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A simple method of measuring the effective SRS coefficient in single-mode optical fibres and the range of its applicability

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Abstract. **This paper describes techniques for measuring the SRS coefficient in a wide spectral range, including the region of small Stokes shifts. A simple, approximate method is proposed for evaluating the SRS coefficient near a gain peak. Spectral dependences of the SRS coefficient are presented for various telecom fibres.**

Keywords: *Raman scattering, SRS coefficient, Raman amplifier, SRS tilt, fibre-optic communication link.*

1. Introduction

The use of distributed Raman amplifiers in combination with standard erbium fibre amplifiers allows one to significantly improve the signal-to-noise ratio in multi- and single-span fibre-optic communication links [1 – 4] owing to the reduction in amplified spontaneous emission noise and nonlinear distortions.

Recent years have seen a substantial increase of interest in the use of Raman amplifiers due to the advent of the multilevel modulation schemes QAM16, QAM64 and even QAM256, which require considerably higher optical signal-to-noise ratios. It is also worth noting the importance of the traditional use of Raman amplifiers and lasers for extending the spectral range of fibre laser sources.

A key parameter for calculating characteristics of such an amplifier is the effective SRS coefficient g_R, which is determined by the characteristics of the fibre that ensures Raman amplification.

Several approaches to measuring the spectral dependence of the SRS coefficient (vs. frequency detuning or Stokes shift) have been proposed to date. This dependence can be derived from the spectral dependence of the gain coefficient of a small signal (for both a wavelength-tunable and a broadband source) counterpropagating with the pump [5 – 7], or from the emission spectrum of amplified spontaneous Raman scattering (ASpRS) [5, 8]. The former approach has the advantage that the SRS coefficient is easy to calculate from the small-signal gain coefficient, because they are related by a well-known formula [9]. However, a signal source operating in the wavelength range of interest is needed for experimental studies in this case.

The latter method has the advantage that measurements can be made even when only one fibre end is accessible. This advantage is especially important if it is necessary to measure the SRS gain coefficient of telecom fibres in existing communication links in going to multilevel modulation schemes for increasing the transmission capacity of the links.

This paper demonstrates that measuring the SRS gain coefficient from ASpRS spectra ensures sufficient accuracy for using such coefficients in measurements of the cable infrastructure of existing fibre-optic communication links. Moreover, this technique makes it possible to measure the SRS coefficient in a wide spectral range and evaluate it with high accuracy at small Stokes shifts in order to take into account the interchannel SRS tilt in dense wavelength division multiplexing (DWDM) links, which has not been proposed previously. We consider a simple, approximate method of estimating the SRS coefficient near a gain peak with the aim of characterising and comparing different fibres in terms of this parameter.

The described technique is used to measure SRS gain spectra of new types of fibre that are employed in optical fibre communication systems.

2. Schematic and technique of SRS gain coefficient measurement from ASpRS spectra

Figure 1 shows a schematic of the experiment in which we measured the emission spectrum of ASpRS counterpropagating with the pump. Pump light is coupled into the fibre under study, of length L. A 99/1 directional coupler was used to monitor the pump power at the fibre input and the emission spectrum of ASpRS counterpropagating with the pump. To properly measure SRS gain spectra in long lengths of isotropic optical fibres, a depolarised pump source should be used. In our experiments, we used a Raman fibre laser operating at a wavelength of 1480 nm or two semiconductor lasers operating at the same wavelength, whose outputs were combined using

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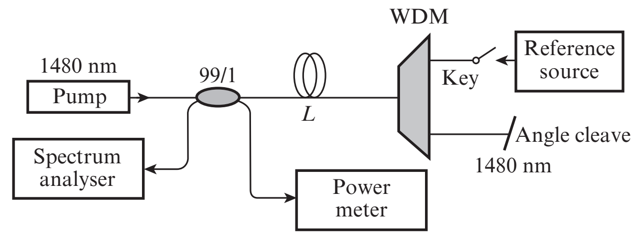

Figure 1. Schematic of SRS coefficient measurement from ASpRS spectra. Closing the key yields a measurement scheme with a reference source.

a polarisation multiplexer. At equal total powers, the measured ASpRS spectra were identical.

