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INTERACTION OF LASER RADIATION WITH MATTER. LASER PLASMA https://doi.org/10.1070/QEL16430

Absorption of resonance laser radiation in ultracold plasma

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Abstract. **The absorption of resonance laser radiation in an expanding ultracold plasma is considered. Initially the optical thickness τ_0 of the medium decreases due to correlation heating of the ions; the subsequent lowering of τ_0 is due to the variation of radial ion velocities. The expanding optically thick medium is shown to become transparent when the ratio between the particle expansion velocity at the sphere boundary and the thermal velocity exceeds the optical thickness τ_0.**

Keywords: *optical transparentisation, resonance laser radiation, expanding ultracold laser-produced plasma.*

1. Introduction

Under ordinary conditions, the propagation of resonance radiation is described by the Bouguer–Lambert–Beer law and the absorption length $l \approx 1/k_0$, where k_0 is the absorption coefficient at the centre radiation wavelength. In its interaction with a medium, high-power resonance radiation induces a saturation effect, which gives rise to gas transparentisation, so that the radiation penetrates the medium to a depth $l_1 \approx I_0(\gamma n)^{-1}$, where γ is the spontaneous decay probability of an excited atom, n is the atom density, and I_0 is the photon flux intensity [1]. As is noteworthy, saturation conditions, when the excited atom density is high, may give rise to gas ionisation caused by the production of an ion and an electron in the collision of two excited atoms with an electron avalanche sustained by the superelastic heating of electrons in their collisions with excited atoms. The atom density becomes lower due to the ionisation, which eventually results in the ionisation-induced transparentisation of the medium and a decrease in extinction length to a value $l_2 \approx I_0[\gamma(n - n_i)]^{-1}$ [2, 3], where n_i is the resultant ion density. In both cases the transparentisation of the medium is caused by the lowering of absorbing particle density.

The present work is concerned with the properties of absorption of low-intensity resonance radiation in expanding media with velocity gradients. These situations occur in astrophysics [4], in laboratory conditions in X-ray and ultraviolet laser research [5, 6] and in the study of ultracold laser-produced plasmas [7]. We consider the spectral characteristics of the absorption of external resonance laser radiation (the optical thickness, the shape and width of the spectral line) in an expanding ultracold plasma (UCP) in the absence of absorption saturation. The resultant calculated data are compared with experimental results [7], and we predict the effect of resonance transparentisation caused by the variation of the absorption spectrum due to the radial ion velocity gradient. It is pertinent to note that the special features of resonance radiation transfer in expanding media were first emphasised by V.V. Sobolev [8] and that the absorption of continuous radiation in expanding media was considered in Refs [9–11].

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2. Absorption model

The UCP of spherical shape was first obtained by step-wise photoionisation of an ensemble of strontium atoms cooled by laser radiation [12]. The temperature T_i of the resultant ions was equal to the cooled-ion temperature and their spatial positions were chaotic. The Coulomb interionic interaction induces their transition to a more equilibrium (ordered) state, part of the potential ion energy being transferred to the kinetic one. This gives rise to correlation heating of the ions [13], and the ion temperature rises from 10^{-2} to 1 K in a time $\tau_c \sim 1/\omega_i$ (ω_i is the plasma ion temperature) [7].

The electron temperature T_e is defined by the excess of photon energy over the ionisation potential of excited atoms. For $T_i \ll T_e$, the electron pressure gives rise to radial ion velocities, which are defined at point $\boldsymbol{r}$ at the instant of time t as [14]

$$\boldsymbol{V}(\boldsymbol{r}, t) \approx \frac{k_B T_e}{m_i \sigma_0^2} t \boldsymbol{r}, \tag{1}$$

where k_B is the Boltzmann constant; m_i is the ion mass; and σ_0 is the characteristic UCP dimension at the initial instant of time. The radial ion motion gives rise to a shift of the resonance absorption frequency ν_0, which takes on the form

$$\nu_0' = \nu_0 \left(1 + \frac{\boldsymbol{V}(\boldsymbol{r}, t)\boldsymbol{l}}{c}\right), \tag{2}$$

where $\boldsymbol{l}$ is the unit vector that defines the direction of propagation of the resonance laser radiation and c is the velocity of light.

