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LIGHT BEAMS https://doi.org/10.1070/QEL16420

Reflection of light beams from a chirped dielectric plane-layered structure

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Abstract. **The reflection of light beams with various widths of the angular spectrum from a chirped plane-layered dielectric structure is numerically studied. For wide beams, the total lateral shift of the reflected beam depends on the position of the photonic barrier inside the structure and the value of the Goos–Hänchen shift. For narrow beams, the region of the interaction with the structure increases, the profile broadens and the reflected beam wavefront becomes phase modulated. It is shown that the light reflected from a quasi-periodic structure with negative spatial dispersion undergoes focusing.**

Keywords: *chirped periodic structure, reflected beams, wavefront modulation, focusing.*

1. Introduction

There is a well-known analogy between the compression and stretching of light temporal pulses and the focusing and defocusing of spatial beams in quasi-periodic dielectric structures [1, 2]. As an example, let us consider the reflection of a monochromatic collimated beam from a plane-layered two-component structure (Bragg mirror), in which the period d increases linearly along the z axis: $d(z) = d_0 + \alpha(z - z_0)$, where z_0 is the coordinate of a certain point inside the structure (Fig. 1). In this case, the longitudinal components of the wave vectors of the plane waves forming the beam's transverse profile satisfy the Bragg condition at various points of the structure: $k_z(z) = k_0 + \beta(z - z_0)$, where $k_0 = \pi/d_0$, $\beta = -\alpha\pi/(2n_0d_0^2)$, and n_0 is the average refractive index.

A wide beam incident along the normal, with a central spectral component satisfying the condition $k(z_0) \approx k_0$, basically preserves its transverse profile and the plane wavefront after reflection. The reflection of a narrow beam, in the spectrum of which the side components play an important role, is characterised by the profile broadening and phase front distortion. As shown in Fig. 1, the parameter α determines the phase modulation sign of the beam's angular spectrum. For example, with the period d decreasing along z ($\alpha < 0$), the chirped Bragg mirror behaves as a medium with negative spatial dispersion, similar to a pair of diffraction gratings [3]. The

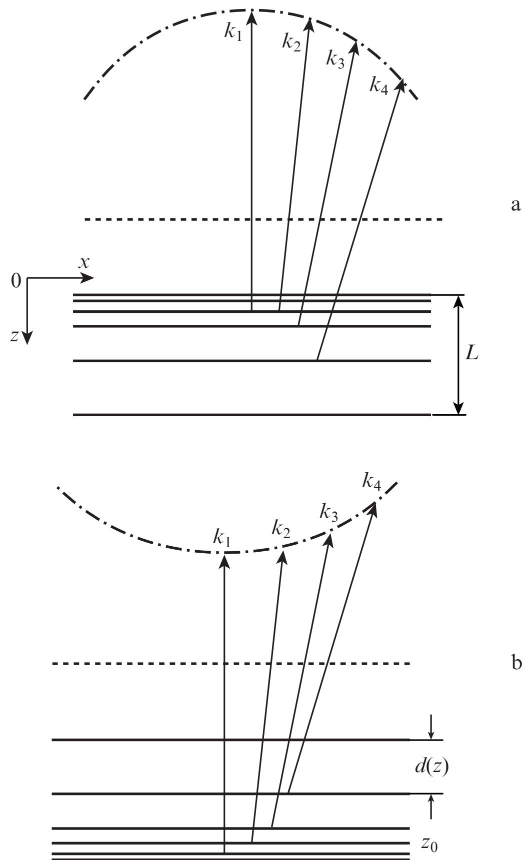

Figure 1. Reflection of various spectral components of a light beam from chirped periodic gratings for (a) $\alpha > 0$ and (b) $\alpha < 0$. The dashed and dash-dotted lines show the phase-front profiles of incident and reflected beams in the $z = 0$ plane, respectively.

beam is focused when propagating after reflection in a medium with positive diffraction.

Note that in the plane wave approximation, pulse reflection from a chirped Bragg mirror is characterised by the fact that different frequency spectrum components acquire different delay times [4]. As a result, the pulse duration increases and a linear frequency modulation appears. The sign of α, which determines an increase or a decrease in the instantaneous frequency inside the pulse, is equivalent to the sign of the phase front steepness of a spatial beam.