To find out how ASpRS spectra and the spectral dependence of the SRS coefficient are related to each other, consider a model for Raman amplification in single-mode optical fibres.

3. Calculational model

Pump–signal interaction in a Raman amplifier obeys the well-known system of rate equations that describe the evolution of the relevant intensities or powers as the light propagates along the optical fibre [9, formula (4.6)]. After a proper change of variables, the rate equations can be rewritten in the form of equations for the pump and signal photon densities:

$$\frac{dn_s}{dz} = \gamma n_p(n_s + n_{spon}) - \alpha_s n_s,$$
$$\frac{dn_p}{dz} = \gamma n_p(n_s + n_{spon}) - \alpha_p n_p, \tag{1}$$

where n_s and n_p are the number densities of signal and pump photons, respectively; γ is the SRS coefficient expressed in inverse metres; α_s and α_p are the signal and pump attenuation coefficients; and n_{spon} is the spontaneous scattering coefficient, dependent on the Stokes shift:

$$n_{spon} = 1 + \Theta(\Delta\nu) = 1 + \left[\exp\left(\frac{h\Delta\nu}{k_B T}\right) - 1\right]^{-1} \tag{2}$$

(here h is the Planck constant; k_B is the Boltzmann constant; T is the absolute temperature; $\Delta\nu$ is the frequency shift between the signal and pump; and $\Theta(\Delta\nu)$ is the average number of photons at thermodynamic equilibrium at temperature T, or the Bose–Einstein coefficient).

To represent Eqn (1) in power units [9], we make the change

$$\gamma n_p \to g_R P_p, \tag{3}$$

where P_p is the pump power and g_R is the SRS coefficient expressed in units of m^{-1} W^{-1}.

In the small-signal approximation, the term containing n_s in the second equation of system (1) can be neglected. The solution then takes the form

$$n_s(L) = n_s(0)\, G_{eff} \exp(-\alpha_s L) + n_n, \tag{4}$$

where $G_{eff} = \exp(\gamma n_p^L L_{eff})$ is the effective gain coefficient; n_n is the number of noise photons:

$$n_n = n_{spon}\left(\frac{\alpha_p}{\gamma n_p^L}\right)^{\alpha_s/\alpha_p} \exp\left(\frac{\gamma n_p^L}{\alpha_p}\right)$$
$$\times\left[\Gamma\left(1 + \frac{\alpha_s}{\alpha_p}, \frac{\gamma n_p^L}{\alpha_p}\exp(-\alpha_p L)\right) - \Gamma\left(1 + \frac{\alpha_s}{\alpha_p}, \frac{\gamma n_p^L}{\alpha_p}\right)\right]; \tag{5}$$

n_p^L is the number of pump photons at the fibre input (counter-propagating with the signal); $\Gamma(a,x)$ is the incomplete gamma function; L is the fibre length; and $L_{eff} = [1 - \exp(-\alpha_p L)]/\alpha_p$ is the effective fibre length.

The power spectral density of ASpRS is related to the number of photons by

$$\frac{\delta P_{ASpRS}}{\delta\nu} = n_n h\nu, \tag{6}$$

where ν is the frequency of the light.

Using relations (5) and (6), the SRS coefficient can be found numerically.

In the small ASpRS signal approximation, according to (4) the SRS coefficient can be evaluated using the simple relation

$$\Delta S\,[\mathrm{dB}] = 4.34 g_R L_{eff} \Delta P_0, \tag{7}$$

where ΔS is the ASpRS difference spectrum for two pump power values differing by ΔP_0. The range of validity of this approximation will be considered below.

