momentum. Due to this effect, after some time a part of atoms from the initial cloud changes the direction of motion for the opposite one, which leads to the splitting of the atomic cloud that can be detected in a real experiment. The numerical experiments with lithium-6 atoms with SE taken into account have shown that by varying the laser radiation intensity in a real experiment one can provide different regimes of atomic motion in the OL, from ballistic flight to chaotic wandering.

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References

1. Kazantsev A.P., Surdutovich G.I., Yakovlev V.P. *Mechanical Action of Light on Atoms* (Singapore, London: World Scientific, 1990).
2. Minogin V.G., Letokhov V.S. *Laser Light Pressure on Atoms* (New York: Gordon and Breach, 1987).
3. Askar'yan G.A. *Sov. Phys. JETP*, **15**, 1088 (1962) [*Zh. Eksp. Teor. Fiz.*, **42**, 1567 (1962)].
4. Letokhov V.S. *JETP Lett.*, **7**, 272 (1968) [*Pis'ma Zh. Eksp. Teor. Fiz.*, **7**, 348 (1968)].
5. Moore F.L., Robinson J.C., Bharucha C., et al. *Phys. Rev. Lett.*, **73**, 2974 (1994).
6. Moore F.L., Robinson J.C., Bharucha C., et al. *Phys. Rev. Lett.*, **75**, 4598 (1995).
7. Steck D.A., Oskay W.H., Raizen M. *Science*, **293**, 274 (2001).
8. Hensinger W.K., Heckenberg N.R., Milburn G.J., Rubinsztein-Dunlop H. *J. Opt. B: Quantum Semiclass. Opt.*, **5**, 83 (2003).
9. Graham R., Schlautmann M., Zoller P. *Phys. Rev. A*, **45**, 19 (1992).
10. Kon'kov L.E, Prants S.V. *JETP Lett.*, **65**, 833 (1997) [*Pis'ma Zh. Eksp. Teor. Fiz.*, **65**, 801 (1997)].
11. Prants S.V. *Phys. Lett. A*, **33**, 225 (1997).
12. Prants S.V., Kon'kov L.E., Kiriluyk I. *Phys. Rev. E*, **60**, 335 (1999).
13. Prants S.V., Sirotkin V.Yu. *Phys. Rev. A*, **64**, 033412 (2001).
14. Argonov V.Yu., Prants S.V. *J. Exp. Theor. Phys.*, **123** (5), 832 (2003) [*Zh. Eksp. Teor. Fiz.*, **123** (5), 946 (2003)].
15. Prants S.V, Uleysky M.Yu., Argonov V.Yu. *Phys. Rev. A*, **73**, 023807 (2006).
16. Argonov V.Yu., Prants S.V. *Phys. Rev. A*, **75**, 063428 (2007).
17. Prants S.V., Kon'kov L.E. *JETP Lett.*, **73** (4), 180 (2001) [*Pis'ma Zh. Eksp. Teor. Fiz.*, **73** (4), 200 (2001)].
18. Argonov V.Yu., Prants S.V. *J. Russ. Laser Res.*, **27**, 360 (2006).
19. Prants S.V. *J. Exp. Theor. Phys.*, **109** (5), 751 (2009) [*Zh. Eksp. Teor. Fiz.*, **136** (5), 872 (2009)].
20. Prants S.V. *Chaos, Solitons & Fractals*, **43**, 1 (2010).
21. Horsley E., Koppell S., Reichl L.E. *Phys. Rev. E*, **89**, 012917 (2014).
22. Boretz Y., Reichl L.E. *Phys. Rev. E*, **91**, 042901 (2015).
23. Argonov V.Yu., Prants S.V. *Phys. Rev. A*, **71**, 053408 (2005).
24. Argonov V.Yu., Prants S.V. *Phys. Rev. A*, **78**, 043413 (2008).
25. Kon'kov L.E., Prants S.V. *J. Math. Phys.*, **37**, 1204 (1996).
26. Carmichael H. *An Open Systems Approach to Quantum Optics* (Berlin: Springer-Verlag, 1993).
27. Molmer K., Castin Y., Dalibard J. *J. Opt. Soc. Am. B*, **10**, 524 (1993).