If for temporal pulses the compression–stretching mechanism is the same both for normal and oblique incidence onto a chirped periodic structure, for a beam with oblique reflection, a lateral shift and a more complex deformation of the

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phase front arise. This case is studied in detail in the present paper using numerical methods. We have calculated the total field using the finite difference time domain method [5], while the spectral characteristics of the structure have been determined by means of the transfer matrix method [6].

One of practical applications of quasi-periodic structures is their use in amplification of short light pulses [7–9]. In this case, a preliminary stretching of the pulse takes place, which allows one to avoid nonlinear distortions during the amplification stage. Then the same structure, but with the opposite sign of the parameter α, is used to restore the previous shape and pulse duration. The transition in this scheme from the waveguide to volume samples requires consideration of diffraction effects.

A study of the transformation of the beam profile reflected from a quasi-periodic dielectric structure is of independent interest since such structures exhibit the properties of metamaterials [10].

2. Reflection of wide beams. The Goos–Hänchen shift

A feature of a chirped dielectric structure is the existence of local forbidden bands not only in the frequency and angular spectra of incident radiation, but also along the longitudinal coordinate z.

Let a plane wave for which the Bragg condition is satisfied at point z_0 be incident at an angle θ_0 onto a quasi-periodic structure of length L. In the case of a small dielectric contrast of the structure, when the permittivity is approximated as [4]

$$\varepsilon(z) = \varepsilon_0 + \Delta\varepsilon\cos\left[\frac{2\pi}{d_0}z + \frac{1}{2}\beta(z - z_0)^2\right], \qquad (1)$$

it is possible to estimate the size of the local spatial forbidden band – the photonic barrier (PB). The barrier boundaries are determined from the condition $\delta = \pm\kappa$, where the Bragg resonance detuning is $\delta = k_0\sqrt{\varepsilon_0}\cos\theta_0 - \pi/d_0 + \beta(z - z_0)$, the Bragg coupling is $\kappa = \Delta\varepsilon k_0/(4\sqrt{\varepsilon_0})$, $\varepsilon_0 = n_0^2$, $\Delta\varepsilon \approx (\varepsilon_1 - \varepsilon_2)/2$, and $\Delta\varepsilon/\varepsilon_0 \ll 1$ is the dielectric constant. The photonic barrier size is defined as $l_{\rm pb} = 2\kappa/|\beta|$. For chirped periodic structures used in practice, the parameter $\beta \sim 10^8 - 10^9$ rad m^{-2}, and the photonic barrier width may constitute hundreds or thousands of periods.

We have investigated two-component quasi-periodic structures of different lengths, consisting of quarter-wavelength layers with refractive indices $n_1 = 1.6$ and $n_2 = 1.4$ at the parameter $\alpha = 2\times10^{-3}$. The angular forbidden band of the structure was located between the incidence angles $\theta_1 = 27°$ and $\theta_2 = 33°$.

The light beam envelope has the form $A(x) = \exp[-(x - x_0)^2/2a^2]$ (here a is the beam aperture), the wavelength is $\lambda = 1$ μm, and the angle of incidence is $\theta_0 = 30°$ (which corresponds to exact Bragg resonance at point $z_0 = L/2$). We have considered the reflection of light beams with wide ($\Delta k_{x,\rm b} \ll \Delta k_{x,\rm bg}$) and narrow ($\Delta k_{x,\rm b} \leqslant \Delta k_{x,\rm bg}$) apertures, where $\Delta k_{x,\rm b}$ is the width of the beam angular spectrum, and $\Delta k_{x,\rm bg}$ is the bandgap width of the quasi-periodic structure.

Figure 2 shows the reflection of a weakly diverging beam ($a/\lambda = 15$) under the conditions when the photonic barrier with $l_{\rm pb} = 8$ μm is located in the $L/2$ vicinity (in this and other Figures the shaded area shows the location and size of the quasi-periodic structure). There is virtually no profile distortion and phase modulation of the light beam. The total lateral shift (segment CD = 35 μm) of the reflected beam at the boundary $z = 0$ is composed of a shift due to the beam propagation in the transparency region of the chirped structure and a shift at the photonic barrier surface (segment AB = 2 μm), or the Goos–Hänchen shift (the G–H shift [11]).