4. Range of validity of the small amplified spontaneous Raman scattering signal approximation

Consider relation (5) for the number of noise photons. Transforming to the new variables $t = \gamma n_p^L/\alpha_p$ and $A = \alpha_s/\alpha_p$ and taking the logarithm of (5), we obtain

$$\ln n_n - \ln n_{spon} = -A \ln t + t$$
$$+ \ln[\Gamma(1 + A, t\exp(-\alpha_p L)) - \Gamma(1 + A, t)]. \tag{8}$$

For $n_p^L \to \infty$ we have $t \to \infty$ and obtain [10]

$$\Gamma(1 + A, t\exp(-\alpha_p L)) \gg \Gamma(1 + A, t),\ \Gamma(a,t) \to \exp(-t)\, t^a,$$
$$\ln(n_n/n_{spon}) = -A \ln t + t + \ln\{[t\exp(-\alpha_p L)]$$
$$\times \exp[-t\exp(-\alpha_p L)]\} = t[1 - \exp(-\alpha_p L)] - \alpha_s L$$
$$= \ln G_{eff} - \alpha_s L. \tag{9}$$

Relation (9) is an asymptotic approximation of (5) at a pump power tending to infinity, which is equivalent to relation (7).

Since in this method the SRS coefficient is calculated from the ASpRS difference spectrum for two distinct pump powers, to assess the uncertainty in our measurements it is necessary to calculate the uncertainty in the derivative of the optical power (or the number of photons) with respect to the effective gain coefficient.

Consider the magnitude of the derivative of the function $\ln n_\mathrm{n}$ with respect to $\ln G_\mathrm{eff}$ for the number of noise photons, n_n, given by (5), obtained from an exact solution, at different values of the total absorption over the fibre length, $\alpha_\mathrm{s}L$ (Fig. 2).

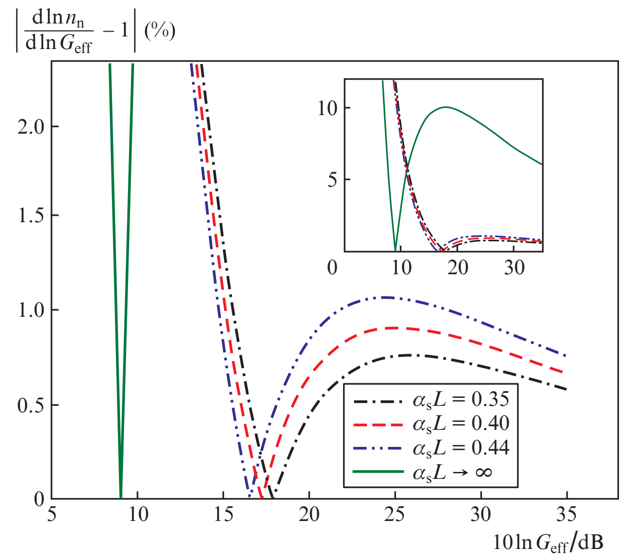

Figure 2. Magnitude of the derivative of the function $\ln(n_\mathrm{n}/G_\mathrm{eff})$ with respect to $\ln G_\mathrm{eff}$ at different values of the loss $\alpha_\mathrm{s}L$ over the fibre length L. Inset: the data plotted on compressed scales.

The data characterise uncertainty in SRS coefficient measurements from the difference between ASpRS spectra. It follows from Fig. 2 that, at low $\alpha_\mathrm{s}L$ values ($\alpha_\mathrm{s}L < 0.5$), uncertainty in g_R measurements does not exceed 1 %, but in such a case a sufficiently large effective gain coefficient should be ensured (above 17 dB). At high $\alpha_\mathrm{s}L$ values ($L \gg L_\mathrm{eff}$), uncertainty in SRS coefficient calculation increases (Fig. 2, solid line).

Figure 3 shows the SRS coefficient measured using a 27.9-km length of G652.D Corning SMF-28 fibre. In this paper, we compare spectral dependences of the SRS coefficient obtained in several ways. First, we use an exact solution for the number of noise photons (5), the ASpRS spectrum and a numerical method [Fig. 3, curve (*1*)]. Second, we use an approximate solution for the number of noise photons (9) and the difference between ASpRS spectra at two distinct pump powers [Fig. 3, curve (*2*)]. It is seen from these data that, on the wings of the graph of the SRS coefficient *vs.* frequency shift, the curves obtained using the exact and approximate solutions diverge, which can be accounted for by the small effective gain coefficient in this region, where the approximate solution is thus inapplicable. Third, we additionally compared the results with the small-signal gain coefficient measured with a reference source [Fig. 3, curve (*3*)].