The absorption coefficient $k(\nu)$ for the resonance laser radiation in the UCP is defined by the Lorentzian (spontaneous decay) and Doppler (the thermal and radial velocities of ion motion) broadenings. Their combined effect is described by the Voigt profile [15]:

$$k(\nu) = k_0 H(a,\nu), \tag{3}$$

$$H(a,\nu) = \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-\xi^2)}{a^2 + \left[\frac{\nu - \nu_0'}{\nu_0}\frac{c}{V_0} - \xi\right]^2}\mathrm{d}\xi, \tag{4}$$

$$a = \frac{\lambda_0(\gamma + \gamma_\mathrm{L})}{4\pi V_0}, \tag{5}$$

$$k_0 = \frac{\lambda_0^3}{8\pi}n_\mathrm{i}\frac{g_2}{g_1}\frac{\gamma}{\sqrt{\pi}}\frac{1}{V_0}. \tag{6}$$

Here, $\lambda_0 = c/\nu_0$ is the resonance wavelength; $V_0 = \sqrt{2k_\mathrm{B}T_\mathrm{i}/m_\mathrm{i}}$ is the thermal ion velocity; g_2 and g_1 are the statistical weights of the excited and ground ion states; and γ is the spontaneous decay probability of the excited ion. Also included in formula (5) is the line width γ_L of the resonance laser radiation.

The spatial ion density distribution in the UCP at the initial point in time replicates the atom density distribution and is defined by the expression

$$n_\mathrm{i}(r) = n_0\exp\left(-\frac{r^2}{2\sigma_0^2}\right), \tag{7}$$

where n_0 is the ion density at the sphere centre.

To calculate the spectral absorption characteristics, we take an auxiliary sphere of radius $R > \sqrt{2}\sigma_0$ and consider the radiation propagation along a chord parallel to the ***l*** direction. The chord position is defined by the angle φ between the ***l*** direction and the direction from the sphere centre towards the point of intersection of the chord with the sphere surface. Then,

$$r^2 = R^2\sin^2\varphi + x^2, \tag{8}$$

where x is the distance along the chord, which varies from $-R\cos\varphi$ to $+R\cos\varphi$.

We take into account expressions (1)–(4), (7), and (8) to write the optical thickness along the chord at the instant of time t as

$$\tau(\nu,\varphi,t) = \int_{-R\cos\varphi}^{R\cos\varphi}k(\nu,\varphi,t)\,\mathrm{d}x = \frac{k_0}{a\pi}\exp(-\alpha^2\sin^2\varphi) \tag{9}$$

$$\times\int_{-R\cos\varphi}^{R\cos\varphi}\int_{-\infty}^{\infty}\frac{\exp\left(-\frac{x^2}{2\sigma_0^2}\right)\exp(-\xi^2)}{1 + \left[\frac{4\pi}{\gamma + \gamma_\mathrm{L}}\left(\nu - \nu_0 - \frac{k_\mathrm{B}T_\mathrm{e}}{m_\mathrm{i}\sigma_0^2}\frac{tx}{\lambda_0}\cos\varphi\right) - \frac{\xi}{a}\right]^2}\mathrm{d}\xi\,\mathrm{d}x$$

where $\alpha = R/\sqrt{2}\sigma_0$.

3. Results of calculations

Our calculations were performed for strontium atoms described by the following set of experimental data [7]: $\lambda_0 = 422$ nm, $\gamma/(2\pi) = 21$ MHz, $\gamma_\mathrm{L}/(2\pi) = 5$ MHz, $n_0 = 2\times10^{10}$ cm^{-3}, $\sigma_0 = 0.6$ mm, a spatial resolution of 0.1 mm; the electron temperature $T_\mathrm{e} = 56$ K, the initial ion temperature $T_\mathrm{i}(0) \approx 10^{-2}$ K and, after correlation heating, $T_\mathrm{i} \approx 1$ K. According to formula (5), parameter $a \approx 0.39/\sqrt{T_\mathrm{i}}$.

We consider the optical thickness $\tau_0(T_\mathrm{i})$ for the line centre ($\nu = \nu_0$) along the diameter ($\varphi = 0$) at the stage of correlation heating, which depends only on the spatially constant ion temperature T_i. The radial ion velocities may be neglected at this stage, and then from expression (9) for $R \to \infty$ we obtain

$$\tau_0[T_\mathrm{i}(t)] = \tau(\nu_0, 0, t)$$

$$\approx \frac{\lambda_0^2}{\sqrt{2}}n_0\sigma_0\frac{g_2}{g_1}\frac{\gamma}{\gamma + \gamma_\mathrm{L}}a\exp(a^2)(1 - \mathrm{erf}\,a), \tag{10}$$

where $\mathrm{erf}\,a$ is the probability integral. Therefore, the optical thickness $\tau_0(t)$ varies in time due to the variation of $T_\mathrm{i}(t)$ [see formula (5)].