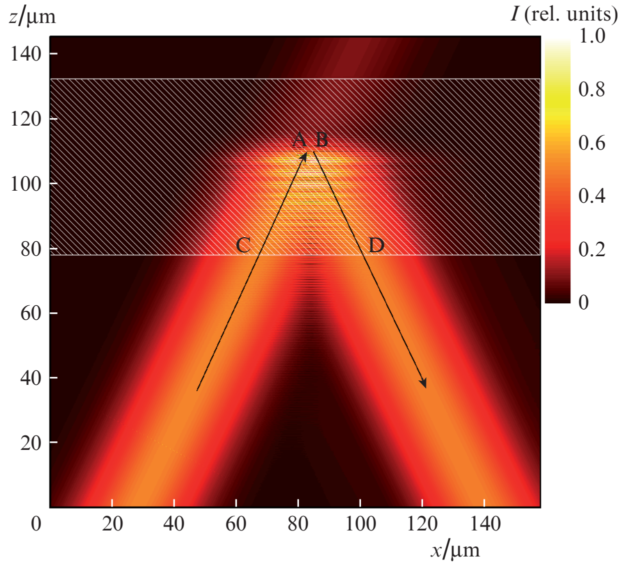

Figure 2. (Colour online) Reflection of a wide light beam ($a/\lambda = 15$) from a photonic barrier located inside the periodic structure ($\alpha = 2\times10^{-3}$, $l_{\rm pb} = 8$ μm). Here and in Figs 3, 4 and 6 the arrows indicate the direction of beam propagation.

Let us consider in more detail the G–H shift using the geometric approach proposed in the monograph [12] to describe the beam shift at the boundary of two homogeneous dielectrics with the refractive indices n_1 and n_2 under the conditions of total internal reflection ($n_1 > n_2$). We may assume that the reflection occurs not from the interface between the dielectrics, but from a plane located at some effective depth $d_{\rm eff}$. If $d_{\rm eff}$ is defined as the imaginary part of a reciprocal of the longitudinal component of the wave vector: $d_{\rm eff} = {\rm Im}(k_z^{-1}) = \left(k_0\sqrt{n_2^2 - n_1^2\sin^2\theta_0}\right)^{-1}$ [here $\theta_0 > \theta_{\rm cr} = \arcsin(n_2/n_1)$ and $\theta_{\rm cr}$ is the critical angle], then the beam lateral shift $\Delta = 2d_{\rm eff}\tan\theta_0$ coincides with the value calculated according to the Artman formula: $\Delta = (k_1\cos\theta)^{-1}{\rm d}\varphi_{\rm r}/{\rm d}\theta$ [13], where $\varphi_{\rm r}$ is the reflection coefficient phase, which has the form

$$r = \left(n_1\cos\theta_0 - {\rm i}\sqrt{n_1^2\sin^2\theta_0 - n_2^2}\,\right)$$

$$\times\left(n_1\cos\theta_0 + {\rm i}\sqrt{n_1^2\sin^2\theta_0 - n_2^2}\,\right)^{-1}.$$

As an intermediate result, we consider the G–H shift in the case of angular incidence of the beam of electromagnetic radiation onto a homogeneous periodic dielectric structure ($\alpha = 0$), when the beam angular spectrum is located inside the forbidden band [14]. As shown in [15], the reflection coefficient phase (it is assumed that $n_1 \approx n_2 \approx \sqrt{\varepsilon_0}$) has the form $\varphi = \arctan(\delta q^{-1}\tanh(\kappa L))$, where $q = \sqrt{\delta^2 - \kappa^2}$. This implies that at $\delta = 0$ the lateral shift $\Delta = -{\rm d}\varphi/{\rm d}k_x = \tanh(\kappa L)\kappa^{-1}\tan\theta$. This expression coincides with the formula $\Delta = 2d_{\rm eff}\tan\theta$ if the effective layer depth is $d_{\rm eff} = 1/2\kappa$.

It is known [15] that the total field inside the structure is a superposition of forward and backward waves which exponentially decay along z coordinate approximately as $\exp(-\kappa z)$.

The effective penetration depth can be estimated numerically by the expression:

$$d_{\rm eff} = \frac{\int_0^L z\,|E_{\rm for}|^2 {\rm d}z}{\int_0^L |E_{\rm for}|^2 {\rm d}z} = \frac{1}{2\kappa}, \qquad (2)$$

where $E_{\rm for}$ is the forward wave amplitude. Thus, the effective penetration depth of radiation is determined by the attenuation of the forward wave's 'centre of gravity'. The same estimate for the penetration depth also retains for chirped periodic structures.