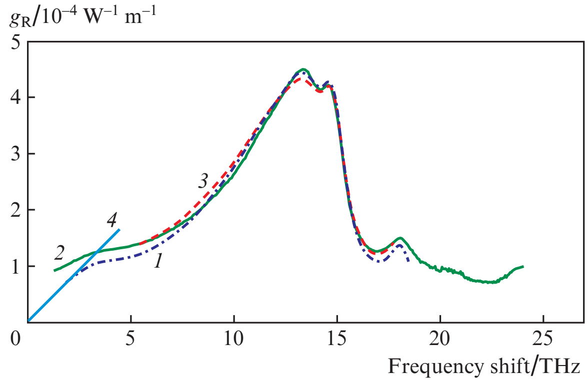

Figure 3. SRS coefficient as a function of frequency shift: the data obtained (*1*) from ASpRS spectra of a Raman amplifier using an exact solution to (5), (*2*) from the ASpRS spectra by an approximate method, (*3*) from the measured gain coefficient of a reference source and (*4*) by direct SRS tilt measurements [11].

The main limitation determining the range of applicability of the approximate SRS coefficient measurement method is the so-called double Rayleigh scattering threshold [9]. Reaching this threshold leads to distributed random feedback lasing. This distorts the ASpRS spectrum and limits the maximum gain coefficient in experiments. The threshold effective SRS gain coefficient is determined by the Rayleigh backscatter coefficient (4.5×10^{-5} km^{-1}) and is about 46 dB in the case of standard fibre [12]. Scaling the maximum in the graph in Fig. 3 to this value, we find that, with the above-mentioned 17-dB tentative level, the SRS coefficient can be measured by the approximate method with required accuracy in the range of Stokes shifts no broader than 7.5–16 THz. Clearly, this numerical estimate is only valid for fibres with a similar chemical composition of their core. Other types of fibre will have a different permissible range (for example, the SRS coefficient of phosphate glasses has maxima near 19.5 and 42 THz [13]). At the same time, in the case of fibres identical in losses and core composition, such an estimate should be independent of the type of fibre. Thus, the obtained range of Stokes shifts is the same for standard telecom fibres having a germanium-containing core and for fibres having a pure silica core.

In examining nonlinear interchannel interaction effects in multichannel fibre-optic communication lines, it is of practical importance to evaluate the SRS coefficient at small Stokes shifts (under 4 THz). In DWDM communication links, stimulated Raman scattering leads to signal amplification in the longer wavelength channels at the expense of the shorter wavelength channels. A widespread approach is fitting the spectral dependence of the SRS coefficient by a straight line, which allows one to derive a simple relation between the slope of this dependence and the tilt of the multichannel spectrum at the output of the fibre in which the signal propagates [14]:

$$\delta(\Delta p) = 4.34\frac{\mathrm{d}g_\mathrm{R}}{\mathrm{d}\nu}\Delta\Omega L_\mathrm{eff}P_\mathrm{s}^0, \qquad (10)$$

where $\delta(\Delta p)$ (dB) is a nonlinear term in the interchannel tilt, defined as the difference in power between the extreme spectral components of the signal; $\mathrm{d}g_\mathrm{R}/\mathrm{d}\nu$ is the slope of the spectral dependence of the SRS coefficient at the origin; $\Delta\Omega$ is the full width of the signal spectrum; and P_s^0 is the total power at the input of the link.

Using (10), the slope can be determined experimentally, by direct SRS tilt measurements in a communication link [12]. Such measurements were made in this study. A straight line with the experimentally determined slope, representing the spectral dependence of the SRS coefficient at small Stokes shifts, is shown in Fig. 3 [line (*4*)]. It is seen that its slope agrees well with that of the line obtained by extrapolating the curve derived from ASpRS spectra to the origin.