We next consider the optical thickness with the inclusion of radial ion velocities. In doing this we assume that the ion temperature T_i initially grows linearly in time from 10^{-2} K to its maximum value $T_\mathrm{i} = 1$ K at tinstant of time $t \approx 0.2$ μs and then remains invariable, which corresponds to experimental data [16]. Figure 1 shows the $\tau_0(t)$ dependence derived from expression (9) (for $\alpha = 2$) with the inclusion of radial ion velocities defined by expression (1).

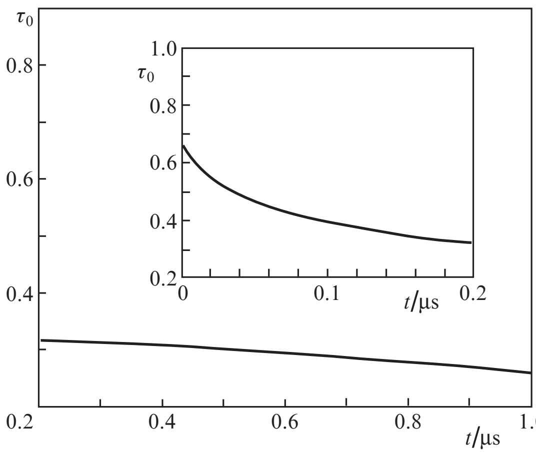

Figure 1. Time dependence of the optical thickness τ_0 with the inclusion of radial velocities. The inset depicts the $\tau_0(t)$ dependence for $0 \leqslant t \leqslant 0.2$ μs (correlation heating).

According to the data of Fig. 1, the optical thickness along the sphere diameter varies from 0.65 for $t = 0$ to 0.26 for $t = 1$ μs. For $t = 0.1$ μs, $\tau_0 = 0.38$. According to the experimental data of Ref. [7], at this instant $\tau_0 = 0.28$. Taking into account that the ion density n_0 is determined with an accuracy of ±(30%–40%) [7], the uncertainty $\Delta\tau_0$ amounts to about ±0.1. Accordingly, the experimental value $\tau_0 \approx 0.28 \pm 0.1$, which is consistent with our calculated data.

During the correlation ion heating time $\tau_\mathrm{c} \approx \sqrt{m_\mathrm{i}/n_0e^2}$ (where e is the electron charge), the radial ion velocity at point $r = \sigma_0$, according to formula (1), increases to

$$V(\sigma_0,\tau_\mathrm{c}) = \frac{\lambda_\mathrm{D}^\mathrm{i}}{\sigma_0}\sqrt{\frac{k_\mathrm{B}T_\mathrm{e}}{m_\mathrm{i}}}, \tag{11}$$

where $\lambda_\mathrm{D}^\mathrm{i}$ is the ion Debye radius. For the above experimental data,

$$\frac{V(\sigma_0,\tau_\mathrm{c})}{V_0} = \frac{\lambda_\mathrm{D}^\mathrm{i}}{\sqrt{2}\sigma_0}\sqrt{\frac{T_\mathrm{e}}{T_\mathrm{i}}} \approx 1.2\times10^{-1}. \tag{12}$$

Therefore, the lowering of the optical thickness during 0.2 μs is primarily due to an increase in T_{i} caused by the correlation heating and is described by expression (10), the subsequent effect being produced by the increase in radial ion velocities.

To understand the physical meaning of the decrease in the optical thickness with increasing radial velocities, we consider the situation for $a \ll 1$. In this case, the Voigt profile transforms to the Doppler profile, so that for $\tau_0(t)$, proceeding from expression (9), we obtain

$$\tau_0(t) \approx \frac{k_0}{\pi}\int_{-R}^{R}\exp\left\{-\frac{x^2}{2\sigma_0^2}\left[1+2\left(\frac{V(\sigma_0,t)}{V_0}\right)^2\right]\right\}\mathrm{d}x$$

$$\approx \sqrt{\frac{2}{\pi}}\frac{k_0\sigma_0}{\sqrt{1+2\left(\frac{V(\sigma_0,t)}{V_0}\right)^2}}. \quad (13)$$

If the radial ion velocity at the sphere boundary $V(\sigma_0,t) \gg V_0$, then

$$\tau_0(t) \approx \frac{1}{\sqrt{\pi}}k_0\sigma_0\frac{V_0}{V(\sigma_0,t)}. \quad (14)$$

For an immobile medium $\tau_0 \sim k_0 \sim 1/V_0$ [see formula (6)], and for an expanding medium $\tau_0 \sim 1/V(\sigma_0,t)$, according to formula (14). Hence it follows that the radial velocity gradient is responsible for the broadening of the absorption line, the lowering of absorption coefficient at the line centre and, accordingly, the transparentisation of the medium.