Figure 3 shows the reflection of a wide light beam under the condition that the photonic barrier is located at the periodic structure edge. The total beam shift is only determined by the G–H shift whose magnitude constitutes several wavelengths of the incident radiation. For a particular structure (segment AB in Fig. 3), as it follows from the geometrical consideration, the G–H shift is $\Delta = 2$ μm, whereas the numerical calculation using the Artman formula gives the value of $\Delta = 1.9$ μm. The estimates given above assume that the reflected beam retains its original profile.

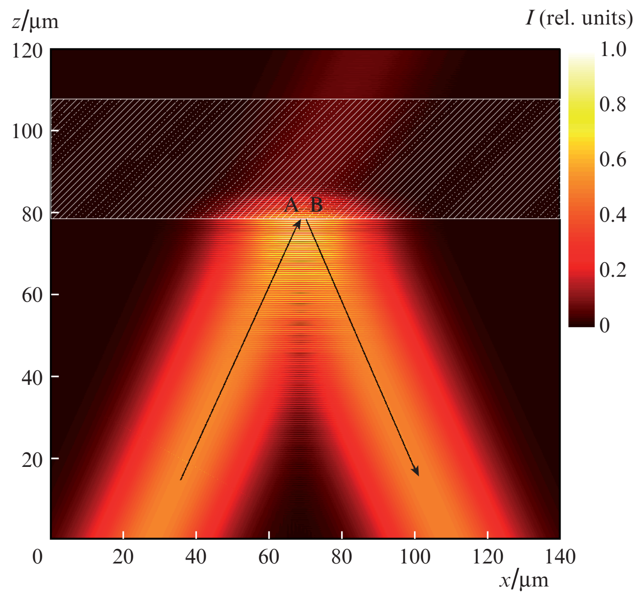

Figure 3. (Colour online) Reflection of a wide beam ($a/\lambda = 15$) from a photonic barrier located at the front boundary of the chirped structure ($\alpha = 2\times10^{-3}$, $l_{\rm pb} = 8$ μm).

The formulas obtained are valid for semi-infinite structures. Actually, this means that the attenuation inside the photonic barrier must be sufficient to ensure that the reflection coefficient is close to unity. This imposes a restriction on the size of the photonic barrier in the case of a chirped grating: the condition $\tanh(\kappa l_{\rm pb}) \approx 1$ implies that $\kappa l_{\rm pb} \geqslant 3$. When a light beam is incident onto a structure with $\kappa l_{\rm pb} < 3$, a noticeable tunnelling of electromagnetic radiation through the PB is observed (Fig. 4). This regime reduces the efficiency of the compression–stretching mechanism of reflected pulses (beams) and is not used in practice.

It also follows from the above estimates that, with the exception of the near-surface layer, we can neglect the G–H shift (segment AB in Fig. 3) against the background of the total beam shift (segment CD in Fig. 2).

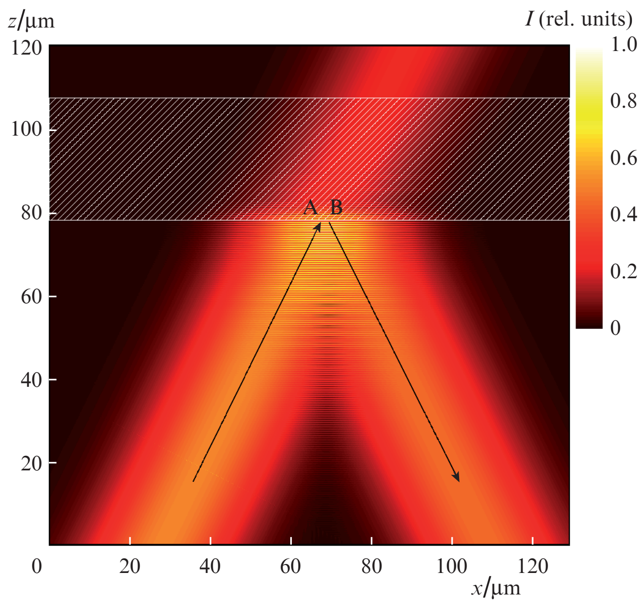

Figure 4. (Colour online) Reflection of a wide beam ($a/\lambda = 15$) at a reduced photonic barrier located at the front boundary of the chirped structure ($\alpha = 6\times10^{-3}$, $l_{\rm pb} = 2.75$ μm).