5. Experimental results

We measured the SRS coefficient in various telecom fibres (Fig. 4): Corning SMF-28 and OFS AllWave ZWP germanosilicate core step-index fibres (ITU-T G.652.D standard, effective mode area $A_{\mathrm{eff}} = 85$ μm^2), OFS TrueWave non-zero dispersion shifted fibre (NZDSF) (ITU-T G.655 standard, $A_{\mathrm{eff}} \approx 52$ μm^2) and large mode area fibres (ITU-T G.654): OFS SLA (130 μm^2) and Corning Vascade EX2000 (112 μm^2). The last type of fibre is a so-called pure silica core fibre (PSCF). The effective mode areas were taken from certificates for the fibres.

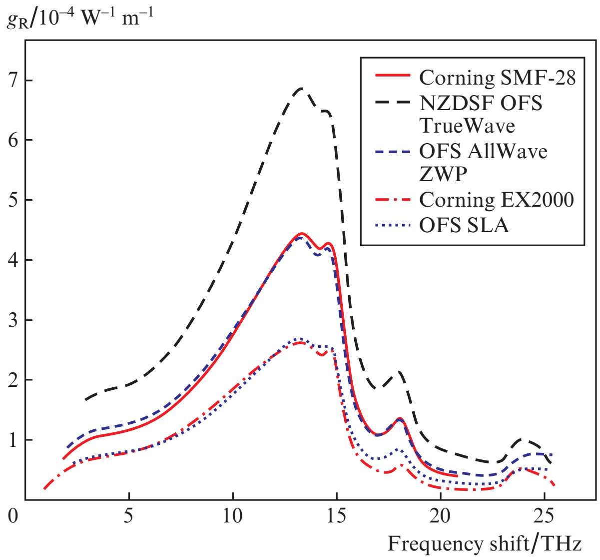

Figure 4. SRS coefficient as a function of frequency shift for various types of fibre.

The obtained dependences of the Raman coefficients are normalised to the effective mode area. Multiplying $g_R(\Delta\nu)$ by A_{eff}, we will obtain similar dependences of the SRS coefficient on the frequency shift for germanium-containing core fibres. The SRS coefficient for EX2000 pure silica core fibre will be smaller than that for germanium-containing core fibres.

Bufetov et al. [5] used three techniques in SRS coefficient measurements. Two of them utilised reference sources: a broadband source and a wavelength-tunable narrow-band source. The third technique was based on analysis of ASpRS spectra. In the case of the broadband reference source, the measurement technique used by Bufetov et al. [5] coincides with that in this study.

The method of measuring SRS coefficients via analysis of ASpRS spectra in Ref. [5] is analogous to the differential technique in this study. In our opinion, in the case of the use of the exact expression for the number of noise photons (5), this method ensures higher measurement accuracy, especially at small detuning from the pump frequency.

Table 1. Maximum SRS coefficients for various types of fibre (HGDF is specialised optical fibre with high GeO$_2$ concentration).

Fibre	Maximum g_R/10^{-4} W^{-1} m^{-1}	Note
Corning SMF-28	4.4	This work
NZDSF OFS TrueWave	6.8	
OFS AllWave ZWP	4.3	
Corning EX2000	2.6	
OFS SLA	2.6	
HGDF	37	Previous results [5]
NZDSF	5.9	
Corning LEAF	4.8	

Table 1 compares the maximum measured SRS coefficients obtained in this study and by Bufetov et al. [5], who determined the SRS coefficients from the expression for the effective gain coefficient of a Raman amplifier:

$$G_{\mathrm{on/off}} = \exp(g_R P_p L_{\mathrm{eff}}). \tag{11}$$

6. Conclusions

In this study, we have obtained a formula describing the emission spectrum of amplified spontaneous Raman scattering in Raman amplifiers. A simplified technique has been proposed for measuring SRS coefficients from ASpRS spectra and the range of its applicability has been determined via comparison with an exact solution to SRS rate equations. For most types of telecom fibre studied, the range of applicability of this technique corresponds to Stokes shifts in the range 7.5 to 16 THz (250–540 cm^{-1}), where the SRS coefficient can be measured with acceptable accuracy.

Comparison with direct SRS tilt measurements in DWDM communication links indicates that the spectral dependences of the SRS coefficient obtained by converting the exact solution for the spectral density of amplified spontaneous Raman scattering allow one to evaluate the slope of the dependence near the origin, which is of practical importance in designing multichannel communication links both using Raman amplifiers and without them.

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