The sphere-integrated optical thickness

$$\tau^{\mathrm{i}}(\nu,t) = \int_0^{\pi/2}\tau(\nu,\varphi,t)2\pi R^2\sin\varphi\cos\varphi\mathrm{d}\varphi = 4k_0\sigma_0^2\alpha^2$$

$$\times\int_0^{\pi/2}\int_{-R\cos\varphi}^{R\cos\varphi}\int_{-\infty}^{\infty}\exp(-\alpha^2\sin^2\varphi)\exp\left(-\frac{x^2}{2\sigma_0^2}\right)\exp(-\xi^2)$$

$$\times\left\{1+\left[\frac{4\pi}{\gamma+\gamma_{\mathrm{L}}}\left(\nu-\nu_0-\frac{k_{\mathrm{B}}T_{\mathrm{e}}}{m_{\mathrm{i}}\sigma_0^2}\frac{\cos\varphi}{\lambda_0}xt\right)-\frac{\xi}{a}\right]^2\right\}^{-1}\mathrm{d}\xi\mathrm{d}x\mathrm{d}\varphi. \quad (15)$$

According to expression (15), at the stage of correlation heating the integral optical thickness for $\nu = \nu_0$ takes on the form

$$\tau_0^{\mathrm{i}}(t) = \tau^{\mathrm{i}}(\nu_0,t) \approx 2\pi\sigma_0^2\tau_0(t). \quad (16)$$

Plotted in Fig. 2 is the dependence of the sphere-integrated optical thickness $\tau_0^{\mathrm{i}}(t)$, which was determined using expression (15) (for $\alpha = 2$) with the inclusion of radial ion velocity variation.

The absorption line shape is determined by the dependence of the optical thickness on the frequency ν. We take advantage of expression (9) and perform integration for $\varphi = 0$ (along the diameter) to obtain the value of $\tau(\nu,t)$, which is plotted in Fig. 3, with $\Delta\nu = \nu - \nu_0$. For $t \leqslant 0.2$ μs the line broadening is caused by the correlation heating of the ions, the subsequent broadening being caused by the radial ion velocity variations.

Figure 4 shows the sphere-integrated spectral line shape obtained using expression (15). At the stage of the correlated heating of the ions, the spectral line widths integrated along the diameter and over the entire sphere are the same. Subsequently the width of absorption line integrated over the sphere is smaller than that integrated along the diameter, which is due to a decrease in the projection of expansion velocity with increase in angle φ.

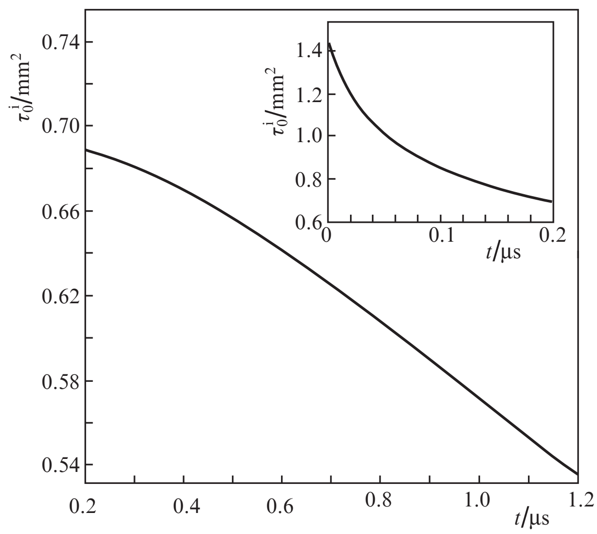

Figure 2. Time dependence of the integral optical thickness τ_0^{i} with the inclusion of radial velocities. The inset shows the dependence $\tau_0^{\mathrm{i}}(t)$ for $t \leqslant 0.2$ μs.

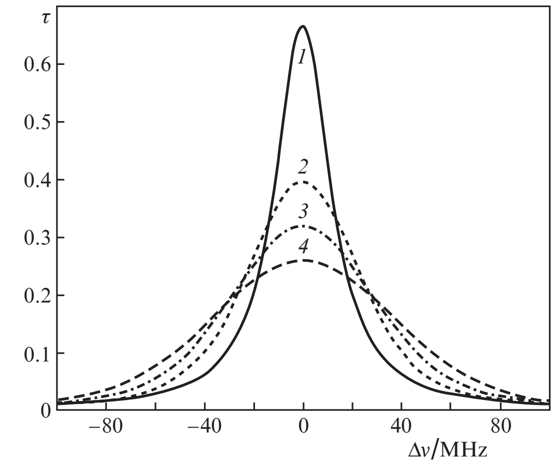

Figure 3. Absorption line profile integrated along the diameter for t = (*1*) 0, (*2*) 0.1, (*3*) 0.2 and (*4*) 1 μs.