3. Reflection of narrow light beams

Consider the reflection of narrow divergent light beams, i.e. the beams possessing a wide angular spectrum ($\Delta k_{x,\,\rm b} \leqslant \Delta k_{x,\,\rm bg}$) with phase modulation. The reflection region of such beams embraces a considerable part of the chirped structure, with the beam profile and its wavefront being distorted.

Since there is no explicit expression for the reflection coefficient, approximate formulas can be used to estimate the lateral shift and angular dispersion. We assume that the reflected beam is formed from the spectral components, each of which is completely reflected from the planes located at various depths. The reflection coefficient of the current spectral component is $R \approx \exp({\rm i}2zk_z)$, where its phase $\varphi = 2zk_z$, and $2z = 2(k_z - k_0)/\beta + z_0$ is the double path that the spectral component has passed in the longitudinal direction when reflected from the plane at the depth z. Defining the shift by the formula $\Delta = -{\rm d}\varphi/{\rm d}k_x$ and using the relationship between the longitudinal and transverse components of the wave vector: $k^2 = k_x^2 + k_z^2$, we obtain approximate formulas for the lateral shift and phase modulation of the beam's spectral components:

$$-\frac{{\rm d}\varphi}{{\rm d}k_x} = \frac{2k_x(k_z - k_0)}{k_z\beta} = 2z\tan\theta, \qquad (3)$$

$$-k\frac{{\rm d}^2\varphi}{{\rm d}k_x^2} = \frac{2}{\beta}\left(k - \frac{k_0}{\cos^3\theta}\right) \qquad (4)$$

(here θ is the current angle).

We may conclude from formula (3) that a stretched reflected beam is formed in the plane $z = 0$, whose spectral components have a characteristic linear lateral shift along x. Figure 5 shows the dependences $\Delta(k_x)$ calculated by formula (3) [curve (*1*)] and using a numerical method [curve (*2*)]. The shape of the curves is almost the same; however, curve (*2*)

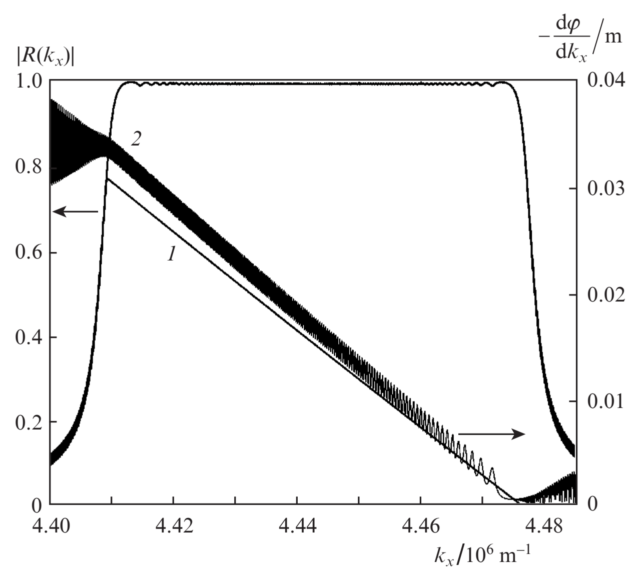

Figure 5. Dependences of the lateral shift on the wave vector's transverse component, obtained with (*1*) the use of formula (3) and (*2*) numerical calculation. The periodic structure parameters: $L = 1.5$ cm, $n_0 = 1.4$, $\alpha = -5\times 10^{-7}$, $\Delta\varepsilon/\varepsilon_0 \approx 10^{-3}$.

contains lateral shift oscillations caused by re-reflections of light radiation inside the periodic structure. The diffraction parameter (4) determines the sign and magnitude of the beam's wavefront curvature.

Figure 6 shows the reflection of a narrow divergent light beam from a chirped structure with $\alpha = -2\times 10^{-3}$. It can be seen that a significant part of the periodic structure is involved in the reflected beam formation. The beam acquires a convergent wavefront in the course of reflection. Along with focusing of part of the reflected beam, additional beam reflections and a more complex field structure in the region of its formation are observed.

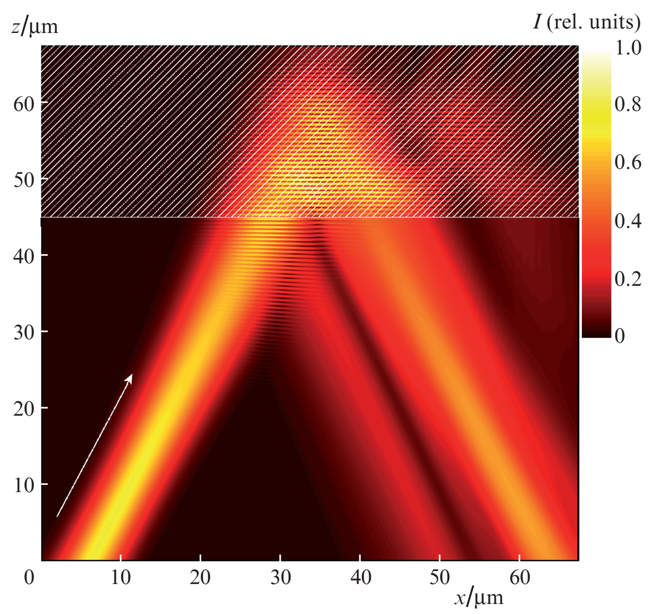

Figure 6. (Colour online) Focusing of the narrow beam reflected ($a/\lambda = 5$) from the chirped periodic structure ($\alpha = -2\times 10^{-3}$).

When the sign of α changes, the light beam, after reflection from the quasi-periodic structure, acquires a phase front corresponding to the divergent beam, and its profile broadens. With further propagation in free space, such a beam will continue to diffract.

The efficiency of using the chirped volume structures for the transformation of light beams depends on the accuracy of matching the spectral characteristics of the compression and stretching devices. Since the dependence of the angular dispersion on the angle in the chirped structures is nonlinear, this limits the use of narrow light beams with a large angular divergence.

The use of sufficiently wide beams makes it possible to improve the adjustment accuracy. As an example, let us consider the compression of a diffracted light beam ($a/\lambda = 5\times 10^3$, $\lambda = 1$ μm, $\theta = 45°$, $-kd^2\varphi/dk_x^2 = 2.83$ m) by approximately 25 times when the beam is reflected from a chirped periodic structure with parameters $L = 1.5$ cm, $\alpha = -5\times 10^{-7}$, $\Delta\varepsilon/\varepsilon_0 \approx 10^{-3}$ (Fig. 7). As a result of phase modulation compensation, the beam restores its original width ($a/\lambda = 200$) at the boundary $z = 0$. The restoration efficiency is 96%, which is related to the beam energy losses due to tunnelling.

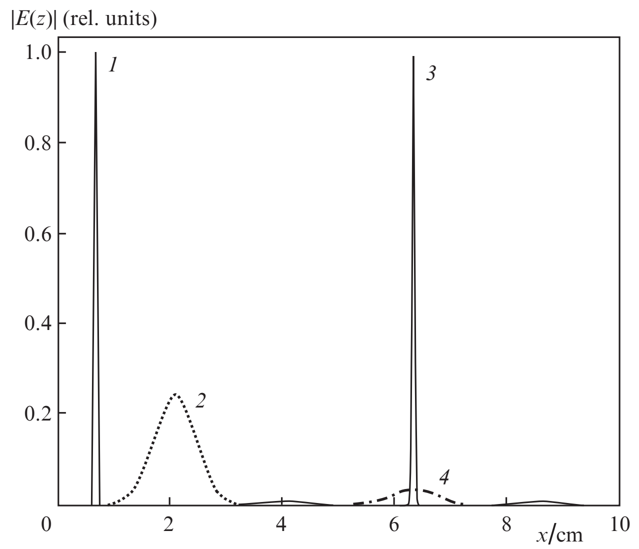

Figure 7. Compensation of phase modulation after reflection from a chirped grating. Shown are the profiles of (*1*) the initial beam, (*2*) broadened beam and (*3*) reflected beam in the $z = 0$ plane, and also (*4*) the transmitted beam in the plane $z = L$

4. Conclusions

We have numerically calculated the reflection of the light Gaussian beams with various angular spectrum widths from a chirped dielectric plane-layered structure. By means of a geometric approach, approximate formulas for the total lateral shift, Goos–Hänchen shift, and phase modulation of the reflected radiation wavefront have been obtained.

The total lateral shift of the reflected beam with a narrow angular spectrum is composed of the Goos–Hänchen shift and the shift associated with radiation propagation in the transparency region of a quasi-periodic structure. The reflection of light beams with a wide angular spectrum is accompanied by a growth of the region of radiation interaction with the structure, and, as a result, the profile is broadened and a phase modulation of the angular spectrum appears. The use of a chirped periodic structure allows one to obtain both converging and divergent light beams during reflection.

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