The experimental [7] and calculated data for the integral optical thickness and the line width are collected in Table 1. The absorption spectral line width $\Delta\nu = 2(\nu_1 - \nu_0)$, where ν_1 corresponds to the frequency at which the absorption is two times lower than at the line centre, i.e. for $\nu = \nu_0$. The experimental and calculated data agree nicely.

We estimate the effect of plasma expansion, which increases the plasma dimension as well as lowers the ion density and the optical thickness. The greatest optical thickness variations caused by plasma expansion take place along the sphere diameter, where the optical thickness $\tau_0 \sim 2n_0(t)\sigma(t)$ and $\sigma(t) = \sigma_0 + \Delta\sigma(t)$. The time dependence $\sigma(t)$ is defined by the expression [7]

$$\sigma^2(t) = \sigma_0^2\left(1+\frac{t^2}{\tau^2}\right), \quad (17)$$

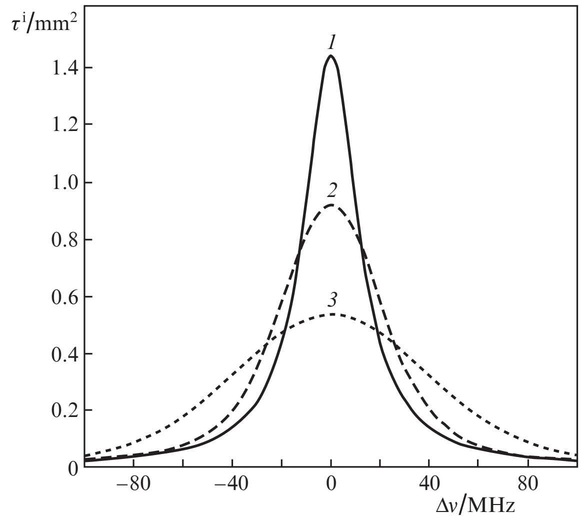

Figure 4. Integral profile of the absorption line for t = (*1*) 0, (*2*) 75 and (*3*) 1200 ns.

Table 1.

t/ns	$\tau_0^{i}(t)$/mm^2		$\Delta\nu$/MHz	
	Experiment	Calculation	Experiment	Calculation
0	–	1.44	–	25
75	0.8	0.92	62	52
1200	0.5	0.52	120	100

$$\tau = \sqrt{\frac{m_i \sigma_0^2}{k_B[T_e(0) + T_i(0)]}} \qquad (18)$$

[$T_e(0)$, $T_i(0)$ are the initial electron and ion temperatures]. Then, the relative variation of the optical thickness due to an increase in sphere size and the lowering of the particle density

$$\frac{\Delta\tau_0}{\tau_0} \approx -2\frac{\Delta\sigma(t)}{\sigma_0}. \qquad (19)$$

We use expression (1) to obtain

$$\frac{\Delta\tau_0}{\tau_0} \approx -\frac{k_B T_e}{m_i \sigma_0} t^2. \qquad (20)$$

The numerical value of the relative lowering of the optical thickness (20) for $t \approx 1$ μs is equal to 0.007, which is 30 times smaller than its value calculated with the inclusion of radial velocities (see Fig. 1), because over a time interval of 0.2–1 μs the relative variation $\Delta\tau_0/\tau_0 \approx 0.2$. That is why the effect of expansion may be neglected for the time interval under our consideration.

4. Conclusions

In the present work we have considered the absorption of external resonance radiation in an expanding ultracold plasma sphere and calculated the optical thickness, the profile, and width of the absorption line. It is shown that at the initial stage of expansion the optical thickness τ_0 of the medium decreases due to the correlation heating of the ions and that the subsequent lowering of τ_0 is caused by the variation of radial ion velocities. The existence of radial ion velocities results in Doppler shifts of the absorption frequency and broadens the absorption line width, thereby lowering the optical thickness τ_0 of the sphere. This is borne out by our analysis of the experimental data of Ref. [7]. The results obtained in this work suggest that optically thick expanding media become transparent to resonance radiation when the ratio between the radial velocity of particle expansion at the sphere boundary and the thermal velocity becomes greater that the initial optical thickness of the medium. Note that the frequency-integrated absorption of continuum radiation increases in expanding media [17].

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